Standard errors of unit weight (two operators): 0.5 and 0.7 micron. In this case there were so many residuals that normal distribution tests could be done. The results are shown in Figure 6.

The results of these glass scale tests show that a very high geometrical quality can be obtained in the determination of absolute scales of a comparator.

References

1. Hallert, B.: Photogrammetry. McGraw-Hill, New York, 1960.
2. Cramér, H.: "The Elements of Probability Theory and Some of Its Applications." Wiley, New York, 1954.
3. Hallert, B.: "Determination of the Geometrical Quality of Comparators for Image Coordinate Measurements." Manuscript. Published as a Gimrada Research Note 1962.

Anaglyphic Calculating Apparatus

LUISA BONFIGLIOLI, DR.,
Asst. Prof. of Descriptive Geometry,
Technion—Israel Inst. of Technology, Haifa

Abstract: *In this paper is described a simple apparatus that may be used instead of a normal slide-rule. It is composed of a special nomogram printed in two complementary colors, of anaglyph viewers and a movable index. With this apparatus it is possible to multiply and to divide some numbers and also to add and to subtract them. This accuracy is equal to that of a normal slide-rule.*

Introduction

As is known, one of the methods of obtaining stereoscopic vision is the anaglyphic process. For this, two photographs of an object, taken from two stations A and B, are printed one on top of the other on the same sheet of paper. The distance from A to B is parallel to the plane of reference of the object. It is called "the Stereoscopic Base." Each photograph is colored in one of two complementary colors e.g. red and green.

The picture is observed through anaglyph viewers with glasses of the same complementary colors as the photographs. Because the glasses are of transparent colored matter, the eye which looks through the red glass perceives the background as red, and the red-printed details merge with this red color. Against that the green-printed details are much more visible and their color is almost black. In a like manner, the eye which looks through the green glass perceives only the red-printed details because the green-printed details merge with the color of the background.

The result is that each eye sees only one of the two photographs; this gives rise to stereoscopic vision—namely the formation of the "*Optical Model.*"

Now if a needle is introduced between the optical model which is being observed and the anaglyph viewers, this single needle is seen by both eyes. Hence, by reason of the stereoscopic property of the binocular view, the needle is seen in its true place above the sheet of paper.

It follows that when the needle point is seen as touching a point of the optical model, the height of this needle point above the picture is equal to the height of the point of the model above the same picture.

As known, the heights of the various points of the optical model above the sheet of the picture are proportional to the horizontal parallax; i.e. the difference of the distances between the two images of the same spatial point and the corresponding axis y.

The calculating apparatus exploits this principle. It is composed of two parts: (I°), a nomogram designed in two complementary colors, and (II°), a movable index which measures the heights of the points of the optical model above the sheet of the nomogram. There must be added anaglyph viewers as an accessory to the two parts. With this apparatus we can perform the four operations of arithmetic—as well as powers of any exponent and square and cubic roots.

Theoretically roots of any index can be found, but because not useful they are not taken into consideration. The operations may be separates or linked together.

The accuracy obtained is equal to that of a normal slide-rule.

Movable Index

The movable index is composed of a rod (Figure 1) and a sliding sleeve to which is soldered a long needle, the point of which is compared to a geometrical point N. A suitable base holds the rod always vertical to the sheet of paper. The sleeve slides smoothly along the rod without play and it can be stopped at any point. This index can be operated in two ways: (I°), the base can be moved and consequently the rod *without changing* the height of the sliding sleeve until N coincides with a certain point P lying on a spatial line 1. In short "We move N toward a point P of a line 1"; (II°), the base can be moved and at the same time slide the sleeve along the rod until N coincides with a point Q in space. In short "We bring N to a point Q".

Anaglyph Viewers

The viewers are spectacles of colored glasses of the same complementary colors as the nomogram. They can be mounted on a suitable frame (Figure 2) or connected to the rod (Figure 1).

Nomogram for Multiplication

Consider the rhomb of Figure 3 in which the graduations on the right and lower side correspond to the values of the multiplicand and multiplier; the numbers at the intersections of the straight lines parallel to the sides of the rhomb—"*Lines of Reference*" are the corresponding products.

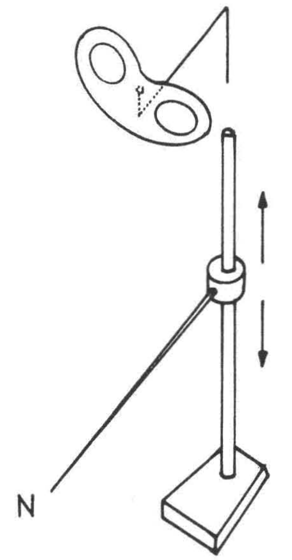


Fig. 1

Suppose that, instead of writing the numbers at the points of intersection, the same points be raised to a height proportional to the value of the number that should be written by them. This elevation can be obtained through using the anaglyphic process by drawing two quadrilaterals, one red-colored on the other green-colored, in such a manner that the graduations of the right and lower sides of both the quadrilaterals will be coincident while the points of intersection of the reference lines are displaced, each from its corresponding point, by a distance parallel to the stereoscopic base and proportional to the value of the number which ought to have been written by the same point. The geometrical construction is very simple (Figure 4). Let A, B, C, D be the vertices of the rhomb of the Figure 3 and D_1 and D_2 two symmetrical points with respect to the line AD. Draw parallel lines to D_1D_2 through the marks of the graduation of DA, DC and connect the points of intersection of the sides D_1A, D_2A and the corresponding points of CB with straight lines "*Reference Lines.*" Similarly connect the points of intersection of the sides D_1C, D_2C with the corresponding points of AB.

In such a manner two grids can be obtained of which the corresponding points; e.g. E_1, E_2 are separated in a direction parallel to D_1D_2. These separations are parallaxes.

If such a nomogram is observed through the spectacles, the two quadrilaterals will be

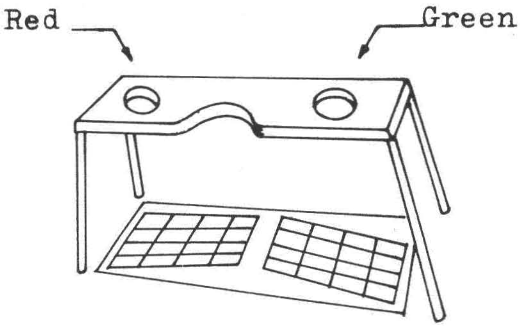


FIG. 2

seen as a warped quadrilateral with the bottom and right sides lying on the sheet of paper. The two points E_1, E_2 will be seen as a single point E^*. The other corresponding points of D_1A and D_2A; D_1C, D_2C will similarly be seen as points of different elevations. By similar notation D^* is the stereoscopic point seen instead of the two points D_1, D_2 . . . etc.). From Figure 4 it is clear that the length of the segment E_1E_2 is equal to the length of D_1D_2 multiplied by the product of the numbers of the reference lines from BC and AB which intersect at E_1 and E_2.

The same parallax is seen once between two corresponding points T_1, T_2 of D_1A and D_2A and another time between two corresponding points S_1, S_2 of D_1C and D_2C.

Hence the points E^*, T^*, S^* are seen at the

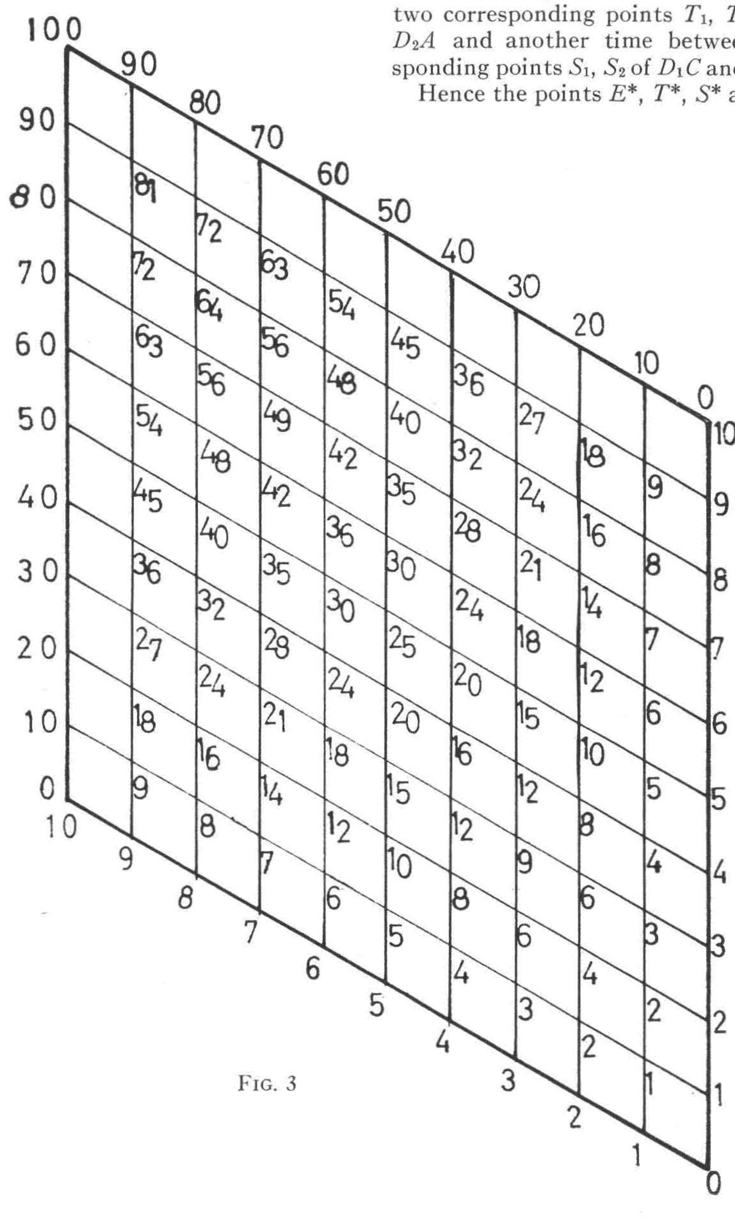


FIG. 3

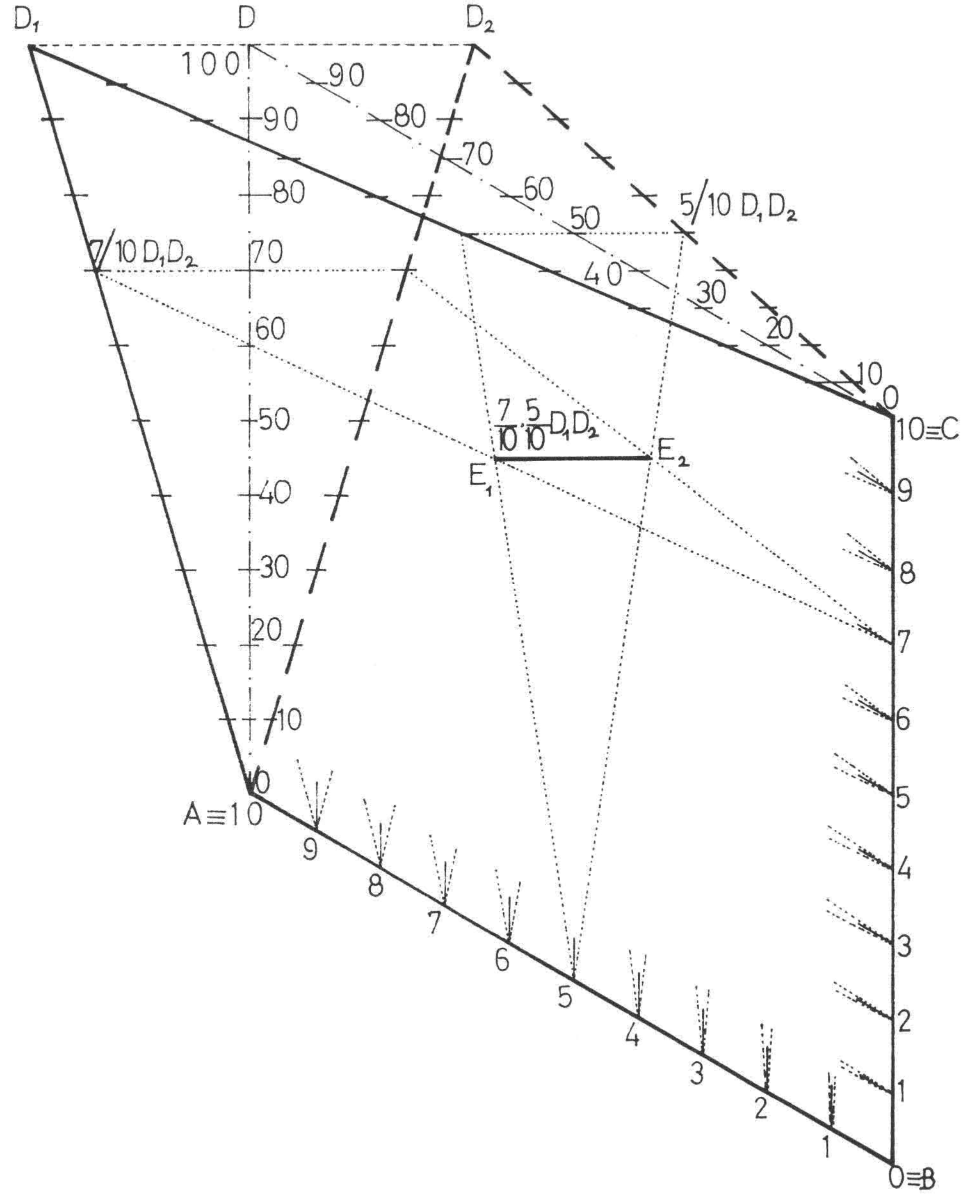


FIG. 4

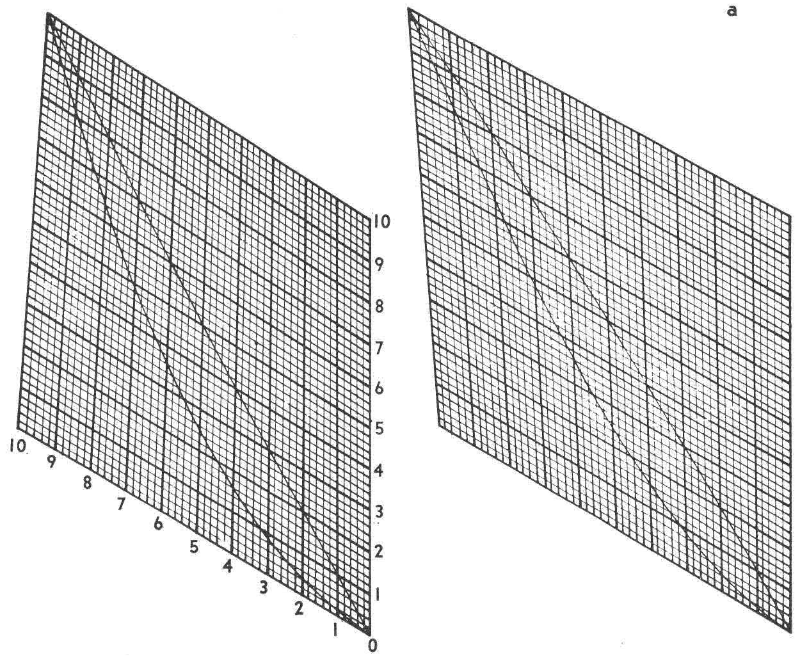
same height above the plane of the grids, and the number of the point T^* or S^* gives us the numerical value of the height of E^* relative to the height of the point D^*.

Figure 5a is a black-and-white stereogram which illustrates the effect of the proposed anaglyphic nomogram. It should be viewed through an ordinary hand stereoscope.[1]

THE USE OF THE NOMOGRAM

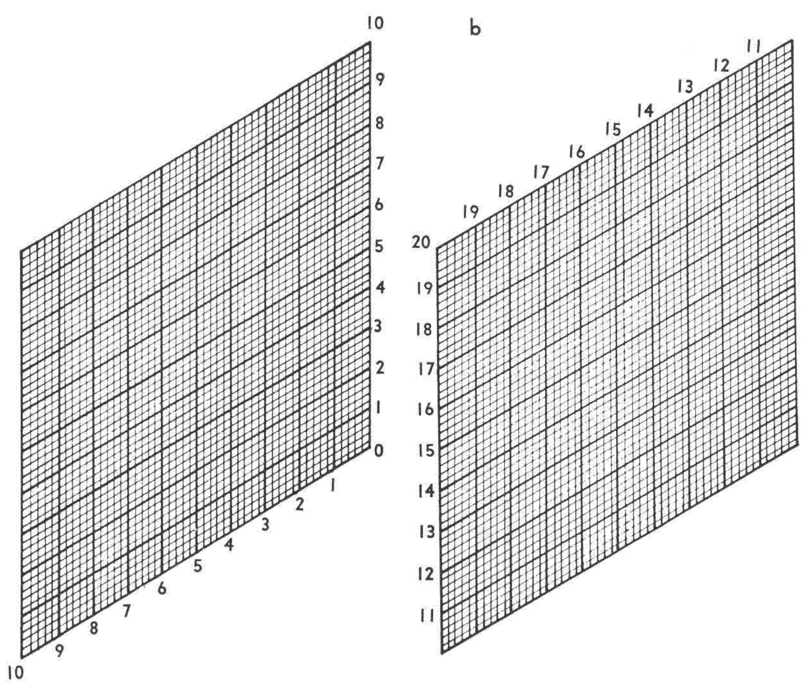
The nomogram is viewed through the spectacles until the optical model is seen very clearly, then N is brought to the point of the optical model lying on the intersection of the reference lines which pass through the marks

[1] It is unfortunate that technical difficulties and high cost prevent the publication in this journal of the actual anaglyphs. The stereograms of Figures 5a and 5b indicate the general appearance of the nomograms, but they cannot illustrate the use of the movable index marker.—EDITOR



Green Red

FIG. 5(a). Stereogram—for multiplication.



Green Red

FIG. 5(b). Stereogram—for addition.

of AB, BC whose numbers equal the values of the multiplicand and multiplier. After that N is moved towards a point P^* of the stereoscopic line AD^*. The number of P^* gives the desired product.

If it is desired to multiply this result by another number n, N is brought to the intersection of the reference line that passes through P^* with the reference line passing through the point of AB whose number is equal to n. N is moved towards a point M^* on the side AD and there is read the number written by M^*. If it is necessary to multiply this result by other numbers m, p, . . . the operations explained above are successively repeated.

It is clear that before beginning the operations described above there must be individually multiplied each of the values of the numbers n, m, p, . . . by a corresponding power 10^s so that each one of these values will be a number less than 10 or equal to it. Suppose that q is the number of the given factors of the multiplication, then the last result must be multiplied by: $18^{-s_1-s_2} \ldots {}^{-s}q+q^{-2}$.

NOMOGRAM FOR ADDITION

This nomogram is based on the same principle of the nomogram for multiplication (Figure 6), but in it there is only a single point without parallax. This is point B. The parallaxes between the points A_3 and A_4, and between C_3 and C_4 are arbitrary but equal, and the parallax between D_3 and D_4 is twice this parallax. A stereogram illustrating this nomogram is given in Figure 5b. Because the two nomograms are related, the relations between the parallaxes are:

$$C_3C_4 = A_3A_4 = \frac{D_1D_2}{10}; \qquad D_3D_4 = \frac{2D_1D_2}{10}.$$

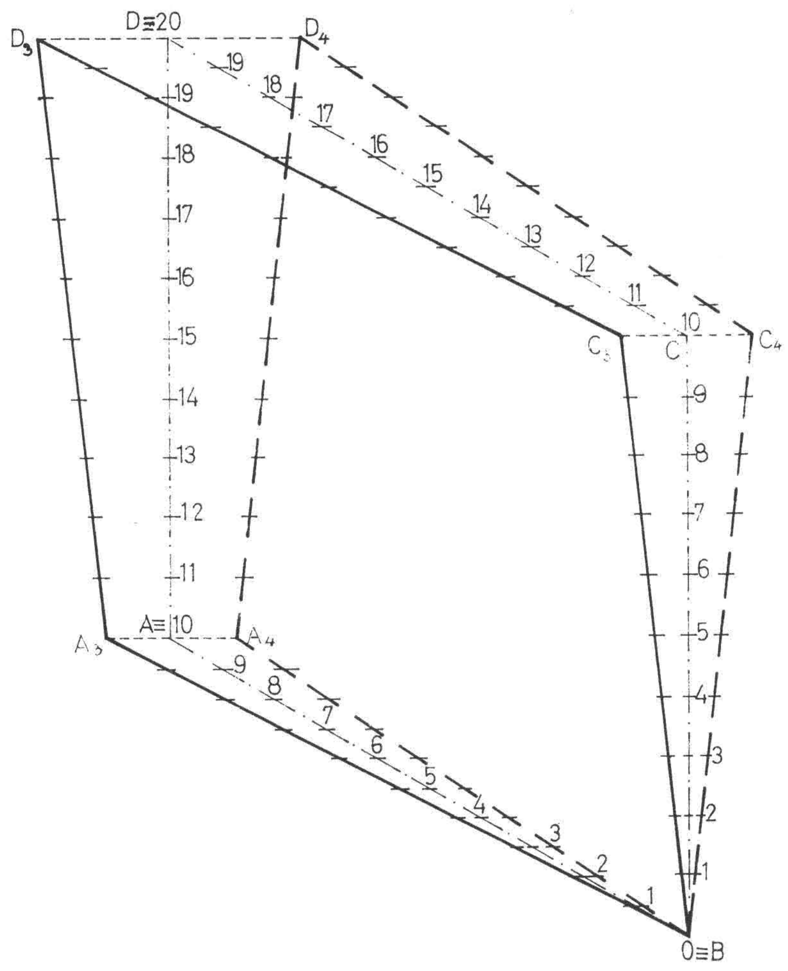


FIG. 6

Use of Nomogram

First, individually multiply the numbers which are to be added by a power 10^s so that each of these numbers will be converted to a number less than 10 or equal to 10. After that, the nomogram is observed through the spectacles until the model appears very clearly.

Then, N is brought to the point of intersection of the reference lines which pass through the points of A^*B and C^*B whose numbers are equal to the first addenda. Finally N is moved towards a point P^* on the side A^*B or A^*D corresponding to the height. The number of P^* is the sum of the two addenda. If desired to add another addendum to this result, N is brought to the point of intersection of the reference line which passes through P^* with the reference line which passes through the point on BC^*, whose number is equal to the third addendum, if P^* lies on the side A^*B. But if P* lies on the side A^*D^* one reference line must pass through a point P^0 of A^*B whose number is equal to 1/10 of the number of P^*, and the other reference line must pass through a point of BC^* whose number is 1/10 of the third addendum. After that N is moved towards a point of A^*B or A^*D^* corresponding to the height. The process is continued in this manner until the last addendum. The last result must be multiplied by 10^{-s+q}; where q is the number of times the partial results have been reduced to 1/10 of their value.

SUBTRACTION

For subtraction the stereoscopic nomogram for addition is used. N is brought to the point of A^*D^* whose number is equal to the minuend. Then N is moved towards a point R of the reference line which passes through the point of A^*B whose number is equal to the subtrahend. The second line of reference that passes through R intersects BC^* at a point, whose number is the remainder. Before these operations, the given numbers must be reduced as explained in the paragraph "Nomogram for Addition."

DIVISION

For division the stereoscopic nomogram for multiplication is used. N is brought to the point of AD^* whose number is equal to the dividend. After that N is moved towards a point R of the reference line that passes through the point of AB whose number is equal to the divisor. The second line of reference that passes through R intersects the side BC at a point whose number is the quotient.

In this case the given numbers must be reduced in accordance with what is written in the paragraph "Nomogram for Multiplication."

POWERS

Again look at the nomogram for multiplication. Lying on the surface of the model there is seen a diagonal and another curve whose ends coincide with the ends of the diagonal. The height of any point of the diagonal is equal to the square of the number written at the end of the model lines which intersect in it.

The height of any point of the curve is equal to a tenth of the cube of the number written at the end of the model line (lying on the lower side of the model) which passes through it. Then:

(A) TO SQUARE, N is brought to the point of intersection of the model line which passes through the point R of AB whose number is equal to the given number and the diagonal. Following that N is moved towards a point of the same elevation on the left side AD^*. The number of this point gives the desired square.

(B) TO CUBE, the procedure is the same as the square, but the point of intersection must lie on the curve. In addition the number written by the last point on the left side must be multiplied by 10.

(C) FOR POWERS As in the case of multiplications of n factors equal to each other.

SQUARE ROOT

The stereoscopic nomogram for multiplication is used. N is brought to a point of AD^* whose number is equal to the given number whose root we seek. After that N is moved towards a point R of the diagonal BD. The equal numbers corresponding to the ends of the two reference lines which pass through R give the desired root.

CUBIC ROOT

We use the stereoscopic nomogram for multiplication. Let "a" be the number whose cubic root we want to find. We bring N to a point of AD^* whose number is equal to $a/10$; after that we move N towards a point R on the curve BD^* (at the left side of the diagonal BD^*).

In this case we consider only the reference line which intersects the side AB. The number of this point of intersection gives the desired cubic root.