*The Precise Evaluation of Lens Distortion**

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ABSTRACT: *Four methods of radial distortion measurement are described. Comparison of results obtained on the same lens using each of the four methods is used to locate systematic errors and to evaluate the reliability of each method. Several sources of error are discussed. It is concluded that a precision of ±2 microns can be achieved using any one of the four methods provided adequate attention is given to the reduction of all potential errors to negligible proportions: that is, the PE_s of D_β does not exceed 2 microns where D_β is the value of distortion obtained at angle β and PE_s is the probable error of a single determination.*

1. INTRODUCTION

THE measurement of radial distortion in the focal-plane of photographic objectives has been the subject of intensive study since the advent of mapping from aerial photographs. This particular aberration is of prime interest to photogrammetrists as its magnitude determines the accuracy with which the final negative maintains the correct relationships among the array of point images making up the photography of the corresponding array of points in the area photographed. It was early realized that the reliability of the quantitative information obtained from a photograph is increased as the distortion in the camera lens is decreased. Consequently, the development of improved lenses was encouraged with the result that succeeding series of new lenses were characterized ever lower values of distortion.

During this period of change, diverse methods of evaluating the distortion of lenses came into use at various laboratories. The reason for such diversity was primarily the availability of given types of measuring instruments in various laboratories. There are now three principal methods of measuring distortion, plus numerous additional methods that are either the inverse of one of the principal methods or a variation thereof. The nodal slide bench is one of the oldest methods; this is a visual method capable of high accuracy and is perhaps the most widely used. The photographic method is more recent and arose out of the desire to make measurements under conditions approximating those of use. The third principal method is the goniometric, which is used to considerable extent in Europe.

Because of the diversity of methods used in evaluation of distortion, it seemed worthwhile to investigate the results of measurements made in a single laboratory on a single lens by a variety of methods, to determine whether or not the values so obtained varied appreciably with the method. This has been done using four different methods and a comparison of the results is reported in this paper.

2.0 METHODS OF MEASUREMENT

The methods are as follows:

1. Photographic—Precision lens testing camera.
2. Visual—Nodal slide optical bench.
3. Visual—Inverse nodal slide on T-bench with small aperture telescope.
4. Visual—Modified goniometric using Wild theodolite.

Methods 1, 2, and 4 have been described at some length in the literature,[1,2,3] while Method 3[4] has been recently developed at this Bureau and is a modification of a very early method[5] of measuring lens distortion. In the following sections, a brief description of each method is given.

2.1 PRECISION LENS TESTING CAMERA. METHOD 1

The precision lens testing camera,[1] shown in Figure 1, was developed at this Bureau by I. C. Gardner and F. A. Case. It is one of the earliest successful devices developed to measure the performance of lenses by photographic means. It consists of a bank of collimators

* This paper was presented at the 1962 St. Louis ACSM-ASP Convention.

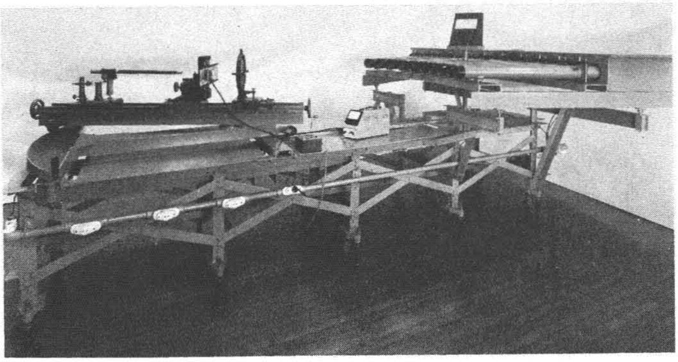

FIG. 1. Precision lens testing camera (Method 1).

spaced at 5° intervals having resolution test charts as reticles. The lens under test is mounted at the center of convergence of the collimator fan, and can be aimed at any one of them by rotation of a carriage which carries the lens holder and camera back whereon the photographic recording plate is mounted. As presently constituted, the lens testing camera has 10 collimators covering a total angle of 45°. When used to test wide-angle lenses, the camera is aimed at one of the extreme collimators (Position I) and the test made. In order to cover a complete diameter, a second test is made with the camera aimed at the collimator at the opposite extreme (Position II). It has been found by experience that the results obtained from two negatives made in this manner are quite reliable.

From the measured separation of the images recorded on the test negative made with a given lens, it is possible to determine both the equivalent focal-length f and the distortion D_β. The determining equation used in evaluating the distortion at a given angle β is

$$D_\beta = r_\beta - f \tan \beta, \quad (1)$$

where r_β is the measured distance separating the images produced by the lens under test in the 0° collimator and that in the collimator inclined at angle β, and f is the value of the equivalent focal-length based on the separation of the 0° and 5° images and that of the 0° and 10° images.

2.2 VISUAL OPTICAL BENCH; DIRECT NODAL SLIDE. METHOD 2

The visual optical bench has long been the basic tool for evaluating the constants of lenses. The one used at this Bureau[2] is shown in Figure 2. For measuring distortion, a collimator, nodal slide lens holder, and micrometer microscope are used. The lens is carefully aligned in the holder and the axial-image formed by the lens under test of the illuminated reticle of the target is brought into coincidence with the object-plane of the viewing

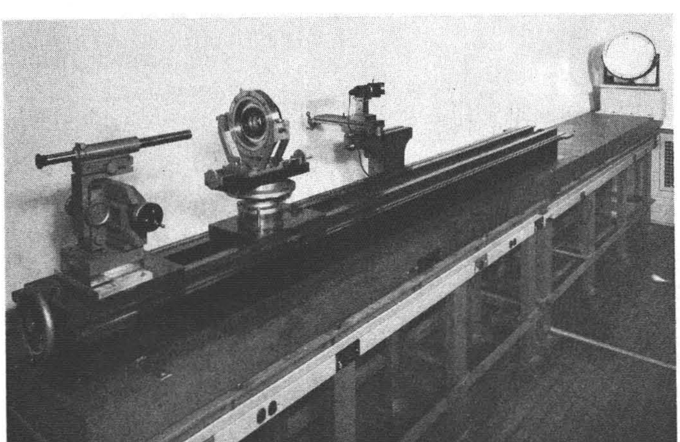

FIG. 2. Visual optical bench. Direct nodal-slide (Method 2).

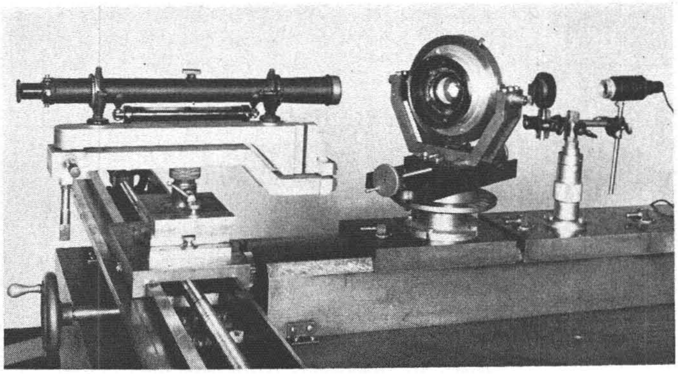

FIG. 3. Inverse nodal-slide (Method 3).

microscope. By a series of successive adjustments, a condition is found for which a small rotation of the lens about a vertical axis does not produce a displacement of the axial-image viewed. The rear nodal-point of the lens is then considered to coincide with the center of vertical rotation of the nodal-slide.

Assuming the equivalent focal-length f to be known, the nodal-slide is rotated by amount β about the vertical axis using the calibrated circle of the nodal-slide to position it exactly. The entire saddle carrying the nodal-slide and lens is then moved away from the microscope toward the collimator by an amount $f(\sec\beta\text{-}1)$. The viewing microscope is shifted laterally to the new position of the image and its dial read and recorded as reading R_β. The nodal-slide is then rotated to position $-\beta$ and a second setting of the microscope, $R_{-\beta}$, is made. These displacements are measured in a plane normal to the chief rays inclined to the axis at angles $+\beta$ and $-\beta$, respectively. The distortion D_β is obtained from the relation

$$D_\beta = \frac{\Delta R_\beta}{2} \sec \beta \qquad (2)$$

where $\Delta R_\beta = R_{+\beta} - R_{-\beta}$.

2.3 INVERSE NODAL SLIDE. METHOD 3

In the inverse nodal-slide method[4], shown in Figure 3, an illuminated target, preferably a point source, is placed at the rear focal-point of the lens under test. Light from the point-source emerges from the lens in a collimated beam which enters the viewing telescope. By a series of adjustments, a condition is found for which a small rotation of the lens about a vertical axis does not produce an angular displacement of the collimated beam observable through the viewing telescope. The rear nodal-point of the lens is then in close coincidence with the center of rotation of the nodal-slide. The nodal-slide bearing the lens under test is then rotated by amount β about the vertical axis, the target reticle is moved away from the zero position by amount $f(\sec\beta - 1)$, the pointing of the viewing telescope is adjusted to compensate for change in direction in the collimated beam emergent from the lens under test, and the setting of the transverse micrometer is recorded. This reading is $\epsilon_{+\beta}$. The process is repeated with the lens rotated to position $-\beta$, and the reading $\epsilon_{-\beta}$ is recorded. The distortion D_β is obtained from the relation

$$D_\beta = \frac{\Delta\epsilon_\beta}{2} f \sec^2 \beta \qquad (3)$$

where $\Delta\epsilon_\beta = \epsilon_{+\beta} - \epsilon_{-\beta}$, and f is the equivalent focal-length of the lens.

2.4 MODIFIED GONIOMETER. METHOD 4

The goniometer method, illustrated in Figure 4, used in this laboratory does not employ a specially constructed camera goniometer such as is used in Europe, but more nearly approximates the method used by Merritt[3]. A calibrated linear-scale is placed in the focal-plane of the lens under test and the scale is viewed through the front of the lens with a precision theodolite. The angular displacements β of selected points on the scale distant r_β from the center line on the calibrated scale are measured using the precision circle of the theodolite. The distortion D_β is obtained from the relation

$$D_\beta = r_\beta - f \tan \beta. \qquad (4)$$

3.0 RESULTS OF MEASUREMENT

An extensive series of measurements in a single wide-angle lens using each of these four methods was made and values of distortion determined with each method. The results of

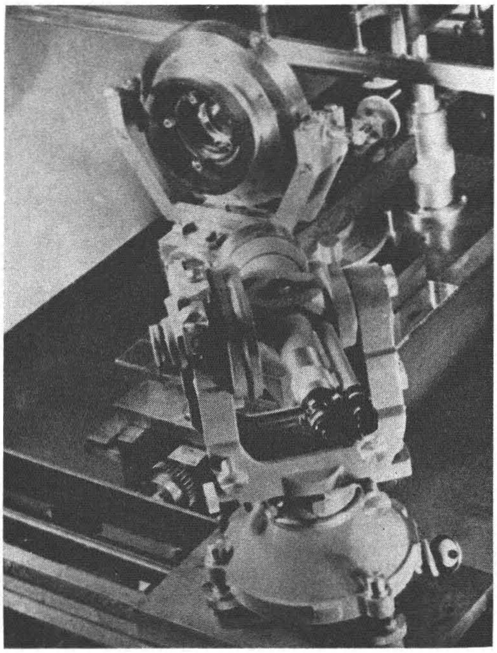

FIG. 4. Modified goniometer (Method 4).

these measurements (for a lens having a focal-length of 150 mm.), were reported in a series of papers[4,6,7] which give a detailed discussion of each. The primary purpose of this paper is to present an overall picture of the problems encountered and the solutions thereof.

3.1 THE CALIBRATED FOCAL LENGTH

In comparing values of distortion obtained by two methods such as Methods 1 and 2, the distortion must be evaluated in such a manner that corresponding values are readily comparable. For example, values of distortion obtained by Methods 1 and 2 are shown in Figure 5. In the upper frame, the values are based upon the equivalent focal-length (EFL) in each case. However, the equivalent focal-length used in Method 2 approximates the paraxial value while that used in Method 1 is that value which makes $D_\beta=0$ at $\beta=$approximately 7.5°. It seems more reasonable therefore to refer the values in both cases to a calibrated focal-length (CFL)[8] such that $D_{35°}=-D_{45°}$; this has the advantage of ensuring that the maximum positive value equals minus the maximum negative value in each case. The effect of this transformation is shown in the lower frame of Figure 5. It is evident that this transformation improved the situation for the 35° to 45° region although the disparity of values of D_β was increased in the 5° to 30° region.

In addition, the referral of the values of D_β to the calibrated focal-length removes from consideration errors in distortion arising from errors in equivalent focal-length. Hence, throughout the paper values of distortion are referred to the calibrated focal-length for each of the four methods.

3.2 PRECISION OF MEASUREMENT

The accuracy of the final accepted values of D_β obtained by any of the four methods is dependent upon the magnitude of random and systematic errors[9] that may affect the measurement. For the most part, the magnitude of random errors may be estimated either on the basis of past experience or from analysis of the original data taken during the course of measurement. The systematic errors are less amenable to evaluation as they arise in many instances from defects of various nature in the equipment itself or from errors in calibration of the measuring devices. It sometimes possible to detect this type of error from analysis of measurements of the same quantity using different methods.

It is clear that the values of the distortion

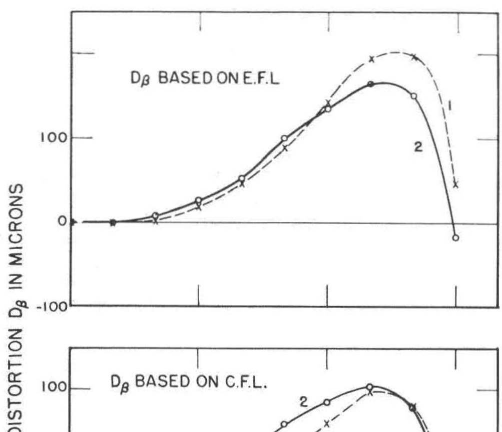

FIG. 5. Comparison of values of distortion, D_β as a function of β by Method 1 (Precision lens testing camera) and Method 2 (Direct nodal-slide bench).

In the upper frame, values of D_β are referred to the equivalent focal-length (EFL) while in the lower frame the values of D_β are referred to the calibrated focal-length (CFL).

obtained with the aid of Equations 1 through 4 will be affected by errors in r, β, ΔR, and $\Delta\epsilon$. In general, values of the probable error of a single determination PE_s do not exceed ± 1 micron for measurements of r and ΔR, and it is believed the systematic error in these values arising from calibration errors does not exceed ± 1 micron. In addition, values of PE_s do not exceed ± 1 second for measurements of β in methods 1 and 4; and values of PE_s do not exceed ± 1 second for measurements of $\Delta\epsilon$ in Method 3. Hence for any one of these methods, the precision of measurement is such that values of PE_s of D_β should be within ± 2 microns throughout the range of $\beta = 0°$ to $\beta = 45°$.

3.3 COMPARISON OF RESULTS FOR THE FOUR METHODS

Following evaluation of the distortion D_β for a preliminary series of measurements made by each of the four methods, an average $\bar{D}_\beta$ was obtained for each value of β using the relation

$$\overline{D_\beta} = \frac{D_1 + D_2 + D_3 + D_4}{4}$$

where the subscripts indicate the method used in determining this particular value of D_β. The

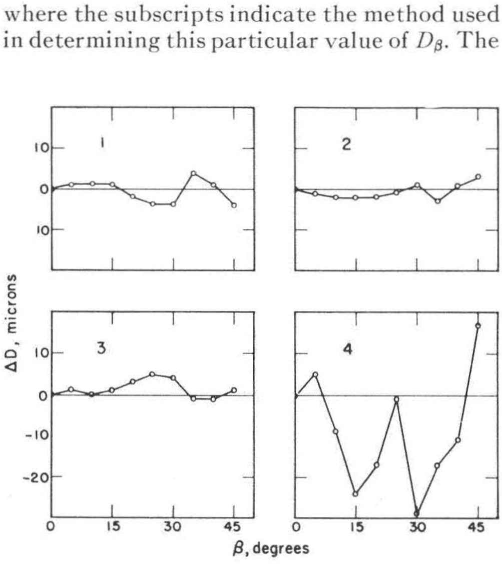

FIG. 6. Variation of the departures ΔD_β from the average value $\overline{D_\beta}$ for the four methods as a function of β for each method.

The numbers in the upper left of each frame specify the method for which the departures $\Delta D_\beta = \overline{D_\beta} - D_{\beta n}$ are shown. Thus, Frame 1 gives the comparison for the precision lens testing camera; Frame 2 for the direct nodal-slide; Frame 3 for the inverse nodal-slide; and Frame 4 for the modified goniometer. This figure shows the magnitudes of the values of ΔD_β prior to the adjustment of various errors.

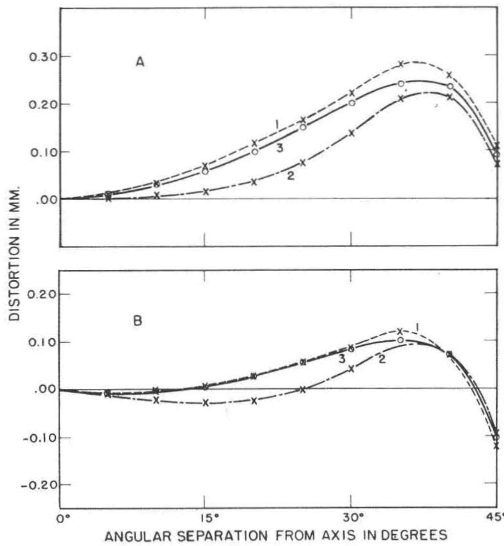

FIG. 7. Variation of distortion for incorrect and correct relative locations of the lens-aperture with respect to the aperture of the viewing telescope as a function of angular separation β from the axis.

The values of the distortion referred to the equivalent focal-length are shown in box A; the values of the distortion referred to the calibrated focal-length are shown in box B. The curves marked 1 show the maximum incorrect values obtained; the curves marked 2 show the minimum incorrect values; while the curves, marked 3, show the normal or correct values of the distortion. The values are for a wide-angle lens having a nominal focal-length of 150 mm and maximum aperture f/6.3.

departure from the average $\Delta D_{\beta n}$ was determined for each method using the relation

$$\Delta\overline{D_{\beta n}} = \overline{D_\beta} - \overline{D_{\beta n}}$$

where n indicates the method number and ranges from 1 to 4. The magnitudes of these departures from the average are shown for each method in Figure 6. It is obvious that the observed departures are considerably in excess of ± 2 microns although well within the tolerance of ± 20 microns claimed as the accuracy of measured values of distortion prior to the advent of nearly distortion-free lenses.

These measurements were all made in one laboratory so that it was easier to isolate possible causes of systematic error and determine their magnitude. After careful study and analysis, it was concluded that the values of D_β obtained by Method 2, the direct nodal-slide method, were perhaps least affected by systematic errors. The departures of the magnitude of the values of ΔD_β for Method 1 can be substantially reduced by correcting for a small amount of plate curvature or

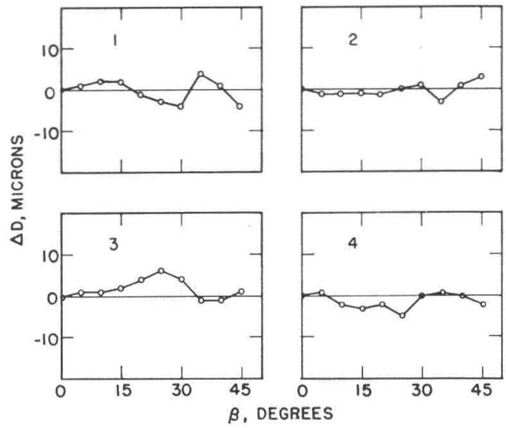

FIG. 8. Variation of ΔD_β with β for each of the four methods after corrections.

differential plate tipping induced by warping of the camera back of the precision lens testing camera.

Analysis of values of distortion by Method 3 showed that substantial changes in distortion values are produced by unsymmetrical apertures. Some idea of the magnitude of this change is given in Figure 7. In fact, the chief value of Method 3 is its usefulness in evaluating errors produced by asymmetrical use of the lens aperture.

The values of D_β determined by using Method 4 were found to be affected by curvature of the scale used in the focal-plane and by asymmetrical use of the aperture.

Additional measurements were made by all methods with special emphasis placed on reducing these causes of error. In addition, adjustment of the final values was made to compensate for residual errors of plate curvature where evident.

Values of departures from the average, $\Delta D_{\beta n}$, for each of the four methods determined after making these corrections are shown in Figure 8. It is clear that a substantial improvement was effected. The average precision index, given as probable error of a single determination PE_s, has a value of ± 2 microns for each method. It is worthy of note that 25 of the 36 values of $\Delta D_{\beta n}$ do not exceed ± 2 microns and only one value of $\Delta D_{\beta n}$ is as great as 6 microns.

4. CONCLUSIONS

On the basis of the findings made in this study, it seems probable that a precision, (PE_s), of ± 2 microns in the measured values of lens distortion can be achieved with any of the four methods provided adequate attention is given to reducing all potential errors to negligible proportions. For all methods it is desirable to measure distances to ± 1 micron and angle to ± 1 second of arc. Errors arising from plate or scale warpage should be reduced to the extent that they do not exceed ± 1 micron. In addition, if the measured values of distortion are to be accurate as well as precise to the nearest 2 microns; it is desirable that the measurements be made by not less than two independent methods and that the corresponding values for each of the two methods agree within ± 2 microns.

Finally, while it may be possible to achieve this level of accuracy in the measurement of distortion along a single radius of a lens, it frequently happens that the value of distortion, D_β, for a given value of β may vary with azimuth to an extent far in excess of the precison of measurement. Thus it would appear that if it be necessary to know the distortion within ± 2 microns, it would be necessary to make measurements along many radii. From such measurements, one could then make a chart similar to a map which would show the magnitude of the distortion at any selected point in the entire picture area.

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