*Accuracy of Photogrammetric Measurements of Curved Surfaces**

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ABSTRACT: *It was not until quite recently that photogrammetry was applied to architecture and archaeology. Its major application is to document facts and conditions about old buildings and temples in the form of stereograms. It is quite common for these buildings to contain great numbers of round columns. Such columns present a problem when photogrammetric reproduction of their shapes and sizes is attempted. This paper explains such problems and gives different solutions.*

INTRODUCTION

As PHOTOGRAMMETRY proves, day after day, to be adaptable to other branches of science, one expects to be confronted with more and more new problems. The kind and nature of such problems depend on the field and the way in which photogrammetry is applied. Among such fields, architecture and archaeology have been lately using photogrammetry in many applications. Of particular importance, it is being used as a powerful tool in recording and documenting facts and conditions about historic buildings and temples. In working with photogrammetry in these two fields an interesting problem arose on which—apparently—not much literature exists. This problem concerns the inaccuracies that result from using measurements from photographs of shapes bounded by curved surfaces. Or, in other words, it deals with the errors introduced when converting a perspective to an orthographic view of such shapes. The purpose of this paper is to discuss these errors and to present theoretical as well as practical solutions.

GENERAL

It is well known that the main principle of stereophotogrammetry—whether aerial or terrestrial—lies in the ability to see stereoscopically. Any difficulty in stereoscopic vision, for any reason, results in inaccuracies of different degrees in the final drawings. In turn, stereoscopy is based on the appearance of every image, to be seen in three-dimensions, clearly enough to be identified in both photographs of the stereopair. For example, the accurate dimensions of a facade of any building could be obtained, to a high degree of accuracy, so long as the edges of this facade appear clearly in each photograph of a stereopair. This fact, however, does not hold true for other parts of the building that are bounded by curved surfaces such as round columns. Such curves, when photographed from two different stations, and positives (or negatives) inserted in the stereo-plotter, do not produce a model of their actual size and shape.

The different shapes of round-surfaced columns may be cylindrical, or conical, or in general, any surface of revolution resulting from revolving a curve in the vertical plane around the axis of the column. The characteristic of all such shapes is that the plan cross-section at any plane is always a circle. For a systematic approach of the problem let us first consider the cylinder.

CYLINDRICAL COLUMNS

In Figure 1 a cross-section of a cylindrical column is shown together with the positions of the two camera stations L and R from which two photographs of the column are taken. The bundle of rays from each photograph touch the profile of the column in lines (denoted in the plan view by points A,B and C,D) which are not the same in each photograph and which do not correspond to the profile in the orthogonal projection. In an orthogonal projection the distance between the boundary lines of the column profile is always equal to the diameter, while the apparent diameter imaged in each photograph ($a'b'$ or $c''d''$ Figure 2) corresponds to a

* This paper has been submitted by the author in the competition for an award in 1962–3.

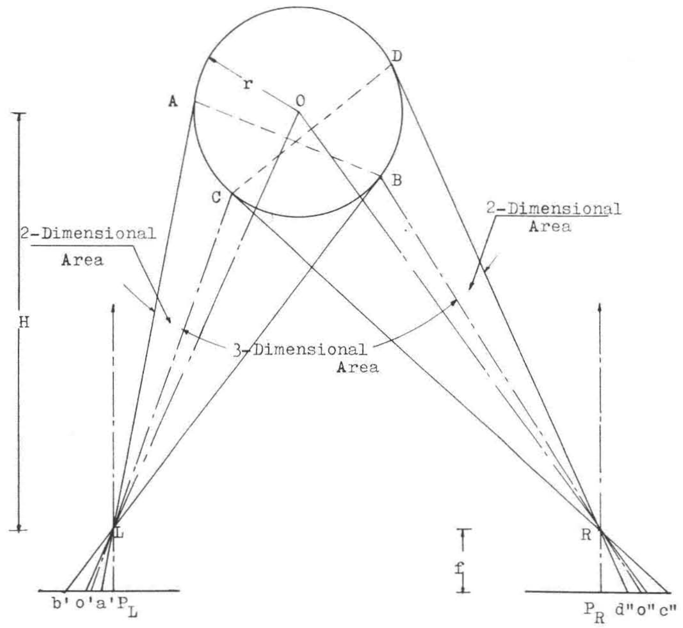

FIG. 1. Plan view of a cylindrical column photographed from two stations.

distance in the column (*AB* or *CD*) which is always less than the true diameter. Also, the distance $a'b'$ is not equal to $c''d''$ according to the relative position of each camera station with respect to the column position

Graphical Determination of the Diameter, Hence the Plan, of a Cylindrical Column

The cross-section of any cylindrical column can be determined when it appears in at least two photographs. From the camera lens of each photograph as a center of projection, two rays originate and touch opposite edges of the profile. From geometry, three lines tangent to a circle are the minimum required to define a circle. In this case there are four tangent rays to the column touching it in points *A*, *B*, *C* and *D* (see Figures 1 and 2). When the interior and exterior camera orientations are known, the positions of the camera stations can be plotted to scale and the four rays reconstructed in plan view. The problem now is reduced to simply drawing a circle touching four lines. This can be done in two ways: (1) A direct, completely graphical solution; (2) A semigraphical solution.

(1) Graphical Solution

The four rays will intersect in four points, three of which (*E*, *F* and *G* in Figure 2) usually lie on the drawing sheet and the fourth (*K*) may fall forward or backward of the column center depending upon the relation between the distance *H* between the base (*LR*) and the center of the column (*O*), and the diameter of the column (*2r*). The bisectors of the angles between every pair of rays (lines *OE*, *OF*, *OG* and *OK* in Figure 2) must intersect in the same point which will be the center of the circle *O*. Whether the four bisectors meet at the same point or not is only a matter of accuracy involved in both the measuring from photographs and the plotting and drawing on the sheet. In general, these bisectors will form a quadrilateral of error *MNPQ*. If the area of this quadrilateral is large enough, the angles at the corners *M*, *N*, *P* and *Q* are bisected and the new bisectors, will—or very closely will—meet at the center point. If it is so small that it will be difficult to construct these new bisectors, then it will be accurate enough to assume the position of the center by estimation. Knowing the center, the radius *r* is simply the length of perpendicular from O to any one of the tangents such as *EA* ($r = OA$).

(2) Semi-Graphical Solution

The semi-graphical method requires that some simple measurements be made. It is based on the theorem that the bisector of any angle in a triangle divides the side opposite to that angle with a ratio that is equal to the ratio between the lengths of the other two

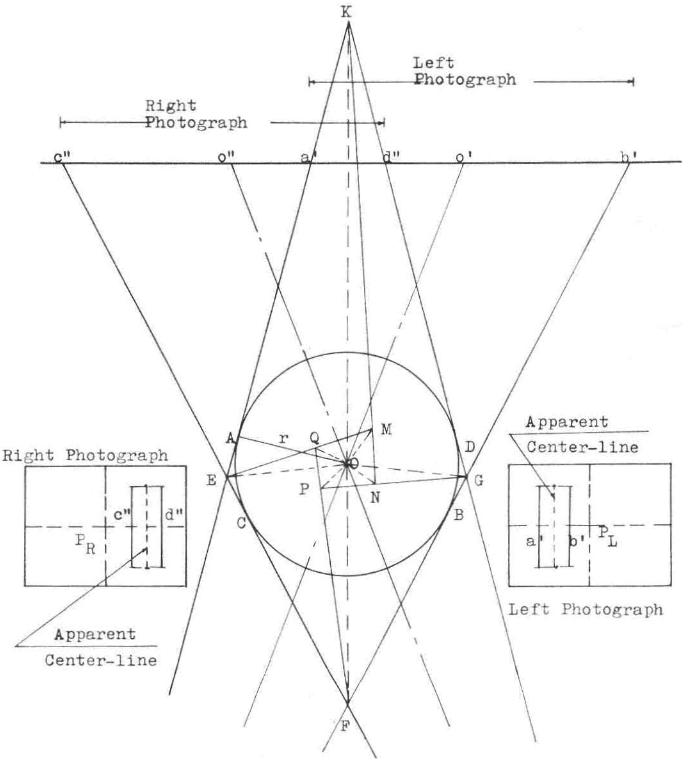

FIG. 2. Graphical determination of the diameter of a cylindrical column from two photographs.

sides. For example, in Figure 1 we have

$$\frac{a'o'}{o'b'} = \frac{La'}{Lb'}$$

for the left photograph, and

$$\frac{c''o''}{o''d''} = \frac{Rc''}{Rd''}$$

for the right photograph. Now, we plot the focal-length of the camera to scale (LP_L) and construct a line perpendicular to it from one end P_L representing the photograph. The distances P_La' and P_Lb' are measured from the photograph. These distances are plotted on the line representing the photograph and in turn the triangle $La'b'$ is constructed. The same is done for the other photograph and the triangle $Rc''d''$ is obtained. Measure the distances La', Lb', Rc'', and Rd'' and compute the ratios

$$\frac{La'}{Lb'} \text{ and } \frac{Rc''}{Rd''}.$$

Locate points o' and o'' on the lines $a'b'$ and $c''d''$ in proportion to these two ratios, respectively. Connecting point o' with L and point o'' with R, we get the center of the circle as the point of intersection of these two lines ($o'L$ and $o''R$). It ought to be mentioned here that the line bisecting the distance $a'b'$ and $c''d''$ on the photographs (Figure 2) although apparently representing the center-line of the column, does not actually represent the image of the center-line which is invisible. The true image of the center-line is closer to point a' than b' in the left photograph, and closer to point d'' than c'' in the right photograph. The true and apparent positions of the center-line image will coincide only in the case when the photographic plane is perpendicular to the plane containing the camera axis and the center-line of the column. This makes $a'o' = o'b'$ and $c''o'' = o''d''$.

CONICAL SHAPED COLUMNS

With the camera axes perpendicular to the column axis, the profile of a cylindrical column appears in the photograph similar to its orthogonal projection i.e., two parallel vertical lines. In contrast to the cylinder, the frustum of a cone, in general, does not appear in the photograph similar to its orthogonal projection. It only resembles the orthogonal case when the plane of the photograph is

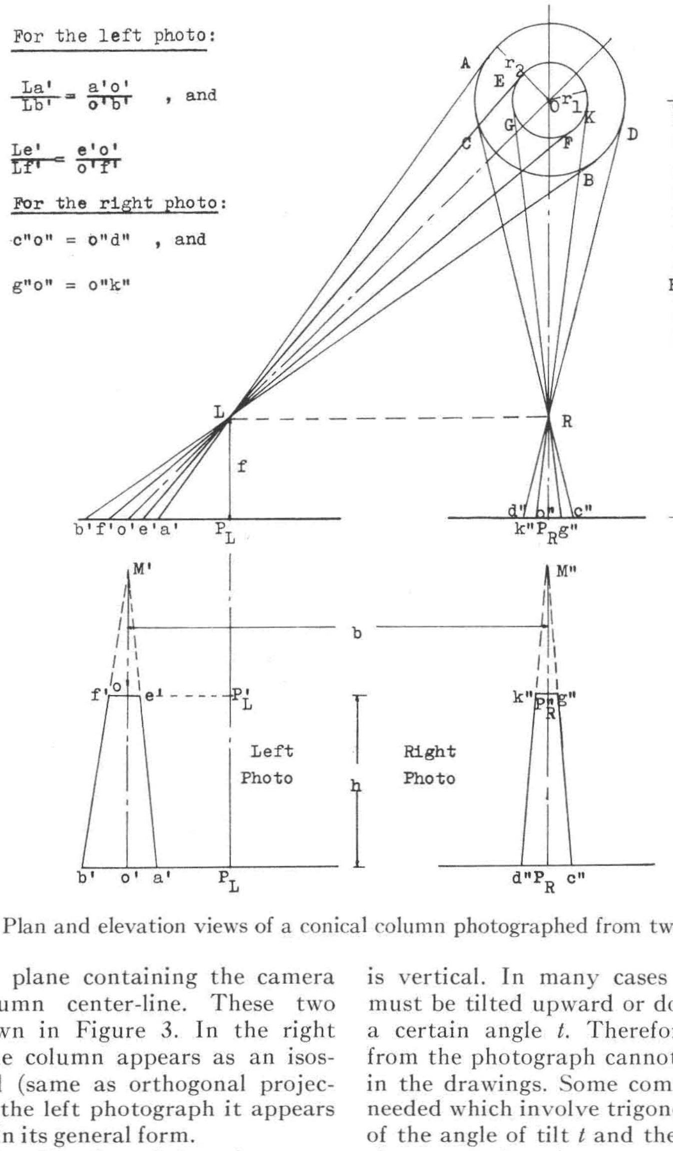

FIG. 3. Plan and elevation views of a conical column photographed from two stations.

normal to the plane containing the camera axis and column center-line. These two cases are shown in Figure 3. In the right photograph the column appears as an isosceles trapezoid (same as orthogonal projection) while in the left photograph it appears as a trapezoid in its general form.

To obtain the plan view of the column, we follow either one of the two methods explained above for the cylindrical case. The difference, though, is that the solution has to be performed twice, once for the upper base and once for the lower base. Consequently, knowing the radii of the bases r_1 and r_2 and computing the height of the cone corresponding to h in the photographs, we can construct the elevation view of the column.

The two graphical methods as stated above are rather simple because of the assumptions of having the camera axis horizontal. In such a case, measurements are taken directly from the photograph and plotted—to scale—on the projection of the photograph on the horizontal plane, which is a line since the photo plane is vertical. In many cases the camera axis must be tilted upward or downward through a certain angle t. Therefore measurements from the photograph cannot be used directly in the drawings. Some computations will be needed which involve trigonometric functions of the angle of tilt t and the focal-length f of the camera lens (or more precisely the principal distance).*

So far, two shapes of columns have been considered, namely the cylindrical and conical. At this point it should be mentioned that the same argument and solutions apply to the columns of more general shape and form, that is columns generated by the rotation of any combination of straight lines and curves around a vertical axis representing the axis of the column (see Figure 4). It would be

* For more complete analysis of the general case, see "Uses of photogrammetry in preserving the antiquities of Nubia" by Edward M. Mikhail, a thesis presented to the Faculty of the Graduate School of Cornell University for the M.S. degree, 1961.

rather unwise to try to develop the mathematical relationships in such cases especially when the camera axis is in a general position with respect to the column to be photographed. Consequently this leads to a more practical solution utilizing the advantage yielded by stereoplotters through reproducing a model of the object (or part of it) in three dimensions. Due to the familiarity of the author with the Wild A-7 Autograph, it will be considered in the following discussion.

USE OF THE A-7 IN PLOTTING CURVED SURFACES

It is clear from Figures 1, 3, and 5 that the area which appears in any photograph is always less than half the circumference of the column. This is because any two rays emanating from a point can never touch the diametrically opposite points of a circle (cross-section of all the columns under discussion). Furthermore, the area covered by a model (or that appearing in 3-*D*, see Figures 1 and 5) is even less than that covered by one photograph. Consequently if two plates are inserted in the stereoplotter, we will not be able to measure any diameter directly, since only a part bounded by a chord and not a diameter will be constructed. The area of the model compared to half the circumferential area, depends mainly on the base length b and the distance H. The shorter the base the larger the area covered, while, in contrast, the longer the distance H the larger the area covered, will be (theoretically at $H=\infty$, the area covered will be half the circumference).

Theoretically speaking, the same principle applies to the human vision. No person can see (with the naked eyes) exactly the diameter of any round column of any shape (cylindrical, conical, or general shape). Each eye perceives a part of the surface always less than half the periphery. Correspondingly, the resulting model formed and translated by the nerves in the brain, will represent a part bounded by a chord. So, one will always see such types of surfaces, smaller than they are. But, since the eye-base (the distance between the eyes) is very small, the difference between surface area of the model and the area perceived by one eye is so small that the two areas can be considered equal. Furthermore, the discrepancy between the apparent diameter and the true diameter depends on the distance between the object viewed and the observer. This difference is usually so very small that it is hardly thought of in practical life. But, as far as theory is concerned, a cylindrical column will be seen exactly, only if its diameter is less than the eye base, and on the assumption that two dimensional areas can be sensed by the human brain as if stereoscopic.

Now, the following question may be raised; since two photographs are not sufficient to construct the diametric area, then how many

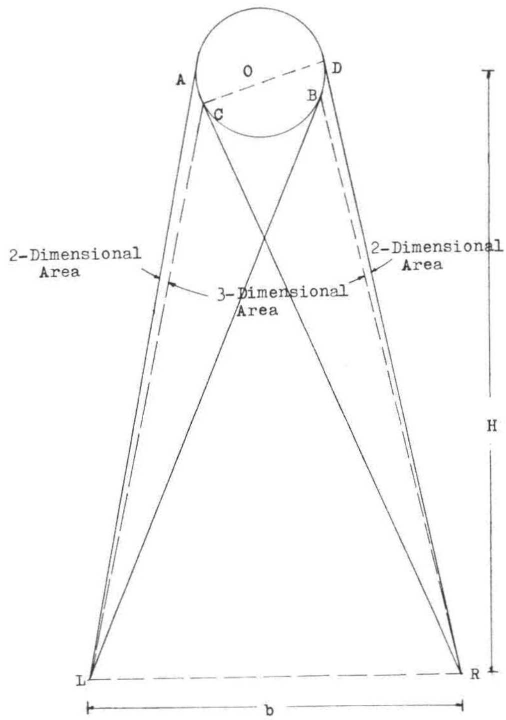

FIG. 5. Effect of b/H ratio on the 3-dimensional area coverage from two photographs.

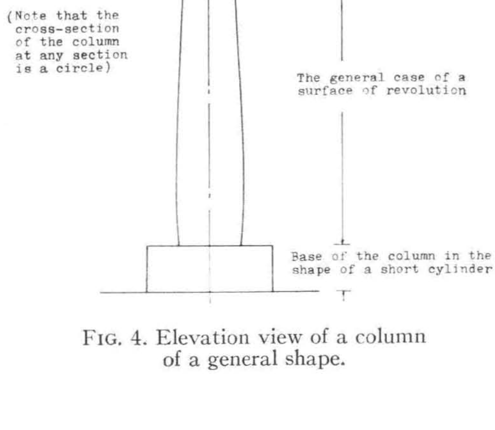

FIG. 4. Elevation view of a column of a general shape.

will do? The answer to this question cannot be given without first stopping to think. One might say hastily that three stations with relatively long bases and short distances will be quite sufficient to construct this area. But, this is absolutely not true, no matter how large the base-to-distance ratio is. This is graphically proven in Figure 6. The plan of the cross-section of a column in the horizontal plane through the camera axis—which is assumed to be horizontal—is photographed from four different stations s_1, s_2, s_3, and s_4. Pairs of rays from these stations touch the circle in points A, B, C, D, E, F, G, and H.

For simplicity, let us assume that we have a Balplex with four projectors available.

We start by inserting the plates No. 1 and No. 2 to get the first model (1–2) absolutely oriented (assuming that we have enough control within the model area to furnish this). This model will cover only the minor arc BE from the circle. When the third plate is inserted, model (2–3) will cover minor arc CF overlapping the first model with the minor arc CE. The area of the two models (i.e., three photos) is represented by the minor arc BF which is less than a semi-circle. This latter arc always should be less than the semi-circle since it is the arc subtended by the rays from the middle station (s_2) of the three used. When the fourth plate is inserted, and the third model oriented, the total is obtained by the three models is the arc BG. In Figure 6, this arc is slightly more than the semi-circular arc BK. Thus, in general, we cannot say that four stations will reproduce the semi-circular area. This again depends upon the lengths of the bases, the distance of the object from these bases, and how they are arranged with respect to the object.

When the main purpose is to determine the true value of the diameter of a column at any horizontal section, we need not have more than two photographs. It is not necessary at all to construct a complete semi-circle in order to measure the diameter instrumentally.

The case discussed in the last paragraph is important only when the column contains data such as inscriptions of different shapes around the whole surface that need be recorded. In such case the whole circumferential area must be properly covered by sufficient photographs for stereoscopic reconstruction.

A universal stereoplotter—such as the Autograph A-7—is the type of instrument that can be used to determine the corrections for the distortions resulting from photographing curved surfaces. As mentioned before, one stereo pair will suffice. The method is based upon the simple fact that three points not on a straight line (in plan) determine one and only one circle. The plan of all round columns is one, or more, circles. Figure 7 shows a column (chosen as a cylinder for simplicity in drawing) whose cross-section at the plane a-a is required. In part "A" of the figure the position of the instrument axes are shown for the "Aerial" case. The model appearing in the stereoplotter will be the segment bounded by the two lines bb and cc. When the instrument is in this position (aerial), the orthogonal

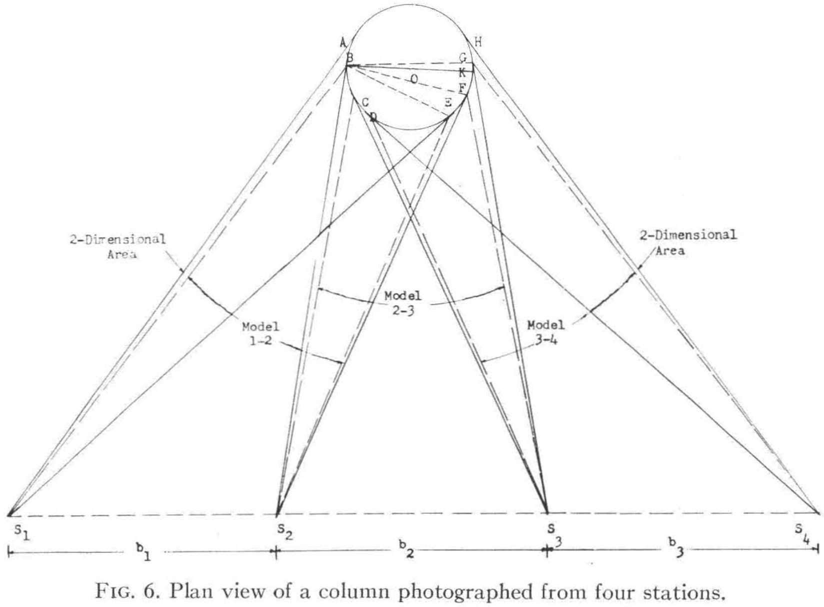

FIG. 6. Plan view of a column photographed from four stations.

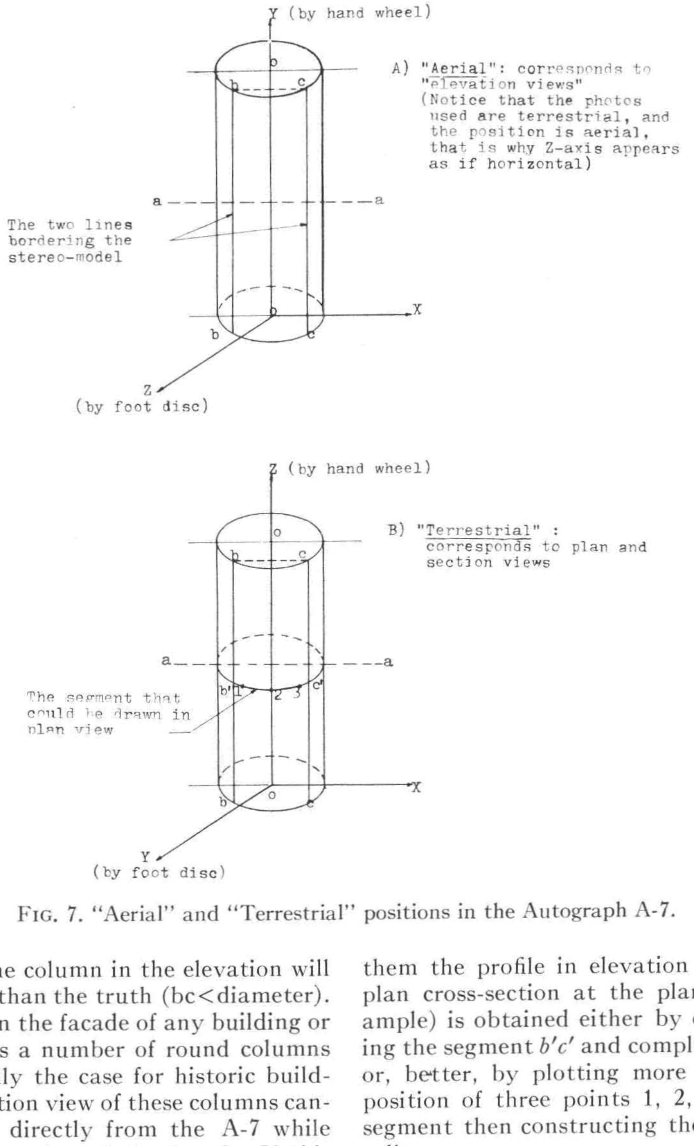

FIG. 7. "Aerial" and "Terrestrial" positions in the Autograph A-7.

projection of the column in the elevation will appear smaller than the truth (bc<diameter). Therefore, when the facade of any building or temple contains a number of round columns (which is usually the case for historic buildings), the elevation view of these columns cannot be plotted directly from the A-7 while plotting the elevation of the facade. If this were done, for example for a number of equal columns, we would get a different diameter for every column, each of which would be less than the actual dimension.

When the gear change is used to shift from the "Aerial" to the "Terrestrial" position—which is a real advantage of the A-7 with respect to architectural photogrammetry—plans and horizontal sections can be drawn. By this shift, movement between Y and Z can be interchanged. The Z movement is then coupled to the hand wheel and the Y counter, and the Y movement to the foot disc and the Z counter. This provides the clue to the problem. First the cross-sections at different horizontal planes are drawn, and then from them the profile in elevation is drawn. The plan cross-section at the plane *a-a* (for example) is obtained either by carefully drawing the segment $b'c'$ and completing the circle, or, better, by plotting more accurately the position of three points 1, 2, and 3 on this segment then constructing the circle graphically.

The reader is now referred to Figure 4 for solving the general case. This figure shows the profile of a column whose shape is not uncommon in ancient temples. To draw the profile of this column a number of horizontal planes are passed through the column at different levels and the diameters of the resulting circles are obtained as explained earlier. The distances between these planes along the axis of the column are computed from the differences between successive readings of the Y counter (corresponding to Z movement when in "Terrestrial" position). These readings are converted from model scale to the plotting scale through the transmission ratio. The values of diameters are obtained and the distances between sections computed, then the elevation can be plotted accurately.

Conclusion

The problem above discussed proves to be important when the photographed objects are mostly round. For the past two years or so photogrammetry has been used in recording the monuments, temples, and tombs of the ancient Egyptians located south of Egypt and north of Sudan which will be inundated by the water rising behind the Aswan High Dam.

The problem as discussed was mostly concerned with columns, yet, it is always met with in other cases such as domes, arches, conical steeples, . . . etc.

Acknowledgment

Thanks are due to Prof. A. J. McNair, head of the Surveying Department, Cornell University, for his valuable suggestions and helpful comments in reviewing this paper.

Futuristic Photo Interpretation

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Technological advances in the field of photogrammetry have been very rapid during the past few years. There have been improvements in cameras, film and stereoscopic mapping instruments. The use of high-speed digital computers has become quite common in the calculations necessary for lens design and in computing coordinates for mapping. New uses of computers are being tried and tested constantly. The results of these improvements in photogrammetric techniques has meant a greater production in mapping from aerial photographs which are taken with essentially distortion-free lenses.

A question asked in this paper and for which an answer will be attempted is: Has photo interpretation kept pace with the advances in photogrammetric techniques? An unqualified answer would have to be in the negative; but some qualifications are necessary. In the first place, information is being derived from photographs taken by satellites, such as TIROS and SAMOS, which give data on conditions in outer space. The interpretation of space photographs is an entirely new concept in photo interpretation and is one which did not exist a few years ago. Another aspect of photo interpretation which is relatively new is the interpretation of radar displays and of thermal photographic systems using energy transmission in the form of electronic or infrared radiation. Great credit must be given to the imagination and persistence of interpreters confronted with photographic records of this type. There can be very little control in the attitude of the photo station in satellite photography, or of the atmospheric conditions at the time of photography. It is nothing short of amazing to consider the amount of information that can be derived from such photographs and images.

The purpose of this paper is to delve into some of the problems which confront the photo-interpreter of today and to look into the future to see what is in store. Before doing that, however, it would be well to review some of the major contributions to the art of photo interpretation. A glance at recent issues of Photogrammetric Engineering is revealing. In the September 1960 issue, Feder discusses the interpretation of terrain from radar displays and the difficulties involved; in the same issue, Ockert discusses the problems associated with satellite photography and its interpretation. Other pertinent articles may be found by Hoffman (1960), Olson (1960) and by Strees (1961). The ones mentioned are by no means the complete list but have been chosen to illustrate the frequency with which such papers have been appearing. Colwell, in 1959, wrote an excellent paper dealing with the future of photo interpretation and photographic systems. On re-reading his paper today, he would find that many of his predictions for the 25 year period in the future have already come true.

Two further developments in photo interpretation should be mentioned at this point. The first is the use of color film and of color reversal film in the interpretation of data for