*angulation* has *greater* strength than *strips*. As a matter of fact, the two-directional triplets are probably the strongest sub-block units in analytical aerotriangulation.


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# *Definition and Determination of Weights of Fundamental Photogrammetric Data and Results**


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THE concept *weight* is used in measuring sciences to indicate the relative importance of basic data or results. Weights are usually defined as inversely proportional to the squares of the standard errors (standard deviations) or the variances of the actual data or results. Distinction is usually made between *á priori* and *á posteriori* weights. The á priori weights are assigned to measurements before they are used in computations of other data, and refer to factors or relations which for some reason introduce different standard errors or standard deviations in the measured data. The á posteriori weights refer to the geometrical quality of data which are determined through computation from measured values, in particular through some kind of adjustment. In order to illustrate the weight concepts, two examples from photogrammetry will be shown.

It has been found from empirical experiments and least square adjustments that the standard errors of unit weight of image-coordinates increase significantly with the radius from the principal point, Figures 1 and 2. This is a quite natural consequence of the facts that the photographic image is a central projection and that the actual image coordinate measurements are of orthogonal nature, Figure 3. All deviations of the image from a plane must therefore cause errors in the orthogonal positions. There are and must always be deviations of the image from a mathematical plane because

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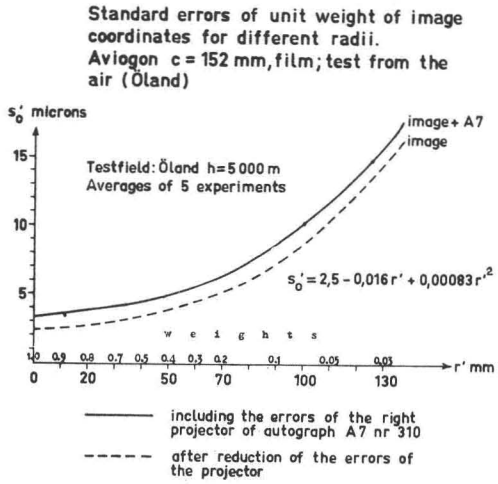
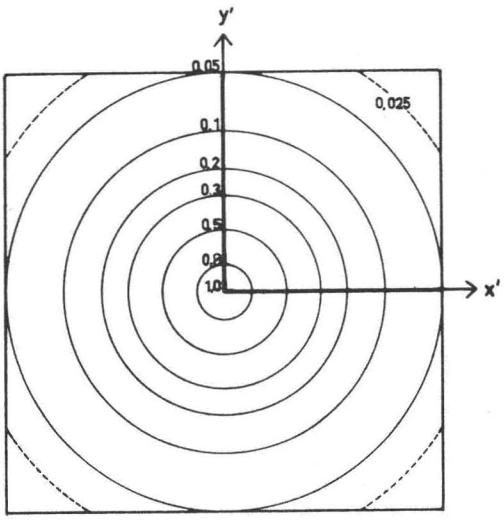



FIG. 1




Distribution of the weights of image coordinates according to averages from the Grimeton and Öland tests. Radial and tangential sources of errors.

FIG. 2

of the impossibility to make a physical surface mathematically flat. The film base and the emulsion can never be exactly constant in thickness, and the photographic contrasts (the silver grains) will always be distributed in depth within the emulsion, Figures 4 and 5. Consequently, before the image coordinate measurements in a comparator are made or before the measuring mark in a stereoscopic plotter is set, it is known that the geometrical quality of the image-coordinates will decrease with the radial distance of the points from the image-center due to the mentioned circumstances. The geometrical errors due to these causes cannot be decreased with the aid of repeated or replicated measurements or in any way, unless the undulations of the surface are known. Therefore, certain á priori weights can be assigned to the image

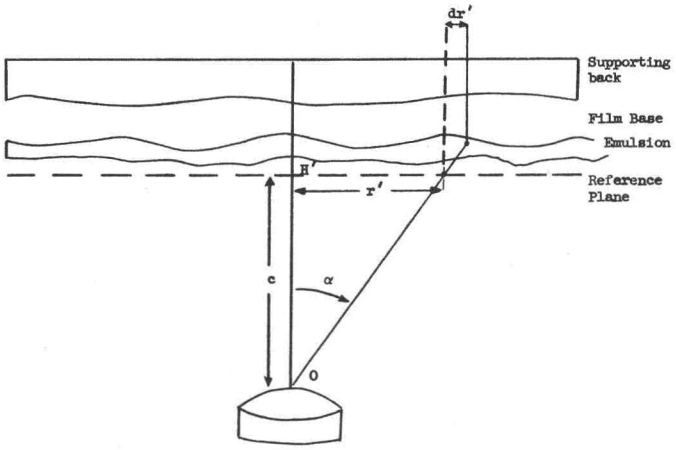



FIG. 3. Schematic illustration of the discrepancies between central projection and orthogonal measurements in case of lacking flatness of the image plane.

coordinates before they are measured. Such weights must be estimated from experimental work and theoretical considerations. In wide-angle photographs, taken from the air under operational conditions, the weight variations between the center and the edges of the image have been sometimes found to reach the relation 50:1, but 20:1 seems to be more normal relation.

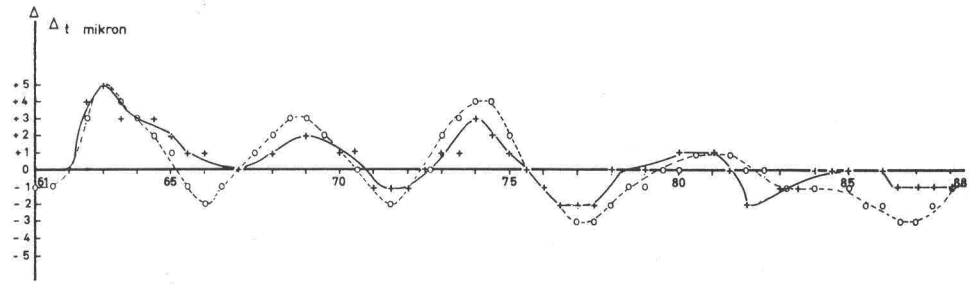



FIG. 4. Thickness variations along a diagonal of a modern film for aerial photography.

After an adjustment of measurements—for instance, the relative orientation or double and single-point resection in space—the parameters will be affected with different standard errors. These errors always should be computed from the adjustment procedure and can be determined with the aid of the laws of error-propagation. The weights are to be determined as inversely proportional to the squares of the standard errors. The theoretical distribution of the weights of coordinates and elevations within a photogrammetric model or along a triangulation strip can be expressed in formulas which are derived from the theoretical standard errors. These standard errors can always be expressed as the product between a standard error of unit weight and the square root of a weight number:

$$s_i = s_0\sqrt{Q_i} \quad \text{or} \quad s_i^2 = s_0^2 Q_i$$

The concept standard error of unit weight refers to the geometrical quality of an observation of the weight one (1) and the weight $P_i$ of a result with the weight number $Q_i$, is therefore $P_i = 1/Q_i$ because the weights are defined to be inversely proportional to the square of the standard errors. Here is the explanation of the terms: standard error of unit weight and weight number.

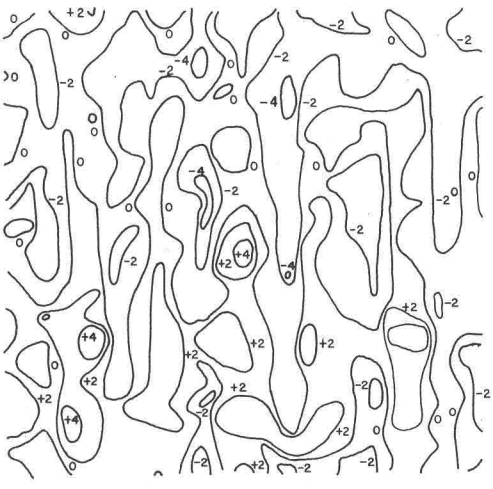


FIG. 5. Thickness variations over the surface of an aerial film.

The concepts of weights are of great importance in practice. In aerial triangulation, with auxiliary data for instance, the elements of orientation should be determined as weighted means between the information from the triangulation and from the auxiliary data. The weights of the elements of the exterior orientation, according to the triangulation and to the auxiliary data, are therefore necessary to the determination. This requires investigations of the error propagation from the fundamental operations, including the camera-calibration, the reconstruction of the bundles of rays and in particular the relative orientation.