Water Depths from Aerial Photographs

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ABSTRACT: *The apparent depth of water as measured with a photogrammetric plotting instrument is subject to a correction based on the index of refraction of the water and the angle of observation as well as the depth of the water itself. A formula is derived for determining a factor by which to multiply the observed depth in order to derive the actual value. The factor varies with the relative location of points in a stereoscopic model, as well as with the separation (overlap; air base) of the photographs and is adversely affected by wave action and other causes.*

INTRODUCTION

THE depth of objects submerged in water can be measured from vertical aerial photographs by means of a photogrammetric plotting instrument provided images of textural detail on the bottom appear in the pictures. This occurs in relatively shallow, clear waters, particularly if the minus-blue filter is removed from the aerial camera or if color film is employed. In a few exceptional instances this office has verified depths to 90-feet in coral areas, and has been able to trace accurately underwater contours to a depth of 60 feet.

However, the depths observed on the plotter are all too small and need to be multiplied by a factor in the neighborhood of 1.5. The cause for the discrepancy is the refractive index effective at the water-air surface. This is the same effect that causes a tank of water to appear shallower than it actually is, and that also causes a straight rod laying obliquely in a water basin to appear to be bent at the water surface.

Several geometric principles are evident from theoretical considerations based generally on placid water conditions, which admittedly are not encountered in practice.

1. The depth of water observed in a plotter needs to be multiplied by a factor which varies from less than 1.4 to more than 1.5 depending on the relative location of the observation in the area photographed.
2. The value of the factor also varies with respect to the relative separation of the two camera positions. Values in the neighborhood of 1.5 occur when the base-altitude ratio is 0.6.
3. The *horizontal* locations of bottom features are plotted correctly with the stereoscopic plotter.
4. The tilt of the aerial camera at the time of exposure has no direct significance.
5. The occurrence of y-parallax is not affected by the refractive conditions.
6. A difference in flight altitude of the two camera stations has an adverse effect on the above principles arising from the fact that the (air) base line is not parallel to the water surface, which is the reference plane for refractive bending. It can be shown that the discrepancy is normally not significant in the presence of more serious sources.
7. It is readily evident that swells and wave surfaces can cause adverse discrepancies arising from the fact that the reference refractive surfaces for the two camera stations are not level as they are in placid basin. This may well cause two types of discrepancies:
   - 7.1 Depths in the corners of the model may vary quite indiscriminately an average value of $\pm 8\%$ of the total depth and occasionally three times as much.
   - 7.2 Y-parallax is also created so that an erroneous orientation solution is

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obtained if images of bottom features are used in the process.

The Factor for Correcting Observed Depths

The derivation of a formula expressing the value of a correction factor is now discussed with reference to Figure 1. Aerial cameras at L_1 and L_2 are considered to be located at a height H above a water area in which images occur on both photographs of an object a at depth h in the water. The object in the water reflects an image upward meeting the water surface at c and making an incidence angle i with the line perpendicular (normal) to the water surface. The image ray is bent (refracted) at the surface, forming a refracted angle r with the same normal line in such a manner that the refracted ray remains in the (vertical) plane that contains a, c, and the normal. The image ray proceeds in a normal manner and registers an image position on photographic film.

Upon reprojection with a stereoscopic plotter, a scaled replica of the original condition is formed, except that an image ray proceeding from L is not refracted at c, but continues instead in a straight line to intersect its companion ray at b at depth h'. A factor F is sought such that $h = F h'$.

The lengths D_1, D_2 and the angles θ_1, θ_2 can be measured. Then from Section Oa, (Figure 1),

$$\tan r = D/H.$$

The formula that expresses the bending principle of Snell's Law is

$$\sin i = (1/n) \sin r.$$

It can also be shown that:

$$\tan r' = \tan r \cos \theta$$
$$\tan i' = \tan i \cos \theta.$$

If k is the distance $C_1'C_2'$,

$$h = k/(\tan i_i' + \tan i_2')$$
$$h' = k/(\tan r_1' + \tan r_2')$$

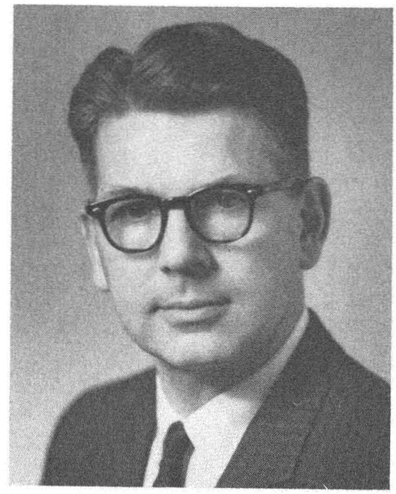
Hence the factor F is

$$F = \frac{h}{h'} = \frac{\tan r_1 \cos \theta_1 + \tan r_2 \cos \theta_2}{\tan i_1 \cos \theta_1 + \tan i_2 \cos \theta_2}$$

which is the expression sought. This is a general equation applicable to any location in any stereoscopic model.

It is significant, however, that the numerator is constant for all locations in a given model.

Figure 2 shows the theoretical values of the factor at 15 places in one-fourth of a stereoscopic model, the values being repeated over the other portions. A base-height ratio of 0.6 was used in the computation and the total length of the model was considered to be twice the base distance. It is noted that the factor varies from 1.365 to 1.538.



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The effect of the variation of these factors is shown in Figure 3 in which the errors in feet are depicted for 60 feet of water if the average value of the factor is applied. It is noted that standard error is essentially 2 feet, but that the error is systematic, causing the corners of the model to be 4.3 feet too deep and the center of the model is 2.9 feet too shallow, giving a maximum spread of 7.2 feet. An easy method for using the correct factor throughout the model consists of using a correction graph on the table [1] of the plotting instrument.

Variation of Correction Factor with Overlap

It was demonstrated during the study that a normal change in the base-height ratio has only a minor effect on both the correction factor and also the error arising from using a constant factor.

For example, in the theoretical computation, the overlap was increased 5% (60% overlap to 63%), which shortened the air base, decreased the base-height ratio from 0.60 to 0.57, but the average correction factor decreased only about 0.4%. The corresponding depths were affected only a few tenths of a foot, and the maximum spread in the error from using a constant factor was reduced only slightly—from 7.2 to 6.9 feet.

In view of the other larger effects, it is therefore concluded that the differences in overlap usually encountered in vertical aerial photography has a negligible effect on the determination of water depths of 60 feet or less, which seems to be the limit at the present state of the art.

Horizontal Location of Submerged Objects

In the theoretical placid basin conditions, the geometry of the problem indicates that the *horizontal* position of a submerged object as determined using a stereoscopic plotter is located without any error arising from the refraction phenomenon. Although the object appears to be shallower than it actually is, its apparent location is vertically above the correct location. This principle has its basis in the fact that light ray for each photograph is refracted in a plane perpendicular to the

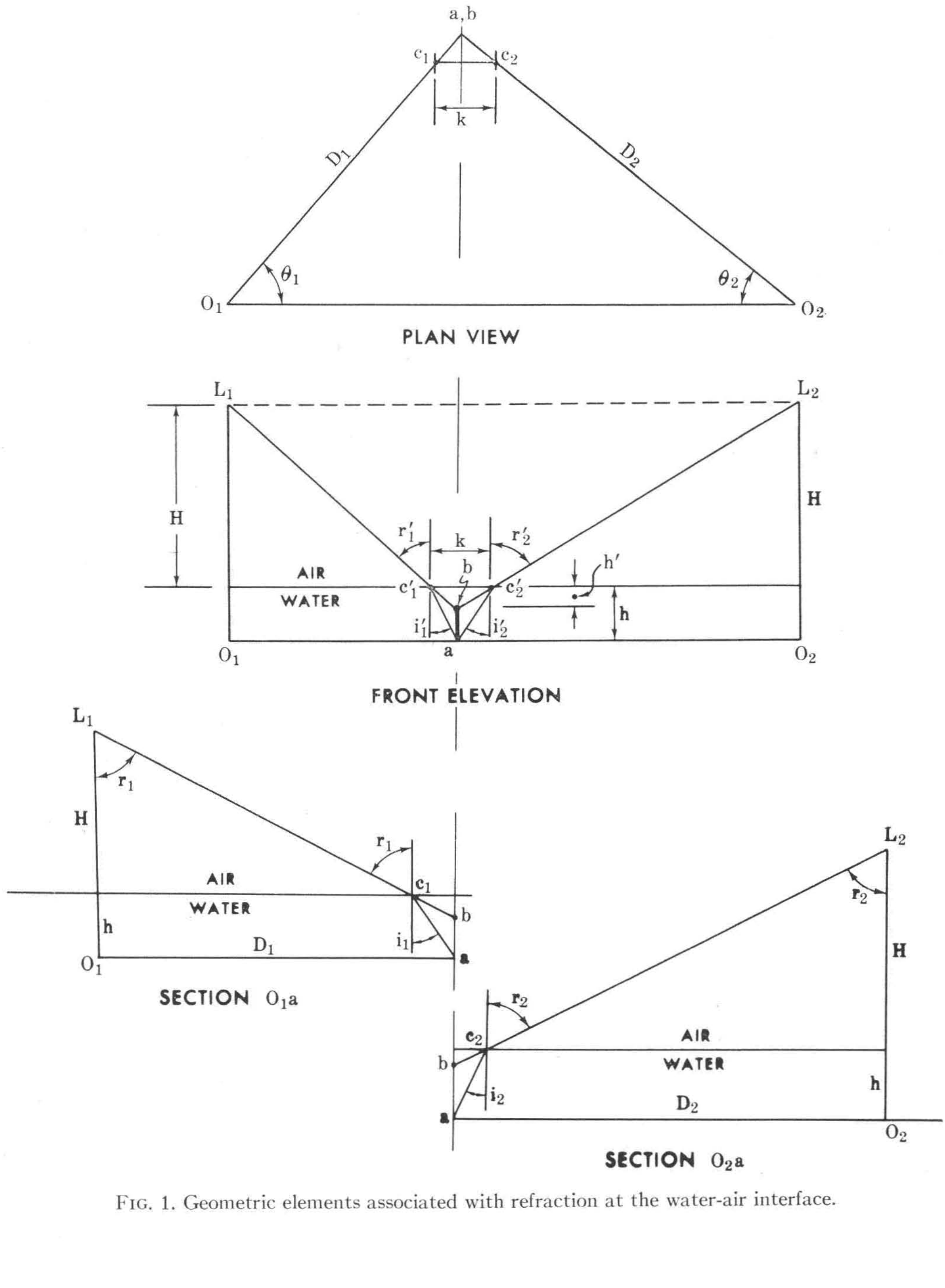

FIG. 1. Geometric elements associated with refraction at the water-air interface.

water surface, and that the two planes reintersect in a line which is also perpendicular to the surface.

Effect of Photographic Tilt

It should be readily evident from the geometry of the problem that the refractive effect is independent from the direction or magnitude of the tilt of the photography. The preliminary procedure of relative and absolute orientation of aerial photographs in a plotting instrument restores the replica cameras in the same orientation as the original cameras. Thus the re-intersection of the image rays is free from the effect of the tilt of the original cameras.

Parallax in the Stereoscopic Model

In the ideal situation of the placid basin of water and equal camera altitudes, the geometry of the problem assures one that no y-parallax is caused by the presence of this systematic water-air refraction. It can be shown in Figure 1, Sections O_1a and O_2a, that the image rays always reintersect at a common point, such as b (which is also on the vertical line through a). Inasmuch as the rays reintersect, no parallax exists.

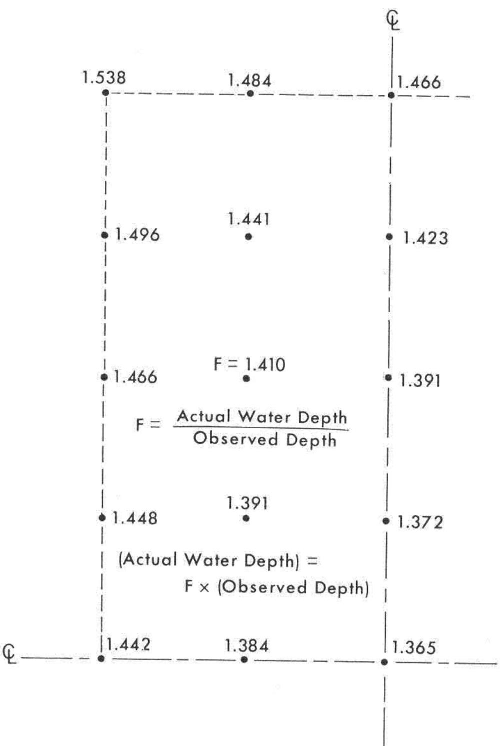

Fig. 2. Variation of value of theoretical refraction factor throughout one-fourth of the stereoscopic model.

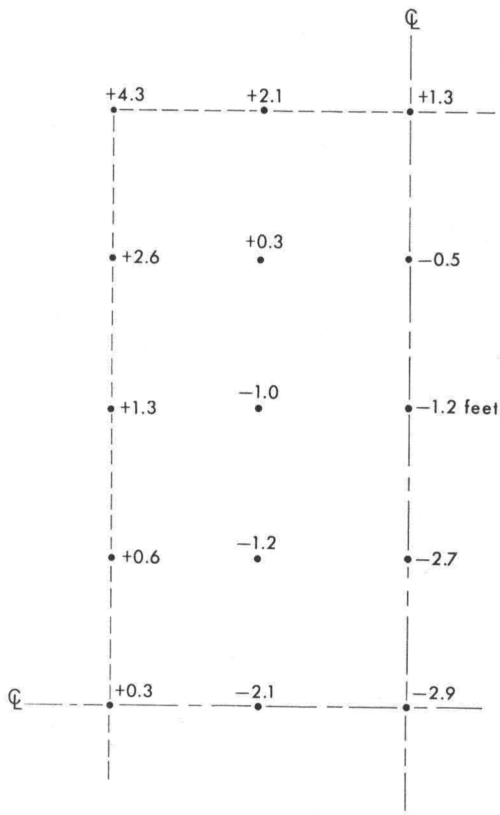

Fig. 3. Theoretical refraction errors in feet of depth in 60 feet of water resulting from using a single average correction factor for the entire stereoscopic model.

That these conditions exist can be pointed out from the geometric figure. First, inasmuch as the water surface is a level plane, the planes (Figure 1, the Sectional views) $O_1ac_1L_1$ and $O_2ac_2L_2$ are both vertical planes that intersect in a vertical line ab. Secondly, the surface (Front Elevation) $L_1L_2c_1c_2$ is a plane because the lines c_1c_2 and L_1L_2 are parallel, inasmuch as c_1 and c_2 are on the level water surface and L_1 and L_2 are considered to be at the same altitude. Consequently this plane $L_1L_2c_1c_2$ intersects the line ab in a unique point. This being the case, y-parallax does not exist from this cause.

Effect of Different Flight Altitudes

The foregoing remarks are based on the assumption that the two photographs are taken at equal flight altitudes. But this condition is only approximated in practice. The result is that the line L_1L_2 connecting the camera stations is no longer parallel to the line c_1c_2, the figure $L_1L_2c_1c_2$ does not constitute a plane, and the lines L_1c_1 and L_2c_2 intersect the line *ab* at different points *b*. The effect is similar to, but less severe than, the wave action effect discussed below.

For an altitude of 5,000 feet, it is reasonable to expect a height difference of 200 feet occasionally, with an average value of approximately 50 feet. The base length L_1L_2 at this altitude is normally 3,000 feet, whence the average difference of 50 feet represents a slope of .01667 or an inclination of essentially 1°. Inasmuch as the probable wave slope error is about 3°, and the two effects accumulate or cancel in random fashion, the combined effect is to increase the wave effect only about 3%. Because the wave effect predominates so completely, the effect of flight altitude is not discussed any further.

Effect of Surface Waves

The theoretical discussions above were based largely on the assumption, for the sake of simplicity, of a placid basin of water. Obviously this is not the case on the ocean; instead the surface is under a continually changing condition of slope that deflects the normal directions at c_1 and c_2 independently, and causes the reprojected image rays to fail to intersect at common point *b* in much the same way as for different flight altitudes discussed immediately above. It can be shown that the depth discrepancy from this cause may be expected to have an *average* value of about 8% of the total depth.

As a further consequence, if bottom features (surface water features are normally not distinguishable) are used by the operator of the stereoscopic instrument in the orientation operations, a warped and incorrectly levelled model seems likely to result. The operator orients the instrument by adjusting the projectors so that common image rays intersect. If the rays in fact do not interesect, but strong adjustments are made to force them to reintersect, some distortion of the stereoscopic model, as well as residual *y*-parallax, may be expected. The magnitude of errors is related to the depth of the water and is relatively independent from the flight altitude. On the other hand the relative magnitude of *y*-parallax *is* a function altitude and the scale of the photograph.

Sverdrup, et al. [2] and Bigelow, et al. [3], indicate the steepness of sea waves. We conclude from these data that the mean inclination of the water surface at any time is in the neighborhood of $\pm 4.5°$ in the direction perpendicular to the wave front. Inasmuch as the photogrammetric direction of the wave front is random, it is assumed that the probable deflection of the refraction normal that is as likely to occur as not at any instant $\pm 3.1°$, which varies in essentially a random manner.

Further, the occurrence of a $+3.1°$ on one ray and a $-3.1°$ on the other causes essentially a zero error. It is only when the two discrepancies fall in the same direction that a significant error results. The likelihood of this occurring is approximately $\frac{1}{2} \cdot 2^{\frac{1}{2}} = .7$, reducing the value 3.1° to 2.2°, which results in a vertical linear error of about ± 4.6 feet in a depth of 60 feet of water. It is therefore concluded that the surface movement of the water causes a discrepancy of about $\pm 8\%$ of the water depth, and that the value may be larger or smaller with about equal likelihood, and that 90% of the time the error is not expected to exceed about 13%.

It should be noted that the numerical values are not rigorous inasmuch as the elevation of the "intersection" of two non-intersecting rays is subjective relative to the operator. The non-intersecting rays have the appearance of having what is called "*y*-parallax," which tends to confuse the operator's view, and his judgement is relied on to estimate the mean value of a would-be intersection.

Simultaneous Photography

Theoretically, simultaneous photography from two aircraft would be quite ideal for determining water depths. In the first place, the water surface becomes a true stereoscopic surface in the plotting instrument useful in two ways: (1) in performing relative orientation and (2) in levelling the stereoscopic model. Therefore the model datum is not warped and depth errors are confined to the error in the image itself and free from the model warpage. Secondly, the sea surface becomes a datum surface furnishing vertical control throughout the model area. Furthermore, inasmuch as the operator could perceive the wave form of the sea surface, he could possibly arrange that his depths be observed only at those places where both image rays pierce the water surface where it is not inclined, such as in a trough.

Another advantage which may be outside the scope of this paper deals with the solar reflection (sunspot) that obscures the bottom detail in a significancy portion of aerial color photographs. If two airplanes are used, the shoreline is no longer required for vertical control, whence the direction of flight can be selected with considerable freedom so as to minimize the sunspot effect.

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[3] Bigelow, Henry B., and Edmonston, W. T., "Wind Waves at Sea Breakers and Surf," U. S. Navy Hydrographic Office Publication No. 602, 1947 (1953).
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*Photographic Interpretation Keys—A Reappraisal**

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INTRODUCTION

THOSE present at the Annual Meeting of the Society in 1955 will recall that the subject of photographic interpretation keys was not only the main topic of a panel, but was also the subject of a lively symposium. At that time, the various aspects of such keys —their concept, format, content and methods of production—held an extremely high priority. This was the apex of a period of burgeoning enthusiasm for photographic interpretation keys, that lasted for perhaps a decade, beginning in 1948.

Looking back to 1948, the concept of keys can be compared with that of a newborn baby, openly admired and cosseted, but never disparaged directly, even when secretly regarded as a moronic homunculus, incapable of useful action. In 1963, this baby has developed into a gangling, fifteen-year-old boy —frequently blasted for his mistakes, seldom praised for anything he does well—but actually performing many useful functions—shoveling snow, for example, or putting chains on his father's car.

Of late, there is a growing tendency among photo interpreters and design engineers to deprecate the need for, and value of, keys. This critical opinion has been expressed so often, that it seems time to examine the purpose, history, general trends and future directions of photo interpretation keys; to find out what has actually happened to this once-flourishing field, and whether hope exists that the program will survive the critics' barrage and attain useful maturity. In doing this, some specific charges against keys will be discussed, and some possible solutions developed which may enhance the effectiveness of future keys.

TERMINOLOGY

In the broadest sense, any interpreted and annotated (or captioned) photograph is a key —thus, a Matthew Brady photograph of General Grant, carefully studied, could have permitted later recognition of that controversial figure by one who had never seen him before. For the purposes of this paper, the definition found in the interservice Photographic Interpretation Handbook (1954) and similar to that in the MANUAL OF PHOTOGRAPHIC INTERPRETATION (1960) will obtain: "Reference material designed to facilitate rapid and accurate identification and determination of significance of objects by the photo interpreter." Note that this definition is intrinsically a broad one; it does not insist that a key be a bound volume, or even that it include graphics. The term thus admits the specialized techniques of disk, punch-card and essay keys, as well as the more common dichotomous and integrated-selective types. Furthermore, it encompasses the various

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† The opinions expressed in this paper are those of the author and do not necessarily reflect official Department of the Army policy.

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