individual is an objective of the ACIC Training Plan, the pressing demand for increased technical proficiency in specific skills frequently requires that a more direct approach be taken to immediate problems. It is fully recognized that while expedients capable of execution in a relatively short period of time may solve today's emergencies, the needs of the future require a systematic and planned approach which will result in the development of a truly professional Cartographer capable of making the most effective and timely use of the new materials and techniques which will result from technological advances.

*Upper Atmospheric Wind Determinations from Stereo-Photography of Rocket Vapor Trails**

WALTER E. BOGE,
U. S. Army Engineer, Geodesy Intelligence and Mapping, Research and Development Agency, Fort Belvoir, Va.

ABSTRACT: *A continuous altitude profile of the velocity vectors of air currents in the upper atmosphere can be obtained by the application of non-topographic photogrammetry. The method presented in this paper consists of taking time sequenced stereo-photographs of vapor trails exhausted by high-altitude rockets with photo-theodolites. By knowing the position and orientation of each terrestrial camera, the displacement vectors of various points on the vapor trail appearing on consecutive sets of stereo-photographs can be determined. An error analysis of the parameters indicates that an accuracy in wind velocity of ±5 miles per hour is attainable. The major advantage of this method is the ability to obtain a wealth of fairly accurate velocity information with considerably less cost and complexity than is necessary with present methods.*

INTRODUCTION

INCREASED information concerning the upper atmosphere is both desired and required by various branches of science. One of the properties of the earth's atmosphere about which more information is wanted is wind velocity. A number of methods already exist for obtaining information about the amount and direction of air currents for various altitudes. However, these methods do not give a continuous profile of velocity; either with respect to altitude or position on the earth. A method is presented in this paper which utilizes an application of photogrammetry to obtain a continuous profile of wind velocities. It is believed that this method can be used to produce more complete and more accurate information about the upper atmosphere.

Aside from the continuous search for more complete knowledge of the earth and its surrounding environment, there are a number of specific reasons for determining more complete information about air currents in the upper atmosphere. For the purpose of discussions in this paper, the upper atmosphere is defined as ranging from about 20 miles above the earth to the point where the atmosphere ceases to exist. Although there is

* This paper was written while the author was attending graduate school at Syracuse University under the sponsorship of GIMRADA.

no definite altitude at which the earth's atmosphere terminates, it will be assumed here that the upper limit of the atmosphere is at approximately 100 miles altitude.

Meteorologists are interested in upper atmospheric air currents with regard to their effect on weather and weather forecasting. Winds at high altitudes are known to affect the weather at the surface to some degree. The extent of this relationship is not fully known due to the limited amount of information about upper atmospheric air currents. Increased knowledge about the amount, the direction and the constancy of these air currents could possibly lead to a better understanding of their effect on weather and to more accurate weather predictions.

The recent increase in the number of tests of nuclear devices in the upper atmosphere has caused much concern about radioactive fallout. It is known that the rate, the intensity and the position of the fallout of radioactive by-products from these nuclear explosions, are, in addition to other properties, affected by the particle size, the air density, and the speed and direction of air currents. Thus an increased knowledge of upper atmospheric properties could reveal important information about radioactive fallout. In addition to the two mentioned, there exist many practical and scientific benefits to be derived from a better understanding of the air currents that exist in the upper atmosphere.

Of the existing methods for determining wind velocities in the upper atmosphere, high altitude balloons have been utilized most frequently. A plastic balloon is inflated with enough helium to lift it to a preselected altitude. The balloon's movements with time are observed and the wind velocity and direction are obtained from these observations. If the wind velocity for another altitude is desired, another balloon must be used. Thus, if any sort of continuous information is desired many balloons would be required. In addition, the capricious movements of the balloon due to air currents are difficult to trace and any degree of accuracy is difficult to attain. Another method which gives an indication of wind velocity is the observation of cloud movements. However, the altitude of the cloud must first be found in order to utilize this observational data. Also, this method cannot be used for very high altitudes because of the lack of natural cloud formations. The use of such methods as explosive grenades in test rockets and the observation of meteor trails as they enter the atmosphere have also been used in the attempt to measure upper atmospheric wind velocities, but will not be discussed here [1].

Proposed Solution

The method proposed in this paper is to determine the velocity of air currents from stereo-photographs taken of vapor trails exhausted by high-altitude rockets. As rockets ascend to and through the upper atmosphere, the hot exhaust gases expelled by the rocket engine condense in the cold air and form a cloud-like trail which follows the path of the rocket. This trail, which is larger in diameter than the rocket due to the expansion of hot exhaust gases, at first conforms exactly to the path that the rocket has taken, then is displaced from its original form by the action of air currents. Since the exhaust gases are very light (i.e. have very small mass), they are quickly accelerated to a velocity equal to that of the surrounding atmosphere.

The displacements of the condensed vapor trail caused by the movement of the surrounding air causes the original smooth trajectory curve to become irregular in shape (see photos). Since the mass of the exhausted gas is essentially constant, the displacements of the condensed vapor will be everywhere directly proportional to the wind velocity. All that is required to determine the velocity of the air at any altitude is to measure the amount of displacement that has occurred in the vapor trail during a known time interval. The displacement vector divided by the time interval then gives a value for the wind velocity. The observation of the vapor trail displacements cannot continue for extended periods of time because it eventually becomes dispersed within the atmosphere by a combination of air turbulence and molecular motion. This dispersion of the vapor trail starts immediately after its formation and continuously makes the trail less and less distinguishable from the rest of the atmosphere. The rate of dispersion is proportional to both the velocity of the air and the turbulence at various altitudes. However, the vapor trail does remain intact for at least a few minutes: long enough for velocity determinations to be made.

The displacements of the vapor trail are determined from measurements made on consecutive sets of stereo-photographs taken of the trail from two vantage points on the ground. By knowing the position of each camera station and the orientation of each camera, the x, y and z positions of various identifiable points on the vapor trail can be computed, if the photo-coordinates of these points are also known. Due to air currents, the x, y and z coordinates of the same point on the vapor trail will be different when computed from photographic measurements taken from another set of photographs exposed at a later time (for example, 30 seconds later). This change in the x, y and z coordinates of a point on the trail is the displacement caused by wind. From this displacement in distance, the amount and direction of the air currents at that altitude can be found. A more detailed description of the procedure for obtaining these displacements for various identifiable points will be given later.

Advantages

The major advantages of this particular solution to the problem are increased accuracy and considerably reduced costs. As stated previously, present methods of determining wind velocities of the upper atmosphere require special equipment for obtaining observational data. The method proposed in this paper requires only that photographs be taken of the vapor trails of rockets that are being shot for other purposes; the cost of the information producing apparatus is free. Thus, the only expenses involved for obtaining observational data are for the purchase and operation of the camera equipment which are comparatively small.

If information of a pending ballistic missile or space probe firing is known beforehand, the camera equipment can be moved to the appropriate exposure stations and be ready when the rocket test is run. If it is deemed necessary to obtain wind information for altitudes where a natural vapor trail does not form, or if special tests on different density trails are desired, this can be achieved by attaching a rider to the test vehicle which will eject a particular vapor as the rocket ascends. For instance, an apparatus for ejecting a continuous yellow sodium vapor trail can be attached to the test vehicle without interfering with the original purpose of the test. This "piggyback" operation will involve only a small cost as compared to the total cost of the rocket test.

Problems

The major problem of this method is to find points on the vapor trail that are identifiable and remain so for an extended time period. Due to the nature of the vapor trail itself, it has a nebulous cloudlike appearance which disintegrates with time. This vagueness tends to make point identification rather difficult. However, due to the long thin shape of the vapor trail, any irregularities in the original shape caused by differential air currents are easily seen. These irregularities tend to remain as the vapor trail is blown about by the wind. Figures 1, 2 and 3 show photographs taken of a rocket vapor trail at 2 minute intervals, approximately 250 miles from the trail with a four-inch focal-length camera. Observation of these photographs will show the irregularities just mentioned.

Another means of identifying points on the vapor trail are points where there is an abrupt change in the direction of the wind. This change in wind direction with elevation causes pointed barbs in the vapor trail. Finally, a factor which tends to reduce the necessity of obtaining extreme accuracy in point identification is that most of the displacements are large enough so that small errors in point identification will not have too great an effect on the wind velocity determination. An analysis of the effect of errors in point identification on the determination of wind velocity will be made later in this paper.

Cameras

In order to maintain any degree of accuracy, the cameras with which the vapor trails are to be photographed must be of precision quality. The focal-lengths should be calibrated and the lenses should have little or no distortion. There are two types of cameras available which have these characteristics and which can be set up so that the camera orientation is known to a high degree of accuracy. These are the ballistic or tracking camera and the photo-theodolite. The ballistic camera such as the Wild BC-4 and the tracking camera such as the Baker-Nunn have a higher inherent accuracy than the photo-theodolites used for terrestrial surveying. In addition they are more versatile as far as range and accuracy of camera orientation settings are concerned. However, in order to achieve the high accuracies of which they are capable, much care and expense must be spent for the provision of environmental domes from which observations are to be made [2], [3]. Thus, these cameras are not as flexible as the photo-theodolite for field usage, since they cannot be moved from location to location as readily. The narrow field of view of the tracking camera is another factor which tends to limit its use for vapor trail photography [3].

On the other hand, the photo-theodolite is much less expensive both in initial and operating costs, more transportable, easier to operate and is considered to have sufficient accuracy for this particular task. The requirements needed to maintain accuracy will be discussed under error analysis. The aspect of flexibility in transporting the photographic equipment is stressed because, as stated earlier, this problem involves the capability of moving to wherever a rocket test is to be fired. Thus, it is deemed that the phototheodolite is most suitable for photographing the rocket vapor trails. If, at some later date, an increase of accuracy is required, the more expensive ballistic camera could easily be utilized for this problem. The Wild is one of the photo-theodolites capable of taking rocket vapor trail photographs. This instrument consists of a T-2 Theodolite and a Wild terrestrial camera. The camera can be rotated around a vertical axis and tilted about a horizontal axis from +12g to −18g in intervals of 6g. The image format is 10 by 15 cm. and the nominal focal-length is 165 mm. [4].

Procedure for Obtaining Data

As an aid to understanding the following discussion on obtaining the photographic data, refer to Figure 4. The positions of the photographic cameras with respect to the vapor trail are almost completely arbitrary. The only stipulation on camera placement is that a stereoscopic image of the entire vapor trail be obtained so that the x, y and z position of any point on the trail can be found. Aside from determining the point of lowest visibility on the vapor trail, the effect of the curvature of the earth will be neglected and all arc distances are assumed to be straight lines. This is done in the interest of brevity of presentation, since the errors introduced by this assumption will be small and will have little effect on the parameter computations. For the purpose of discussing parameters, a Cartesian coordinate system having an origin on the base line will be used. If actual measurements are to be made, a geocentric or topocentric coordinate system can easily be adopted.

As stated previously, the wind velocity is to be determined for a range in altitude from about 20 miles to 100 miles above the earth. If the cameras are placed approximately 200 miles from the vapor trail (it is assumed here that at the outset the vapor trail will be nearly vertical), the lowest point on the trail which could be seen due to the earth's curvature and refraction will be

$$\frac{57\ (200\ \text{mi})^2}{5280\ \text{ft/mi}} = 4.3\ \text{miles}$$

The vertical angular coverage for the Wild photo-theodolite (10×15 cm. format, 16.5 cm. focal-length) is

$$\arctan\frac{5.0\ \text{cm.}}{16.5\ \text{cm.}} = 16.8\ \text{degrees}$$

on either side of the camera axis. Thus, if the

FIG. 1. Photograph of vapor trail—Time 1825 hours.

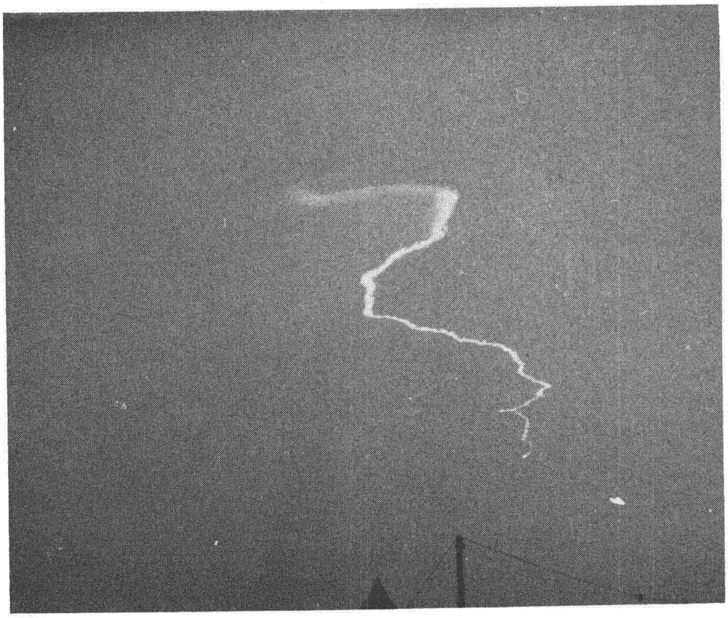

FIG. 2. Photograph of vapor trail—Time 1827 hours.

camera is given a tilt angle of +12 grads (10.8°), the camera axis will intersect the vapor trail at approximately

$$4.3 + 200 \tan (10.8°) = 42.5 \text{ miles}$$

altitude (The change of refraction with altitude angle is neglected for the parameter computations since its effect will be small.) If now the 16.8 degrees above and below the line of sight are included, the maximum elevation of the trail which could be imaged would be

$$4.3 + 200 \tan (10.8° + 16.8°) = 109 \text{ miles}$$

and the minimum would be limited by the earth's surface. In order to photograph points of higher elevation, either the camera must be moved further away from the vapor trail or given a greater tilt angle. Since the above distance chosen covers the preset altitude coverage requirements and the tilt is the maximum that can be utilized with this instrument, a closer distance between the camera station and the vapor trail would not be practical.

In order to get stereoscopic coverage of the vapor trail, the entire trail must appear in both photographs. If a base is established between the two camera stations and the cameras pointed perpendicular to this base (the normal case), there is a maximum distance that this base can have, and still retain the vapor trail image with both cameras. The geometry of stereo-photography is such that the larger the base the smaller the error in depth determination for a certain error in parallax measurement. Thus, it would be ideal to get the largest base possible. If the perpendicular distance from the base line to the vapor trail is 200 miles (y-distance) and the horizontal angular coverage of each camera on either side of the camera axis is

$$\arctan \frac{7.5 \text{ cm.}}{16.5 \text{ cm.}} = 24.4 \text{ degrees,}$$

then the maximum the base (b) could be for stereoscopic coverage is

$$2(200) \tan 24.4° = 180 \text{ miles.}$$

This produces a b/y ratio (base ratio) for the normal case of stereoscopic photography of $180/200 = .9$. However, this ratio would be an absolute maximum. A more practical base ratio, which would insure complete stereoscopic coverage, would be in the neighborhood of .6 or .7. A base ratio of .6 would require a base length of 200 (.6) or 120 miles.

It is not necessary that both cameras be pointed perpendicularly to the base line. Actually, any known camera pointings would be tolerable as long as complete stereoscopic coverage is achieved and a minimum base ratio is maintained. The normal case is being used here for clarity of presentation.

One of the important aspects of the solution of this problem is that the orientation and position of each camera be accurately known at the instant of exposure. Since the facility where the rocket test is to be fired and the planned trajectory of the missile will be known beforehand, the general location of where each camera station should be located can be determined. Within the general locations, convenient sites can be chosen for setting up the photo-theodolites. The x, y and z position of the stations can be determined by usual surveying methods. In addition an azimuth line for each station must be established. Since the trajectory that the rocket is supposed to take will be known beforehand, the azimuth to which each camera should be pointed to get maximum coverage can be determined.

Before exposures of the rocket trail are started, each camera is leveled by the plate bubbles on the instrument and the proper elevation (tilt) angle of the camera is set. Since both cameras have to be exposed at the same instant, means must be provided for communication between the two camera stations and for a master timing signal to reach each station. At either a pre-arranged time based on the scheduled firing time of the rocket or at a time established by communication between the camera operators, both cameras are exposed simultaneously at certain time intervals. The interval between exposures should be long enough to change plates in the cameras and to allow for a change in the position of the vapor trail. After a few rocket firings, experience should determine the optimum interval between exposures.

Since almost all rocket tests are performed in the early morning just before sunrise or in the early evening just after sunset, there will generally be sufficient sunlight striking the vapor trail to make it photographable. In addition, there is just sufficient darkness for stars to be seen in the background. Thus, there are usually two ways in which the orientation of the camera can be obtained. One is the method just described—leveling the instrument by the plate bubbles and getting the camera azimuth from a line of known azimuth. The second method is to use the photo-coordinates of the star background and the time of exposure to determine the camera orientation. The second method, though more accurate, involves more measurements and computations. Until shown that the second method is more beneficial, it is assumed that use of the plate bubbles will produce sufficient accuracy for this problem.

Processing Data

Once the photography of the vapor trail has been obtained, along with the times of exposures and the position and orientation data of the cameras, the job remaining is to determine the x, y and z positions of various identifiable points on each set of stereo-photographs. This can be done either instru-

Fig. 3. Photograph of vapor trail—Time 1829 hours.

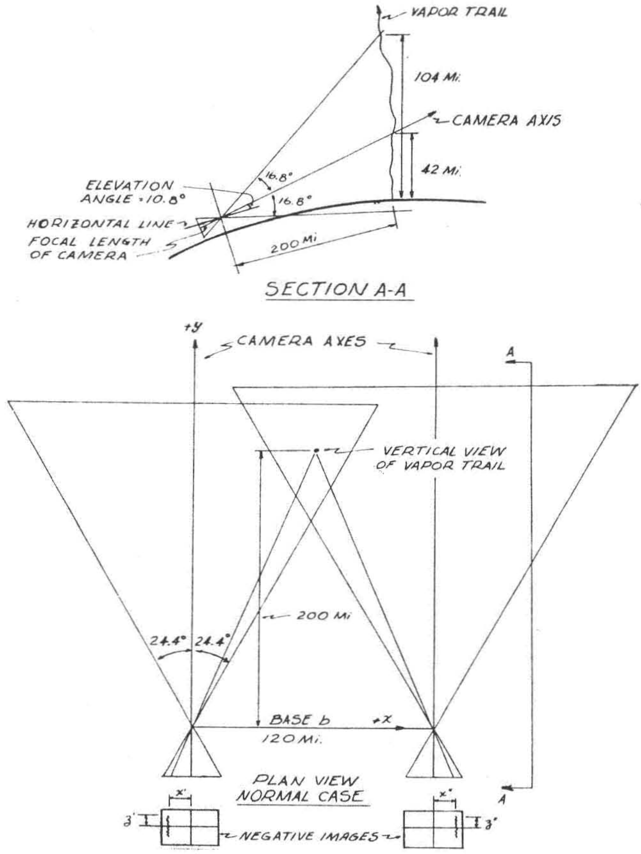

FIG. 4. Stereoscopic coverage of vapor trail.

mentally or analytically. Before either approach is taken, all the photographs should be analyzed to find all the possible points for which the coordinates can be found. Only points which can be identified in at least two sets of stereo-photos should be used. The use of stereoscopic plotting instruments to obtain the required coordinate data of the chosen points can be rather awkward if the photography is taken in other than the normal case (i.e. the cameras not perpendicular to the base), since many plotting instruments equipped to handle terrestrial photography will not accept other than normal case photography. For this situation a universal plotter such as the Wild A-7 or the Zeiss C-8 would have to be used. Since these instruments are both expensive and limited in supply, the processing of data could be delayed and be very expensive. For the normal case of photography there are a number of instruments such as the Wild A-4 which could be utilized for determining the coordinates of points on the vapor trail.

The analytical approach for determining the x, y and z coordinates of points on the vapor trail requires that the photo-coordinates of the chosen points be measured on either a mono- or stereo-comparator and substituted into a space intersection equation. This equation solves for the coordinates of the intersection of two lines in space. The direction of each line, originating from each of the camera stations and passing through the respective photographic images of the point in question, is determined from the photo-coordinates of the images, the camera orientations and the camera constants. For a few point coordinate determinations, this could be performed by hand computations. But, if the space intersection of many points are required, a digital computer is an advantageous tool.

Once a general program for the digital computer solving the space intersection equation for any one point is written, it could be used for many point coordinate solutions. Using a high speed computer to solve for the coordinates of many points appearing on a number of stereo-photographs has a number of benefits over the instrumental solution of this problem. Aside from the fact that the computer solution is faster, many additional computations can be built into the computer program that cannot be handled by plotting instruments. For example, the effects of the earth's curvature, refraction, lens distortion, etc., can be incorporated into the computer program to give more accurate results. In addition, the computer can use the calculated coordinates of points on the vapor trail to arrive at a determination of the wind velocities, all within the same program. Since the digital computer is much more flexible than any plotting instrument for determining the coordinates of the points on the vapor trail, the analytical approach is considered much superior to the instrumental solution of the point coordinates.

Error Analysis

In order to better understand the accuracy of wind velocities obtainable with this method, a hypothetical situation will be developed for which the errors will be investigated. It is assumed that a determination of the wind velocity to an accuracy of 3 miles per hour in each coordinate direction will be sufficient for any present applications. The velocity is determined from the equation $v=s/t$, where s is the displacement of the point of the vapor trail along any of the coordinate axes in the time interval t. If small errors ds and dt are assumed to exist in the measurement of displacement and time, the error in the velocity determination due to these errors will be

$$dv = \frac{ds}{t} + \frac{sdt}{t^2}.$$

(The positive sign is used to obtain the maximum error.) If the error in the time interval t is small (.01 second or less), the error in the velocity determination will almost entirely depend on the error in displacement ds. For example, if $s=5{,}000$ ft., $ds=400$ ft., $t=100$ seconds, and $dt=.01$ second, the error in the velocity will be

$$dv = \frac{400}{100} + \frac{(5000)\,(.01)}{(100)^2} = 4.00 + .005$$
$$= 4.005 \text{ ft. per sec.}$$

or 2.74 miles per hour. This is within the velocity error limit set as reasonable. As can be seen, if the error in the time interval is held to a reasonable value, it will have little effect on the velocity determination.

Based on the above, in order to hold the velocity error in each coordinate direction to less than 3 m.p.h., the error in the determination of the displacement of a point on the vapor trail should be less than 400 ft. for the 100 second time interval between stereo-photographs. Naturally, if a longer interval is used, the maximum allowable error in displacement will be increased. However, this 400 ft. error will be used as the criterion for other accuracies to be discussed.

Using Hallert's [4] notation for photogrammetric quantities, the evaluation of errors in position due to errors in measured quantities can be determined quite easily for the normal case.

By the application of geometry (refer to Figure 4), the equations for the coordinates of a point on the vapor trail in terms of measured quantities are:

$$y = \frac{bc}{p}, \qquad x = \frac{bx'}{p}, \qquad z = \frac{bz'}{p}.$$

Small errors db, dc, dx' and dz' are assumed to be present in the base b, the focal-length c, the parallax p, and the photo-coordinates of a point on one of the photographs x' and z' respectively. By differentiating, the total error in y due to these small errors is

$$dy = y\left(\frac{db}{b} + \frac{dc}{c} + \frac{dp}{p}\right)$$

Assuming that b and c can be measured quite accurately, the terms db/b and dc/c will be small compared to the term dp/p. Thus, the major portion of the error in the computation of the y coordinate is approximately equal to

$$dy = y\frac{dp}{p}.$$

The approximate value for y stated before was 200 miles. For a large base ratio the images of the vapor trail will appear towards the side edges of each photograph, thus making the value of p fairly large. For convenience, it will be taken as 100 mm. Under normal circumstances, the standard error in parallax measurements will lie in the vicinity of .01 mm. Since for this problem point identification may be difficult, three times this value or .03 mm. would appear to be a reasonable error to expect. Substituting these values into the above equation gives an error in the y coordinate of

$$dy = \frac{(200)\,(.03)}{100} = .06 \text{ mile}$$

or about 320 feet. This is within the allowable 400 foot error stated above. Using the same reasoning for the errors in the x and z coordinates, the errors in these directions will be approximately

$$dx = \frac{xdp}{p} \quad \text{and} \quad dz = \frac{zdp}{p}.$$

Since the coordinate values for x and z will be 100 miles or less, the errors in these values will be less than

$$\frac{(100)\,(.03)}{100} = .03 \text{ miles}$$

or about 160 feet. Again within the 400 foot limit. Thus, the error in wind velocity determination for each coordinate direction can be expected to be 3 m.p.h. or less, and the resultant velocity should be able to be determined to a 5 m.p.h. accuracy.

A summary of the results of this limited error analysis for normal case photography follows. Using a photographic base approximately 120 miles long located about 200 miles from the vapor trail and taking photography with Wild photo-theodolites, the resultant wind velocity can be determined to an accuracy of 5 miles per hour. This result was achieved by using what was considered by the author to be reasonable estimates of various errors that can be expected to exist. It is pointed out that estimates of errors other than those used, or measured quantities different than those assumed could produce results that are either considerably better or considerably worse than those achieved here.

Conclusions

Considering the present and future requirements for more information about upper atmospheric air currents, a need exists for the development of techniques which will produce this information. It is felt that the photogrammetric method presented in this paper can be utilized to satisfy these requirements. This method of determining wind velocities has the advantages of being inexpensive, accurate, versatile and fairly simple. In addition, if any further refinements are found to be needed, they can easily be incorporated into the method presented. Based on the use of photo-theodolites, simple mono-comparators, and a digital computer program; the limited error analysis performed, indicates that continuous profiles of the velocities of air currents in the upper atmosphere can be determined to an accuracy of ± 5 m.p.h. Either the development of a new computer program for handling computations or the adaptation of an existing program would certainly be desirable before attempting to determine a large number of wind velocities using this method. However, for experimental purposes, the use of hand computations would not be too burdensome in order to test the validity of using stereo-photography of rocket vapor trails for obtaining better information about upper atmospheric air currents.

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