TABLE III

INTENSITY MEASUREMENTS OF LIGHT PROJECTED THROUGH *Transparent Glass Plates*

Point No.	*Light Intensity on Platen—Foot Candles*					
	Projector No. 1—Light Set High			*Projector No. 2—Light Set Low*		
	A *Blue-Green Filter*	*B* *Polaroid Filter*	*B/A*	*C* *Red Filter*	*D* *Polaroid Filter*	*D/C*
1	1.30	2.25	1.73	1.13	1.75	1.55
2	1.93	3.00	1.55	2.23	3.50	1.57
3	2.00	3.00	1.50	2.75	4.08	1.48
4	1.75	2.50	1.43	2.20	3.18	1.45
5	1.13	1.53	1.35	1.38	1.95	1.41
6	1.93	3.75	1.94	1.00	1.43	1.43
7	2.95	5.45	1.85	1.75	2.75	1.57
8	3.38	5.75	1.70	2.13	3.45	1.62
9	2.63	4.45	1.69	1.60	2.80	1.75
10	1.48	2.58	1.74	1.10	1.93	1.75
11	1.88	3.50	1.86	0.58	0.75	1.29
12	3.38	6.25	1.85	0.95	1.58	1.66
13	4.13	7.30	1.77	1.25	1.95	1.56
14	3.28	5.58	1.70	1.10	1.83	1.66
15	1.25	2.55	2.04	0.70	1.43	2.04

The equipment used in this test was not highly precise and the data shown are only roughly approximate. At very low readings, estimated interpolations between divisions on the meter scale undoubtedly caused rather high percentages of error. Nevertheless, for light intensities of over ½ foot candle, the data are considered sufficiently reliable to indicate a fair comparison between color and polaroid filters.

In general, this test showed that considerably more usable light is made available on the platen with polaroid filters than with color filters.

Determination of Weights of Parallax Observations for Numerical Relative Orientation

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(*Abstract is on the next page*).

INTRODUCTION

THE consideration of weighted parallax observations in determining the relative orientation of a photogrammetric model has not so far been undertaken by the photogrammetrists very seriously. Only some stray work has been done in this respect. The author is aware of the method of Jerie[1] which is aimed at giving a comparatively more precise solution of the ω-tilt in a mountainous model. The next step towards a more rational solution of the problem was given by Kasper,[2] who assumed that the parallaxes (observations) in the picture-plane have the same weight everywhere for a particular model. This consideration is a nearer approach to reality for the Wild-and Santoni-type instruments where the movements of the measuring marks are in planes parallel to the pictures. However, Kasper's views cannot be considered as universal and in no way the final in this respect. The author, under the able guidance of Prof. Dr. Brandenberger, has given a considerable amount of thinking to this problem.

ABSTRACT: *This paper is a report on the theoretical investigations leading to the determination of weights for parallax observations for numerical relative orientation of a photogrammetric model. The geometry of the formation of a stereo-model and the physical aspects that are relevant are considered. A natural conclusion is also drawn with practical example.*

In trying to find out the correct approach to this problem, the author observes that the weight of an individual parallax observation depends on the following relevant factors:

(a) The attitude of intersection of the individual rays coming from the two corresponding pictures, at the point of observation.

(b) The obliquity of the epipolar plane through the point of intersection (on which the two intersecting rays lie in reality) with respect to the horizontal plane (along which the parallaxes are observed).

(c) Change in the scale of the detail and thereby the change in the dimension of the measuring mark with respect to the detail at the particular location of the point in the individual pictures, and thereafter, jointly at different locations in the model. This is more important in a mountainous model.

(d) Different photographic resolution and other image qualities at different points in each picture and ultimately at different locations in the model.
All of the above four factors can be precisely determined from the location of the point of observation in the model with respect to the individual pictures (or, cameras in the restitution instrument).

(e) The restitution instrument and its peculiarities.

(f) The operator and his personal observational capabilities.

While we have yet to come to any conclusion regarding the instrument and the operator, the other items are undoubtedly the most important ones. The author will try to analyse each of the relevant factors and finally formulate the weights due to each as well as their joint effect. Only vertical or near-vertical photography will be considered, assuming that the two air-stations are at the same flying height. Concerning signs, notations, location of the points in the model, etc., used in this paper, see Figures I and II.

DETAIL CONSIDERATIONS

A. WEIGHT DUE TO DIFFERENT ATTITUDES OF INTERSECTION OF THE RAYS

From the much-accepted principles of intersection of two rays in Geodesy (Land Surveying), the weight of an intersection is given by (see also references nos. 3 and 4)—

$$g = \frac{\sin^2 \gamma}{\alpha^2 + \beta^2}$$

where

g is the weight,

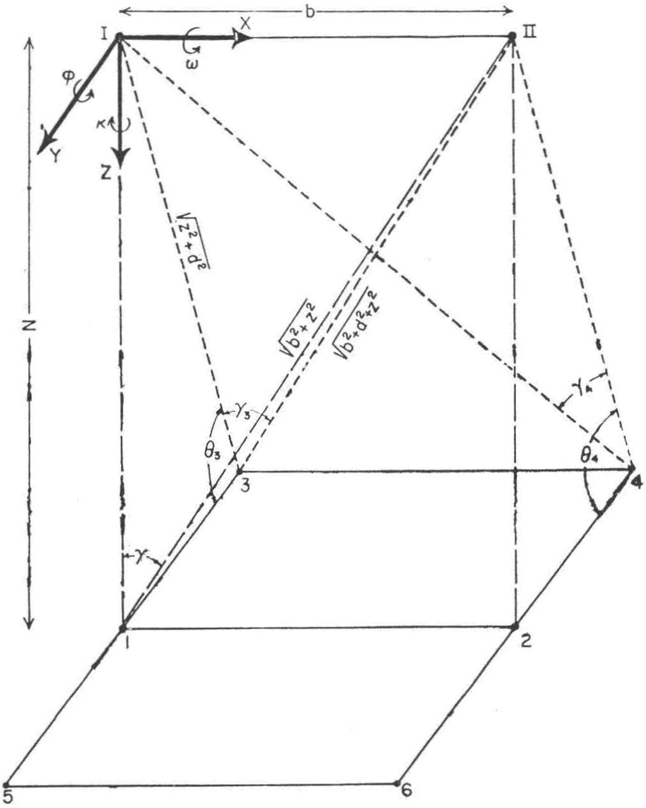


FIG. I

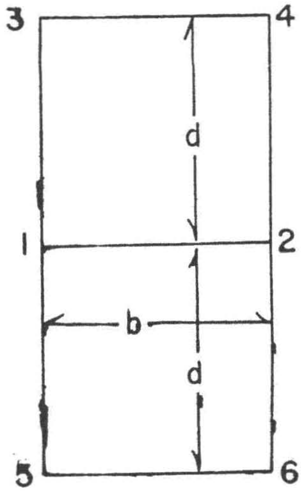


FIG. II

γ is the angle of intersection,

α, β are the lengths of the two intersecting rays.

With the help of the above formula, in the photogrammetric model, the weight at any point becomes

$$g = \frac{\sin^2 \gamma}{d^2 + Z^2 + b^2 + d^2 + Z^2} = \frac{b^2}{(b^2 + d^2 + Z^2)(2d^2 + b^2 + 2Z^2)}$$

where

b is the model base,

d is the distance of the point from base,

and

Z is the distance of the plane of projection from the projection center.

The above expression gives the general form of such weights. As an example, considering standard photography on flat terrain, where $b/Z = \frac{2}{3}$ and also considering $b = d$ (these are very common in practice) we obtain the following weights:

At points 1 and 2 $\cdots$ $0.056/b^2$

and at points 3, 4, 5 and 6 $\cdots$ $0.031/b^2$

For the sake of standardization, assuming unit weights at points 1 and 2, we get: when

$$g_1 = g_2 = 1.0$$

$$g_3 = g_4 = g_5 = g_6 = 0.55$$

B. WEIGHT DUE TO OBLIQUITY OF THE EPIPOLAR PLANE

In the optical projection type instruments, the obliquity and the disadvantage due to it in the observations is apparent as the parallaxes are measured always along the horizontal plane with the help of horizontally placed (apparently horizontal in some instruments) measuring marks. Even in the Wild or Santoni instruments the obliquity comes into consideration because the picture is constituted of details projected in perspective and thus apparently distorted with respect to the measuring mark, which always stays parallel to the plane of the picture. Thus the obliquity of the epipolar plane has a certain direct influence on the precision of the measurement of parallax at a point. The weight of the parallax-observation due to this should vary directly as the sine of the angle of obliquity. The amount of obliquity can always be computed very easily. As an example, considering standard photography and also $b=d$, the obliquity at points 1 and 2 is 90 degrees and $\sin 90° = 1.0$ whereas for points 3, 4, 5 and 6, the obliquity is given by θ where

$$\tan\theta = \frac{Z}{d} = \frac{Z}{b} = \frac{3}{2} \quad \text{and} \quad \sin\theta = 0.83$$

gives the weight directly.

C. WEIGHT DUE TO THE CHANGE IN THE SCALE OF THE DETAIL, ETC.

With vertical photography, if the terrain is flat, there is no change in the scale of the details in the picture. There is then no problem. The problem arises only when the terrain is mountainous. Then nearer to the camera-station a detail-point is at the moment of exposure, the larger is its scale in the picture. Larger scale means comparatively better appreciation of the parallax difference. Thus in a mountainous model the weight should depend on the Z distance of the individual point, and it should be proportional to the reciprocal of the individual Z distance. In normal terrain with relief about 10% of the flying height, the weight will, thus, be within 1.0 and 0.9.

D. WEIGHT DUE TO PHOTOGRAPHIC RESOLUTION

This is a very difficult item, inasmuch as it varies from camera to camera, depending on the photographic material used. Atmospheric haze, and refraction, roll, pitch, vibration and motion of the aircraft also have some effect in this respect.

The photographic resolving-power directly influences the precision of the observation of individual points. Considering the fact that in relative orientation we are concerned with parallaxes, which are nothing but the differences between the coordinates of the same point from the two cameras (e.g., y-parallax $= {}^{y}p_{\text{I}} - {}^{y}p_{\text{II}}$), the weight of an observed parallax will be directly proportional to the difference of the resolutions at the particular point. Here the algebraic sign of the difference of the resolutions is of no significance and the weights are considered without any regard to the sign.

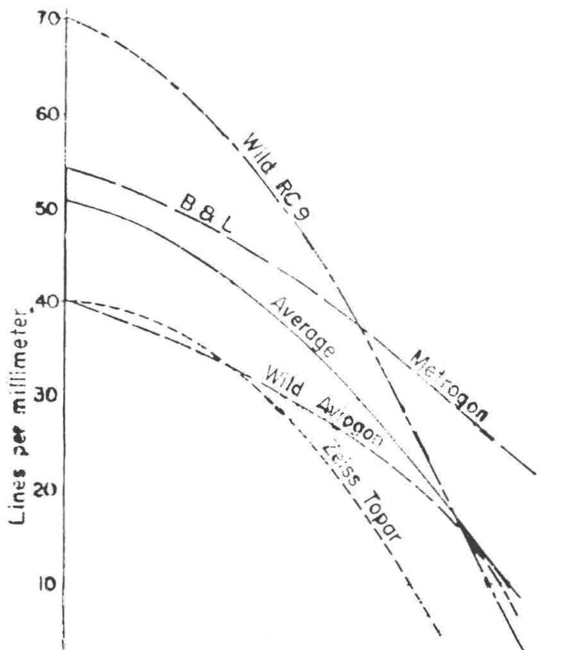
As an example considering the resolving power of a typical wide-angle lens under operational conditions (viz., Wild Aviogon, f = 115 mm., 1: 5.6; also see graph I showing average resolution against field angle), assuming standard photography on a flat terrain ($b/Z = \frac{2}{3}$) and $b=d$, photographic resolution at the individual points will then be as follows:

From the left side camera, (I)		From the right side camera, (II)	
field-angle for point 1	$=0°$	field-angle for point 2	$=0°$
field-angle for points 2, 3 and 5	$=33°$	field-angle for points 1, 4 and 6	$=33°$
field-angle for points 4 and 6	$=42°$	field-angle for points 3 and 5	$=42°$

From these, with the help of Graph I, we obtain the following resolutions from individual pictures and the resulting weights for the observed parallaxes at the respective points:

Point in Model	*Resolutions from*		*Weights equalling to the difference, I–II, disregarding sign*	*Weights, standardized, assuming unit weights at points 1 and 2*
	Camera I	*Camera II*		
1	40	24	16	1.00
2	24	40	16	1.00
3	24	15	9	0.56
4	15	24	9	0.56
5	24	15	9	0.56
6	15	24	9	0.56

Other significant factors determining image qualities are lens-distortion, shifting of emulsion and shrinkage of the picture base. Regular radial distortion due to the taking lens is corrected in all precise instruments by one way or the other and is



GRAPH I (Showing resolving powers)

thus left out of the present study. It has then no practical effect on the parallax observations. Shifting of emulsion, if and when it exists, is very irregular and is beyond the scope of any consideration of systematic differences. Regular shrinkage of the photo-material is taken care of by changing the effective principal distance in the restitution instrument. Actually, when more precision is desired of a numerical relative orientation, one should always use material having the least possible distortion (e.g., glass plates). In reality, a large amount of regular distortion tends to change the entire cone of the projected image and in that case relative orientation yields a model with considerable distortion.

FINAL WEIGHT

It is extremely difficult to determine the order of importance of the four major factors discussed above. Further, it is of no practical significance to determine the relative importance amongst them. Thus a general average of the four weights is considered to be appropriately the total effective weight for each parallax observation at a point. As an example, as was being considered all throughout the present study, in the case of standard photography on a flat (or very nearly flat) terrain with a model where $b=d$, the final effective weights are given by:

Points	*Weights due to factor*				*Final average Weight (standardized)*
	A	*B*	*C*	*D*	
1	1.0	1.0	1.0	1.0	1.0
2	1.0	1.0	1.0	1.0	1.0
3	0.55	0.83	1.0	0.56	0.74
4	0.55	0.83	1.0	0.56	0.74
5	0.55	0.83	1.0	0.56	0.74
6	0.55	0.83	1.0	0.50	0.74

Finally, for the numerical relative orientation with weighted parallax observations, we may consider the following general equation:

$$v = \Delta by_{\mathrm{I}} - \frac{Y}{Z}\Delta bz_{\mathrm{I}} + X\cdot\Delta\kappa_{\mathrm{I}} - Z\left(1+\frac{Y^2}{Z^2}\right)\Delta\omega_{\mathrm{I}} + \frac{XY}{Z}\Delta\phi_{\mathrm{I}} - \Delta by_{\mathrm{II}} + \frac{Y}{Z}\Delta bz_{\mathrm{II}}$$
$$- (X-b)\Delta\kappa_{\mathrm{II}} + Z\left(1+\frac{Y^2}{Z^2}\right)\cdot\Delta\omega_{\mathrm{II}} - \frac{(X-b)}{Z}\cdot\Delta\phi_{\mathrm{II}} - p_y$$

where V is correction, and p_y is the parallax.

Then the weighted working correction will be given by:

$$\sqrt{P}\cdot v = \sqrt{P}\cdot\Delta by_{\mathrm{I}} - \sqrt{P}\cdot\frac{Y}{Z}\cdot\Delta bz_{\mathrm{I}} + \sqrt{P}\cdot X\cdot\Delta\kappa_{\mathrm{I}} - \sqrt{P}\cdot Z\cdot\left(1+\frac{Y^2}{Z^2}\right)\cdot\Delta\omega_{\mathrm{I}}$$
$$+ \sqrt{P}\cdot\frac{X\cdot Y}{Z}\cdot\Delta\phi_{\mathrm{I}} - \sqrt{P}\cdot\Delta by_{\mathrm{II}} + \sqrt{P}\cdot\frac{Y}{Z}\cdot\Delta bz_{\mathrm{II}} - \sqrt{P}\cdot(X-b)\Delta\kappa_{\mathrm{II}}$$
$$+ \sqrt{P}\cdot Z\cdot\left(1+\frac{Y^2}{Z^2}\right)\Delta\omega_{\mathrm{II}} - \sqrt{P}\cdot\frac{(X-b)Y}{Z}\cdot\Delta\phi_{\mathrm{II}} - \sqrt{P}\cdot p_y$$

where P is the weight.

The correction to the individual orientation elements will then follow directly from the normal equations formed on the basis of these observation equations.

CONCLUSION

It is has been found from practical experiments in the laboratory with fifteen different models in various scales and in various types of terrain that the weights as arrived at in the studies above give very satisfactory results in numerical relative orientation of the models using y-parallax observations.

However, it will not be out of place to mention further that with these considerations a numerical relative orientation of a model becomes slightly complicated. This remark holds also for the formulas. An experienced practitioner automatically weighs the y-parallaxes in performing an optical-mechanical (or call it empirical) relative orientation in normal practice.

For further theoretical considerations, the above ideas could be extended to nine, fifteen or more number of points in the model in case a more precise relative orientation is aimed at.

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Projective Nets Corrected for Radial Distortion for Graphical Rectification in Aerial Photogrammetry

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I. INTRODUCTION

RECTIFICATION in aerial photogrammetry, as defined by Hallert [1], implies that an image, photographed by a non-vertical camera over a plane terrain (not necessarily horizontal), may be transferred onto a map, i.e. an orthogonal projection of the terrain on a certain scale in a horizontal plane. Rectification may be performed with numerical, graphical, or optical-mechanical methods.

Methods of graphical rectification are based on constructing two projective nets in map and image. In the map the net is, generally, chosen of regular figures, e.g. squares [1, p. 107], or parallelograms [2, p. 357]. This net is then transferred to the image, which is assumed to be a strict central projection, i.e. free from distortion. More accurate results may, therefore, be obtained if the above process is corrected for distortion, which is mainly radial.

In this paper, two projective nets in map and image are so selected, that they can be corrected for the radial distortion. In the image, the net is taken to consist of rays passing through the principal point H', and concentric circles about H'. The corresponding net in the map consists then of rays through the point H (which corresponds to H') and of ellipses. If the image is considered as a strict central projection, these ellipses will then be the orthogonal projection, on the horizontal plane of the map, of the ellipses of intersection of the plane terrain with the coaxial cones of revolution, whose common axis is the camera axis and whose bases are the concentric circles in the image. This net is corrected for the radial distortion by displacing these ellipses in the map accordingly, as will be shown in IV. The method is here applied to the simple case of an oblique image and a horizontal plane terrain. Numerical data are assumed and the results are represented graphically. The method is, however, applicable to more general cases.

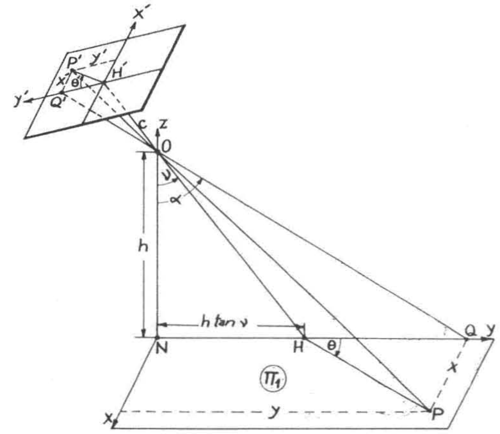

FIG. 1