*Film Stability Investigation**†

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ABSTRACT: *This study was initiated to investigate the errors in photographic coordinates resulting from random film shrinkage. The research included calibrating a base grid, exposing the base grid on two types of film, measuring the shrinkages of the film at different ages from development and reducing the data analytically. It was found that the magnitude of random film shrinkages was similar, both throughout a given photograph and along a given roll of film.*

On Type 1A—Class A Kodak Plus-X Aerographic Film (Acetate Butyrate Base) the random film shrinkages for coordinate values of 2 inches and 4 inches were ±3.6 microns and ±8.6 microns respectively. It was also found that random film shrinkages on Kodak special Plus-X Aerial Film (Estar Base) SO-135 for coordinate values of 2 inches and 4 inches were ±4.4 microns and ±4.6 microns respectively. The two standard deviation level is given in the above figures.

INTRODUCTION

THIS study was conducted to determine the systematic and random film shrinkages present in aerial film. Type 1A-Class A Kodak Plus-X Aerographic Film (Acetate Butyrate Base) was selected for the majority of the work, however, a limited investigation was also conducted with Kodak Special Plus-X Aerial Film (Estar Base), SO-135.

Many measurements were taken and it was possible to compare film shrinkage values between different areas on a given exposure, between different exposures along a given film strip, and between corresponding values for three different ages of the film. The film age study applied only to the Type 1A film.

PROCEDURES

All film shrinkage measurements were performed by experienced operators on a Mann Comparator Type 422C, which has a least reading of 0.001 mm. The comparator was housed in a temperature and humidity controlled room.

A calibration of a base grid imaged on a one-quarter inch by 9 inch by 9 inch glass plate was performed in order to provide a precise reference system which would be readily available for film shrinkage determinations under various conditions. The reference system was a one-half grid whose x and y axes are lettered A to Q and 1 to 17 respectively. The relative coordinates of all intersections were determined from nine sets of readings made independently by three operators. The grid was calibrated over a given region of the comparator x and y screws. Therefore, if subsequent plates are oriented over the same regions of the screws, any undetected errors in the comparator due to such factors as non-perpendicularity of the comparator axes, weave of the ways, periodic errors, and secular errors would not affect the film shrinkage results.

The exposing and processing of the film during this study were controlled closely in

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order to qualify the results. The calibrated plate was exposed on the film, emulsion to emulsion, using a LogEtronic Printer to insure uniformity. Particular attention was given to film flatness. No artificial heat or automatic dryers were used in drying the film. Twelve exposures of the calibrated grid were made on each of the two types of film. The film strips were approximately 65 feet long with the first exposure taking place five feet from the beginning of the strip. The remaining exposures were four feet apart with the last exposure being approximately five feet from the end of the film strip. As with the film, extreme caution was used throughout the exposure and processing of the plates. The processed film was then exposed, emulsion to emulsion, on one-quarter inch photogrammetric glass plates. Again the film was placed flatly against the plate and the LogEtronic Printer was used to insure a uniform exposure. The plates were allowed to dry without artificial heat. The developed film was stored in the photographic laboratory with a temperature of $73° \pm 2°$ and a relative humidity of $50\% \pm 5\%$. A summary of the film and plate preparation is given in Figure 1.

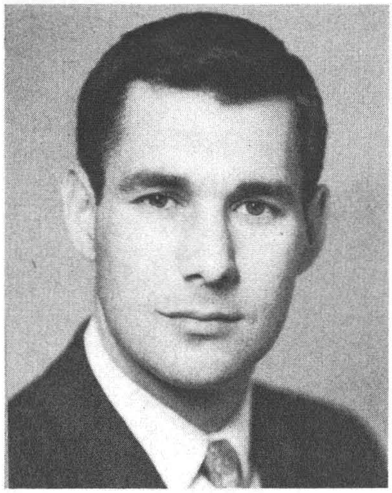
During this study two basic approaches were used in determining the systematic and random film shrinkages. One solution was the brute force approach where many measurements were made over the entire 9 inch by 9 inch plate. The other approach was to make a much smaller number of measurements, taking advantage of the method of least squares, to give a best fit solution. Both methods were used and the results were very similar. Only the later approach will be discussed in this paper.

Figure 2 shows the 26 intersections that were measured on each film shrinkage plate. Each of two operators made two complete passes thru each plate giving four independent readings for each intersection. The grid lines appeared white on a dark grey background, and the reticle used was a doublelined black cross.

The film shrinkage computations were performed over six different regions of each of the 16 plates. Referring to Figure 2, these regions are given as the entire plate, quadrant I, quadrant II, quadrant III, quadrant IV, and subquadrant I.

The following information was determined from each computation.

a) systematic film shrinkage in x, width of film (dV_A)



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b) systematic film shrinkage in y, length of film (dV_B)
c) standard error of dV_A ($\pm mdV_A$)
d) standard error of dV_B ($\pm mdV_B$)
e) error propagation for a typical Δx due to errors in dV_A, dV_B, $d\alpha$ ($\pm md\Delta x$)
f) error propagation for a typical Δy due to errors in dV_A, dV_B, $d\alpha$ ($\pm md\Delta y$)

Refer to APPENDIX A for the derivation of the film shrinkage computation.

RESULTS

The majority of the measurements were made on Type 1A—Class A Kodak Plus-X Aerographic Film. Twelve plates of this type were measured and 72 individual adjustments were computed.

It was first determined whether or not the film shrinkage values varied substantially between different areas of any given exposure. The average shrinkage values for all 12 plates are listed in Table 1.

The random shrinkages in x ($\pm mdV_A$) and y ($\pm mdV_B$) varied as much between themselves in a given area as they did between different areas; however the magnitude of film shrinkage was generally consistent over the entire $9'' \times 9''$ format for the above type film. The values $\pm md\Delta x$ and $\pm md\Delta y$ represent the error propagation in x and y respectively over the specified intervals. These intervals may be thought of as distances between reseau intersections, and a curve can be drawn based on the average of these values. See Figure 3. All indications suggest that

1A

One contact print of precision grid exposed on $\frac{1}{4}''$ glass plate. This plate was calibrated.

1B

Twelve exposures of calibrated grid (*1A*) made on A 65′ roll of Type 1A—Class A Kodak Plus-X Aerographic Film. (Acetate Butyrate Base) Film exposed and developed on same day.

1C

Exposures 1, 4, 8, 11 on Type *1A* film above (*1B*) were printed on $\frac{1}{4}''$ glass plates. Age of film from development was 11 days.

1D

Exposures 1, 4, 8, 11 on Type *1A* film above (*1B*) were printed on $\frac{1}{4}''$ glass plates. Age of film from development was 45 days.

1E

Exposures 1, 4, 8, 11 on Type *1A* film above (*1B*) were printed on $\frac{1}{4}''$ glass plates. Age of film from development was 109 days.

2B

Twelve exposures of calibrated grid (*1A*) made on a 65′ roll of Kodak Special Plus-X Aerial Film (Estar Base), SO-135.

2C

Exposures 1, 4, 9, 12 on SO-135 film above (*2B*) were printed on $\frac{1}{4}''$ glass plates. Age of film from development was 7 days.

FIG. 1. Summary of film and plate exposures.

a grid reseau with one inch spacing would be sufficient to reduce the film shrinkage error propagation to less than one micron at the 68% level.

The film shrinkage values of Table 1 are the averages pertaining to three different film ages. In Table 2 it can be seen that age has a significant effect on the film shrinkage values. The error propagation in x ($\pm md\Delta x$) and y ($\pm md\Delta y$), as well as the standard error of unit weight, is reduced considerably at the age of 45 days; however, the values do not improve at the age of 109 days. In fact most of them tend to increase slightly.

The aging effects on systematic shrinkage are shown in Figure 4.

There has always been some question as to how consistent the film shrinkage is along a given roll of film. Table 3 lists the shrinkage values of four plates scattered along a roll of film. These figures are the averages of the results obtained for the film ages of 11, 45, and 109 days.

Overall there was very little film shrinkage difference between random exposures along a given roll of the above type film. In individual circumstances there were spreads in either $\pm mdV_A$ or $\pm mdV_B$ of as much as four microns between plates but these instances are very rare. When the figures in Table 3 were broken down into age classes the results were very similar.

The effects of film aging are summarized in Table 4.

By referring to Table 4 it can be seen that the values for $\pm mdV_A$, $\pm mdV_B$, and the standard error of one unit weight ($\pm\mu$) improved considerably between the ages of 11 and 45 days. At age 109 days these values appear to be getting larger; quite possibly a permanent aging effect is beginning at this point. Thus there are strong indications that more stable measurements can be obtained from this type of film if it has been allowed to age approximately 45 days after development.

A limited study was also conducted on Kodak Special Plus-X Aerial Film (Estar-Base), SO-135. Four plates were measured and 24 adjustment computations were performed. Refer to Table 5 for the results for different areas of the plate.

The error propagation as a result of random film shrinkage is shown plotted in Figure 3. The curve appears to be leveling off at approximately ± 2 microns, and a reseau with one inch squares would give shrinkage errors of this magnitude.

A comparison of Tables 2a and 5 reveals

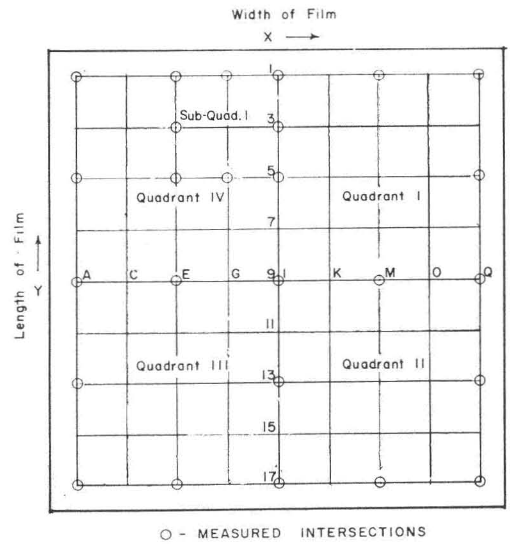

FIG. 2. Measured intersections and study areas for each plate.

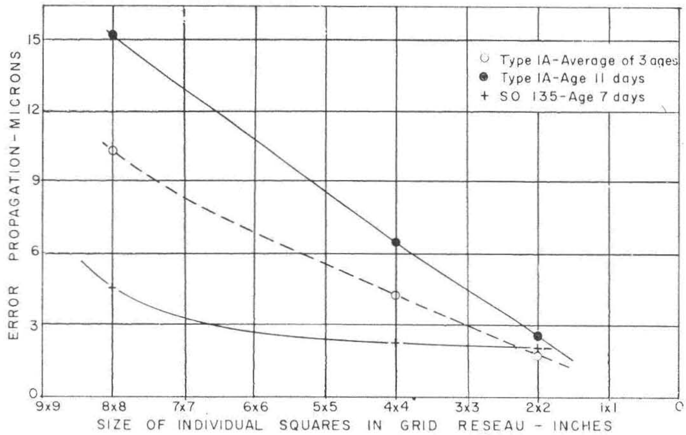

FIG. 3. Film shrinkage error propagation.

significant differences in the error propagation values ($\pm mdV_A$ and $\pm mdV_B$) between the Type 1A (Acetate Butyrate Base) film and the SO-135 (Estar Base) film. The SO-135 film has a decided advantage unless reseau photography is used, and the reseau spacing is at least as small as two inches.

Table 6 indicates the consistency of shrinkage along a given roll of SO-135 (Estar Base) film.

A comparison of Tables 6 and 7 show that the percentage-wise consistency of the random shrinkage along a roll of film is slightly better for the Type 1A film than it is for the SO-135 film. This is true between the different exposures as well as between the x and y coordinates.

CONCLUSIONS AND RECOMMENDATIONS

The following conclusions are based on Type 1A—Class A Kodak Plus-X Aerographic Film (Acetate Butyrate Base) processed and measured under the stated conditions.

1. The random shrinkages appear to be slightly larger across the width of the film. ($\pm mdV_A$)

2. The random film shrinkage values are generally consistent throughout the 9″×9″ format.

3. The random errors in the above type film decreased when approximately 45 days had elapsed between developing and photographic printing. Further studies would be necessary to determine whether this is generally true or simply a characteristic of this experiment.

4. A grid reseau with a one inch spacing would be sufficient to reduce the film shrinkage error propagation for any coordinates on the photograph to less than ±2 microns at the 95% level.

5. The random film shrinkages for different exposures on a given roll of film are similar. This conclusion is the same for all film ages.

6. The magnitude of systematic change for this roll of film decreased very rapidly during the first three months of age. Affined changes

TABLE 1

AVERAGE FILM SHRINKAGE VALUES FOR VARIOUS AREAS OF THE EXPOSURE FORMAT
TYPE 1A—CLASS A KODAK PLUS-X AEROGRAPHIC FILM

Area of Plate	dV_A *microns /in.*	dV_B *microns /in.*	$\pm mdV_A$ *microns /in.*	$\pm mdV_B$ *microns /in.*	$\pm md\Delta x$ *microns*	Δx & Δy *interval in inches*	$\pm md\Delta y$ *microns*	$\pm\mu$-*std. error of unit wt.*
Entire Plate	51.1	46.6	2.21	1.38	11.70	8	8.83	14.09
1st Quad	50.2	48.0	1.44	1.09	3.21	4	3.66	4.71
2nd Quad	50.5	46.8	2.35	1.82	5.67	4	5.66	7.87
3rd Quad	51.2	45.9	1.68	1.26	3.09	4	3.68	4.53
4th Quad	51.0	47.1	3.02	1.51	4.93	4	4.67	6.55
Sub-Quad 1	51.5	48.1	2.05	1.50	1.83	2	1.82	2.50

TABLE 2

AGING EFFECTS ON FILM SHRINKAGE VALUES FOR VARIOUS AREAS OF THE EXPOSURE FORMAT TYPE 1A—CLASS A KODAK PLUS-X AEROGRAPHIC FILM

Area of Plate	dV_A *microns /in.*	dV_B *microns /in.*	$\pm mdV_A$ *microns /in.*	$\pm mdV_B$ *microns /in.*	$\pm md\Delta x$ *microns*	Δx & Δy *interval in inches*	$\pm md\Delta y$ *microns*	$\pm\mu$-*std. error of unit wt.*
a) *Film Age—11 Days from Development*								
Entire Plate	72.1	61.0	3.02	1.77	18.05	8	12.25	20.02
1st Quad	68.8	62.0	1.66	1.70	5.88	4	7.88	9.12
2nd Quad	70.4	61.9	3.10	2.15	7.40	4	8.98	11.32
3rd Quad	71.0	61.6	1.95	1.84	3.82	4	4.48	5.82
4th Quad	69.8	61.5	3.18	2.04	7.58	4	6.42	9.62
Sub-Quad 1	69.6	64.3	2.28	2.36	2.45	2	2.92	3.60
b) *Film Age—45 Days from Development*								
Entire Plate	47.8	44.7	1.93	1.00	7.70	8	6.85	10.88
1st Quad	47.5	46.2	1.04	0.72	1.80	4	1.40	2.23
2nd Quad	47.6	44.8	2.31	1.44	4.85	4	3.80	6.02
3rd Quad	48.2	43.2	1.35	0.98	1.95	4	3.38	4.20
4th Quad	49.9	45.0	3.05	0.93	3.32	4	4.18	5.05
Sub-Quad 1	49.9	45.1	1.35	1.08	1.20	2	1.22	1.75
c) *Film Age—109 Days from Development*								
Entire Plate	33.4	34.1	1.68	1.37	9.32	8	7.40	11.38
1st Quad	34.2	35.8	1.61	0.85	1.95	4	1.70	2.68
2nd Quad	33.6	33.8	1.64	1.87	4.75	4	4.17	6.28
3rd Quad	34.5	32.8	1.74	0.96	2.48	4	3.18	3.58
4th Quad	33.3	34.9	2.82	1.56	3.87	4	3.40	4.98
Sub-Quad 1	34.9	34.9	2.53	1.06	1.82	2	1.32	2.15

(difference between x and y changes) were around 10 microns per inch immediately after development, but became negligible at the age of approximately three months.

7. Non-reseau photography should not be used with Type 1A film for the solution of precise analytical photogrammetric problems. When the coordinate distance between two points are from one to four inches in length they will contain random film shrinkage errors which will range from ± 2 microns to ± 9 microns at the 95% level.

Conclusions drawn from the studies on Kodak Special Plus-X Aerial Film (Estar Base), SO-135 are as follows:

1. The random shrinkages along the length of the film are smaller than those along the width of the film.

2. Film shrinkages are generally consistent among the different regions of the 9″×9″ format.

3. When non-reseau photography is used the random film shrinkage errors in coordinates will be approximately ± 5 microns at the 95% level. In this case the distance between images has little effect on the error.

4. The random shrinkages along a given roll of film are generally consistent.

5. The limited study on this film type indicates that a grid reseau with a one inch

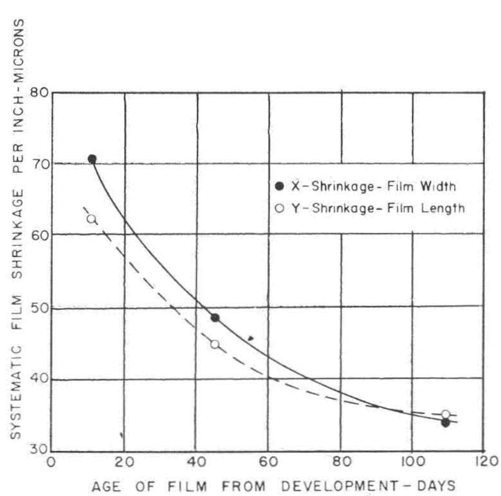

FIG. 4. Aging effects on systematic film shrinkage for Type 1A Class A Kodak Plus-X Aerographic film.

TABLE 3

CONSISTENCY OF SHRINKAGE VALUES ALONG A GIVEN ROLL OF TYPE 1A—CLASS A KODAK PLUS-X AEROGRAPHIC FILM

Plate No.	dV_A *microns /in.*	dV_B *microns /in.*	$\pm md V_A$ *microns /in.*	$\pm md V_B$ *microns /in.*
1	47.8	46.4	2.59	1.24
4	52.1	48.8	1.87	1.42
8	51.8	45.7	2.06	1.64
11	52.1	47.6	1.98	1.46

spacing would be sufficient to reduce the film shrinkage error propagation of any coordinates on the photograph to approximately ±4 microns at the 95% level.

When both types of film are considered the conclusions are as follows:

1. The results of this limited investigation show that for general purpose non-reseau photography, where dimensional stability is of importance, the SO-135 (Estar Base) film appears to be superior.

2. Based on Figure 3 and extrapolations of the curves it would appear that the Type 1A film has an advantage for use with grid reseaus which have a spacing smaller than one inch. Further studies are needed using more samples of film and a finer grid reseau.

TABLE 4

AGING EFFECTS ON FILM SHRINKAGE VALUES
TYPE 1A—CLASS A KODAK PLUS-X AEROGRAPHIC FILM

Age (days from exposure and development)	dV_A *microns /in.*	dV_B *microns /in.*	$\pm md V_A$ *microns /in.*	$\pm md V_B$ *microns /in.*	$\pm\mu$*-std. error of unit wt.*
11	70.3	62.0	2.53	1.98	9.92
45	48.5	44.8	1.84	1.03	5.04
109	34.0	34.4	2.00	1.28	5.18

TABLE 5

AVERAGE FILM SHRINKAGE VALUES FOR VARIOUS AREAS OF THE EXPOSURE FORMAT
KODAK SPECIAL PLUS-X AERIAL FILM (ESTAR-BASE), SO-135
FILM AGE—7 DAYS FROM DEVELOPMENT

Area of Plate	dV_A *microns /in.*	dV_B *microns /in.*	$\pm md V_A$ *microns /in.*	$\pm md V_B$ *microns /in.*	$\pm md\Delta x$ *microns*	Δx & Δy *interval in inches*	$\pm md\Delta y$ *microns*	$\pm\mu$*-std. error of unit wt.*
Entire Plate	6.6	10.4	2.56	0.86	5.15	8	4.15	6.12
1st Quad	5.1	10.4	3.67	1.30	2.95	4	2.28	3.88
2nd Quad	4.1	6.2	1.83	0.86	1.93	4	2.38	3.15
3rd Quad	8.9	9.3	5.30	1.73	1.40	4	1.28	1.92
4th Quad	7.8	11.5	1.79	0.80	3.70	4	2.45	4.08
Sub-Quad 1	7.2	10.7	2.56	1.35	2.30	2	2.10	2.92

TABLE 6

CONSISTENCY OF SHRINKAGE VALUES ALONG A GIVEN ROLL OF KODAK SPECIAL PLUS-X AERIAL FILM (ESTAR BASE) SO-135
FILM AGE—7 DAYS FROM DEVELOPMENT

Plate No.	dV_A *microns /in.*	dV_B *microns /in.*	$\pm md V_A$ *microns /in.*	$\pm md V_B$ *microns /in.*
1	7.3	12.1	2.31	0.54
4	4.8	8.7	4.14	1.76
9	5.3	7.6	2.73	1.61
12	9.0	12.6	3.94	0.69

The film shrinkage study has led to the following recommendations:

1. In the future there is a strong probability that the film will be exposed, developed, printed, and measured in a very short period of time. In order for this to become at all possible in precision photogrammetry a reseau of some type must be employed. The irregular random shrinkage which appears

TABLE 7

CONSISTENCY OF SHRINKAGE VALUES ALONG A GIVEN ROLL OF TYPE 1A—CLASS A KODAK PLUS-X AEROGRAPHIC FILM

FILM AGE—11 DAYS FROM DEVELOPMENT

Plate No.	dV_A *microns /in.*	dV_B *microns /in.*	$\pm md V_A$ *microns /in.*	$\pm md V_B$ *microns /in.*
1	73.0	66.7	2.87	2.18
4	75.1	60.6	1.84	2.23
8	67.5	56.8	2.84	1.17
11	65.5	56.8	2.58	2.23

to be present immediately after developing, can then be corrected in most cases.

2. It is recommended that all film which is used as described above, undergo a film shrinkage examination. This study has pointed out that reliable film shrinkage results can be obtained from a small number of measurements on each plate, and therefore the film can be subjected to an operational shrinkage test at nominal cost.

3. The results of this study are based on controlled environmental conditions which may not always be possible. It is therefore recommended that serious thought be given to the operating conditions to which the film is subjected. Additional shrinkage studies should be conducted for the extreme conditions to determine if certain restrictions are needed.

ACKNOWLEDGEMENTS

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APPENDIX A

DERIVATION OF THE FILM SHRINKAGE COMPUTATION

The following symbolic notation will be used in the derivation of the film shrinkage computation.

x^1, y^1—The calibrated coordinates of the grid reseau
x, y—The machine coordinates of the grid reseau
Δx^1, Δy^1—Differences as taken from the calibrated grid
Δx, Δy—Differences as taken from the machine coordinates
V_A, V_B—Scale factors to correct for systematic film shrinkage in the width and length of the film respectively
α—The angle of rotation between the machine and photographic coordinate systems

Further symbolic notation will be given later in the derivation.

The transformation of plane coordinates between two systems having a common origin may be expressed in the following manner (2):

$$x_{ij} = x_{ij} \cos \alpha - y_{ij} \sin \alpha \tag{1}$$

$$y_{ij} = y_{ij} \cos \alpha + x_{ij} \sin \alpha \tag{2}$$

Now then the differences in reseau grid intersections are subject to film shrinkage. The systematic shrinkage may be treated independently in the x and y directions by correcting the applicable differences in the calibrated grid system with scale factors.

$$\Delta x(i, j - m, n) = \Delta x^1(i, j - m, n) \cos \alpha \cdot V_A - \Delta y^1(i, j - m, n) \sin \alpha \cdot V_B \tag{3}$$

$$\Delta y(i, j - m, n) = \Delta y^1(i, j - m, n) \cos \alpha \cdot V_B + \Delta x^1(i, j - m, n) \sin \alpha \cdot V_A \tag{4}$$

The above equations contain the measured quantities, the calibrated quantities, and the unknowns. Since there are three unknowns, V_A, V_B, and α, at least three equations will be necessary to achieve a unique solution. When a greater number of equations are available an adjustment is possible which yields superior knowledge of the applicable random errors.

TABLE A-1
TABLE OF COEFFICIENTS

Residuals	*a* (dV_A)	*b* (dV_B)	*c* $(d\alpha)$	-1 $(\Delta x^1-\Delta x)$ $(\Delta y^1-\Delta y)$
$v\Delta x_1$	Δx_1	0	$-\Delta y_1$	$\Delta x^1{}_1-\Delta x_1$
$v\Delta x_2$	$\Delta x^1{}_2$	0	$-\Delta y_2$	$\Delta x^1{}_2-\Delta x_2$
$v\Delta x_3$	$\Delta x^1{}_3$	0	$-\Delta y_3$	$\Delta x^1{}_3-\Delta x_3$
$v\Delta y_1$	0	$\Delta y^1{}_1$	Δx_1	$\Delta y^1{}_1-\Delta y_1$
$v\Delta y_2$	0	$\Delta y^1{}_2$	Δx_2	$\Delta y^1{}_2-\Delta y_2$
$v\Delta y_3$	0	$\Delta y^1{}_3$	Δx_3	$\Delta y^1{}_3-\Delta y_3$

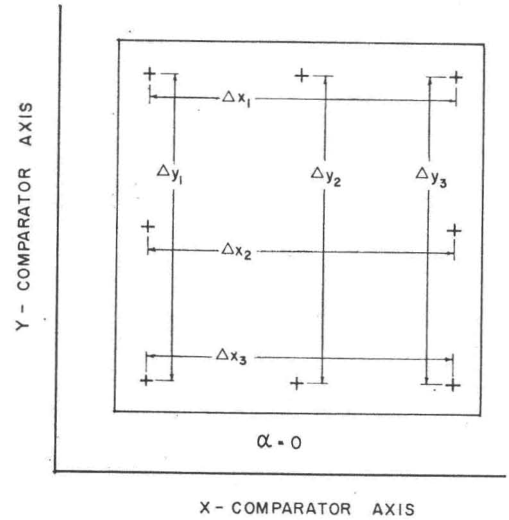

FIG. A-1. Sample of adjustment values.

The condition equations (3) and (4) now appear as follows (2):

$$\Delta x = \Delta x^1 \cdot V_A \cdot \cos\alpha - \Delta y^1 \cdot V_B \cdot \sin\alpha \tag{5}$$

$$\Delta y = \Delta y^1 \cdot V_B \cdot \cos\alpha + \Delta x^1 \cdot V_A \cdot \sin\alpha \tag{6}$$

After the partial derivatives are taken with respect to the three unknowns, the total differentials of equations (5) and (6) become:

$$d\Delta x = \Delta x^1 \cdot \cos\alpha \cdot dV_A - \Delta y^1 \cdot \sin\alpha \cdot dV_B - \Delta y \cdot d\alpha \tag{7}$$

$$d\Delta y = \Delta x^1 \cdot \sin\alpha \cdot dV_A + \Delta y^1 \cdot \cos\alpha \cdot dV_B + \Delta x \cdot d\alpha \tag{8}$$

These in effect are the observation equations where:

$d\Delta x,\ d\Delta y$ = Corrections $(\Delta x^1 - \Delta x)$, $(\Delta y^1 - \Delta y)$

$dV_A,\ dV_B$ = Corrections to assumed scale (1.0) in the x and y directions.

$d\alpha$ = Correction to the assumed value of 0 for α.

Assuming $\alpha=0$ the correction equations become:

$$v_{\Delta x} = \Delta x^1 \cdot dV_A - \Delta y \cdot d\alpha - d\Delta x \tag{9}$$

$$v_{\Delta y} = \Delta y^1 \cdot dV_B + \Delta x \cdot d\alpha - d\Delta y \tag{10}$$

It is now possible to perform an adjustment resulting in a best fit solution for the values of dV_A, dV_B and $d\alpha$ based on both calibrated and measured data.

The method of Rubin-Cholesky was chosen for the adjustment since this method provides not only a solution for the unknowns but also the standard errors of these adjusted values (3).

The coefficients of the unknowns are given in Table A-1. Table A-2 lists the results of the adjustment using the abbreviated notation of Table A-1. Figure A-1 shows typical values which might be used in an adjustment of this type.

The standard error of one observation of unit weight may be computed as follows:

$$\mu = \left[\frac{[vv]}{n-u}\right]^{1/2} = \left[\frac{[vv]}{3}\right]^{1/2} \tag{11}$$

This is in accord with the subject adjustment where there are six observation equations and three unknowns. The standard errors of the adjusted values may then be found according to equations (12), (13), and (14).

$$mdV_A = \mu(QdV_AdV_A)^{1/2} \tag{12}$$

$$mdV_B = \mu(QdV_BdV_B)^{1/2} \tag{13}$$

$$md\alpha = \mu(Qd\alpha d\alpha)^{1/2} \tag{14}$$

Using the values that have been computed thus far it is now possible to obtain the error that exists over a given Δx or Δy distance resulting from errors in dV_A, dV_B, and $d\alpha$.

TABLE A-2
RESULTS OF ADJUSTMENT

Quantity	*Adjusted Results*
dV_A	$\frac{[a1]}{[aa]} + \left[[c1] - \frac{[ac][a1]}{[aa]} - \frac{[bc][b1]}{[bb]}\right]\left[-\frac{[ac]}{[aa]}\right]\left[[cc] - \frac{[ac]^2}{[aa]} - \frac{[bc]^2}{[bb]}\right]^{-1}$
dV_B	$\frac{[b1]}{[bb]} + \left[[c1] - \frac{[ac][a1]}{[aa]} - \frac{[bc][b1]}{[bb]}\right]\left[-\frac{[bc]}{[bb]}\right]\left[[cc] - \frac{[ac]^2}{[aa]} - \frac{[bc]^2}{[bb]}\right]^{-1}$
$d\alpha$	$\left[[c1] - \frac{[ac[a1]}{[aa]} - \frac{[bc][b1]}{[bb]}\right]\left[[cc] - \frac{[ac]^2}{[aa]} - \frac{[bc]^2}{[bb]}\right]^{-1}$
QdV_AdV_A*	$[aa]^{-1} + \left[\frac{[ac]^2}{[aa]^2}\right]\left[[cc] - \frac{[ac]^2}{[aa]} - \frac{[bc]^2}{[bb]}\right]^{-1}$
QdV_BdV_B*	$[bb]^{-1} + \left[\frac{[bc]^2}{[bb]^2}\right]\left[[cc] - \frac{[ac]^2}{[aa]} - \frac{[bc]^2}{[bb]}\right]^{-1}$
$Qd\alpha d\alpha$*	$\left[[cc] - \frac{[ac]^2}{[aa]} - \frac{[bc]^2}{[bb]}\right]^{-1}$
QdV_AdV_B†	$\left[-\frac{[ac]}{[aa]}\right]\left[-\frac{[bc]}{[bb]}\right]\left[[cc] - \frac{[ac]^2}{[aa]} - \frac{[bc]^2}{[bb]}\right]^{-1}$
$QdV_Ad\alpha$†	$\left[-\frac{[ac]}{[aa]}\right]\left[[cc] - \frac{[ac]^2}{[aa]} - \frac{[bc]^2}{[bb]}\right]^{-1}$
$QdV_Bd\alpha$†	$\left[-\frac{[bc]}{[bb]}\right]\left[[cc] - \frac{[ac]^2}{[aa]} - \frac{[bc]^2}{[bb]}\right]^{-1}$
$[vv]$	$[11] - \frac{[ac]^2}{[aa]} - \frac{[bc]^2}{[bb]} - \left[[c1] - \frac{[ac][a1]}{[aa]} - \frac{[bc][b1]}{[bb]}\right]^2\left[[cc] - \frac{[ac]^2}{[aa]} - \frac{[bc]^2}{[bb]}\right]^{-1}$

* Weight numbers of dV_A, dV_B and $d\alpha$.
† Correlation numbers of dV_AdV_B, $dV_Ad\alpha$, and $dV_Bd\alpha$.

After reduction and modification and the assumption that $\alpha=0$ we obtain the general law of error propagation in the x and y directions (4) (5).

$$md\Delta x^2 = \mu^2[(\Delta x^1)^2 QdV_A dV_A + (-\Delta y)^2 Qd\alpha d\alpha + 2[(\Delta x^1)(-\Delta y)(QdV_A d\alpha)]] \quad (15)$$

$$md\Delta y^2 = \mu^2[(\Delta y^1)^2 QdV_B dV_B + \Delta x^2 \cdot Qd\alpha d\alpha + 2[(\Delta y^1)(\Delta x)(QdV_B d\alpha)]] \quad (16)$$

In the above equations QdV_AdV_A, QdV_BdV_B, and $Qd\alpha d\alpha$ are weight numbers and QdV_AdV_B, $QdV_Ad\alpha$, and $QdV_Bd\alpha$ are correlation numbers.

References

1. Brock, R. H., and Faulds, A. H., "Photogrammetric Investigation On Atmospheric Refraction and Film Shrinkage," for Rome Air Development Center, Syracuse University Research Institute, CE743-6110F, November, 1961.
2. Merchant, D. C., and Brock, R. H., "A Comparative Analysis of Photo Coordinates Measured Stereoscopically and Monocularly," CE841-6112F Syracuse University Research Institute, Syracuse 10, New York, December, 1961.
3. Hallert, Bertil, "Basic Factors Limiting The Accuracy of Mapping and Aerotriangulation with Photogrammetric Procedures," Mapping and Charting Research Laboratories Technical Paper, The Ohio State University, 1954.
4. Hallert, Bertil, "Photogrammetry," McGraw-Hill, New York, 1960, Appendix B.
5. Deming, W. Edwards, "Statistical Adjustment of Data," Wiley, New York, 1943.

*A New Approach to Aerial Map Data Acquisition and a Global Operational Concept**

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A New Technique for Aerial Mapping

A new mapping system is presently under development in the USAF. The Boeing RC-135 which is very similar to the commercial airline Jet 707 will soon be delivered as the USAF's new aerial mapping and surveying vehicle.

This high-speed jet aircraft will provide an extremely stable platform for the most intricate and sophisticated system ever developed for aerial surveying and cartographic mapping.

This new aircraft will reach an altitude greater than 50,000 feet. At a true air speed of about 600 miles-per-hour plus its long endurance will result in exceptionally high mission accomplishments.

The mapping and surveying system being developed will consist of a new 6″ focal-length mapping camera with a resolution of better than 40 lines-per-mm; an inertial platform so precise that it will record the true vertical of each exposure to less than 30″ of arc; and a new electronic surveying system that will give instantaneous ranging of distance from the aircraft to as many as 4 widely separated ground stations. These plus many other innovations will be incorporated in this new system. The result expected from the new system is to secure in addition to the mapping photography, most of the control data required for large-scale maps (1/50,000).

The article attempts to explain some of the intricacies of this sophisticated system, and then gives the views of the author on a global operational concept to rapidly acquire the map deficiencies that face the world today.

A New Concept for Aerial Mapping

Map making is big business. The combined efforts of the Army, Air Force, Navy, U. S. Geological Survey, USC&GS, Department of Agriculture, plus civilian contractors coordinate their talents in the massive efforts of accurately portraying the minute details of the earth's surface.

Map detail is derived from data collected

* Presented at the 1962 ACSM-ASP Meeting in St. Louis, Missouri.