The Use of the A9-B9 System in Aerial Surveying

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INTRODUCTION

ATMOSPHERIC conditions, extra costs incurred by high altitude photography and the ceiling of the aircraft are important restrictions on the height from which aerial photographs for mapping purposes can be made. Therefore, it is not always practical to try to increase the general efficiency and the economy of aerial mapping by simply increasing the flying height. Another way of achieving the same objective is to use cameras with wider angular fields. This results in larger photographic ground coverage, leads to a reduction of flying time and, consequently, of the amount of photographic material used, as well as of the density of required ground control. In addition, a wider angular field of the camera increases the base-height ratio of two successive photographs. This offers more favorable intersection angles of corresponding light rays, so that a higher accuracy in photogrammetrically determined elevations can be expected.

Of course, a substantial widening of the angular field of a lens entails certain difficulties. Imperfections in the flatness of the film and irregularities in the emulsion will have a stronger effect on the final results of photogrammetric measurements and mapping. A certain decrease in image quality is another consequence that must be considered.

Since the Second World War, superwide-angle photography has been successfully used in the small-scale mapping of the U.S.S.R. For this purpose, a superwide-angle lens with an angular field of 122°, called Russar, has been developed. This lens is supposed to be practically free from distortion, and the brightness of the photographic image is apparently only 10% lower in the corners than in the center. A high quality superwide-angle lens has also been developed in Switzerland by the Wild Company. This lens, called Super Aviogon, has an angular field of 120° for a 23×23 cm. negative and a focal-length of 88 mm. A corresponding lens, the Super Infragon, has been especially designed for infrared photography, and in order to accommodate these lenses, a fully automatic aerial film camera, the Wild RC9, has been constructed.

The introduction of superwide-angle photographs raises the question of accompanying plotting equipment. On the assumption that the triangulation operations could be carried out analytically, for the actual detail plotting a simple and relatively inexpensive plotter could be designed. In view of the fact however, that the analytical triangulation methods and the electronic computations are not commonly known and used yet, there was a strong argument for a more conventional solution, such as a first-order plotter on which aerial triangulation of superwide-angle photographs could be performed. For the design of such a plotter two conventional ways could be followed:

(a) an instrument based on optical projection,
(b) an instrument based on mechanical projection.

Whatever system is used, a conventional plotter designed for triangulation of original size superwide-angle photographs results in a large and expensive instrument. In order to overcome this disadvantage the Wild Company, which developed the high-quality Super Aviogon and Super Infragon lenses, chose to solve this problem by designing a plotter that accepts photographs reduced to half the original negative size. This plotter, the Wild A9 Autograph, has a maximum picture size of 115 mm.×115 mm., the focal distances varying from 40 to 77 mm. Therefore, wide-angle and superwide-angle photographs can be used if they are reduced to half size. For aerial triangulation the A9 Autograph is provided with the means to interchange the left and the right pictures in the eyepiece, to rotate the optical image by Dove prisms and to set the base in inner and outer positions. The coordinates of the triangulated points can be recorded manually or automatically (magnetic tape, cards, punch tape or typewritten records).

TABLE I

STEREOSCOPIC GRID MEASUREMENTS ON THE A9

Model No.	*Focal length f, in mm.*	*Projection distance h, in mm.*	*Base*		*Base ratio b/h*	*Number of points*	*Mean square errors*			
							In scale of A9 grid plates		*In scale of original negatives*	
			In mm.	*Position*			m_h	m_{p_x}	m_h	m_{p_x}
1	44	100	89	out	0.89	91	6 μ	5.3 μ	12 μ	10.6 μ
2	44	100	89	in	0.89	91	6	5.3	12	10.6
3	44	50	44	out	0.89	73	7	6.2	14	12.4
4	44	50	44	in	0.89	91	5	4.5	10	9.0
5	76.5	100	66	out	0.66	66	7	4.6	14	9.2
6	76.5	100	66	in	0.66	66	5	3.3	10	6.6
7	76.5	50	33	out	0.66	66	6	4.0	12	8.0
8	76.5	50	33	in	0.66	66	9	5.9	18	11.8

A special reduction printer has been built for the reduction of the diapositives. This reduction can be combined with elimination of lens distortion and compensation for earth curvature and atmospheric refraction.

It would not be efficient to use a costly instrument, designed for aerial triangulation, for ordinary plotting operations. Therefore, a simple plotter, the Wild B9 Aviograph, was designed as an auxiliary plotter to be used in conjunction with the Wild A9. The same reduced diapositives are used on both instruments. The principle of the Wild B9 Aviograph is again based on the mechanical solution combined with an orthogonal observation system. Four or five Wild B9 plotters could be used with one Wild A9 to obtain an economical superwide-angle plotting system.

No doubt, the availability of high quality superwide-angle photographs constitutes an extremely important element in modern mapping techniques and it was the purpose of our investigations to supply some concrete information that could assess better this new element and would be of assistance in the planning of mapping projects for which use of the superwide-angle photographs is considered.

GRID MEASUREMENTS

Various grid tests have been performed in the laboratories of the National Research Council for the purpose of determining the instrumental accuracies of the Wild A9 and B9 plotters.

A set of monocular grid measurements in the A9, using focal distances of 44 mm. and 76.5 mm., and a projection distance of 100 mm., showed an average mean square error of $m_x = \pm 6\mu$ and $m_y = \pm 5\mu$ at the scale of the grid plates. Stereoscopic measurements of eight grid models with different combinations of focal distances (44 mm. and 76.5 mm.), projection distances (100 mm. and 50 mm.) and with the base in outer and inner positions gave a mean square error of $m_h = \pm 6\mu$ in the scale of the grid plates (see Table I). For these measurements the Z-scale had been improved by attaching a vernier that divided the smallest interval of the scale into 30 parts.

For the determination of the planimetric accuracy of the B9 Aviograph, the same grid plates as those used in the A9 were employed, and the grid points were plotted on dimensionally stable, aluminum-mounted paper, using the Z-column as well as the reduction pantograph, in which a reducing scale of 2:1 had been set. The measurements were performed for a focal length of 44 mm., a projection distance of 132 mm. and a base ratio $b/h = 1.14$. The coordinates of the plotted grid points were read on the A7 coordinatograph and the differences between the theoretical and the plotted coordinates were computed. The mean square errors of the coordinates were $m_x = m_y = \pm 0.03$ mm. Since possible errors of the A7 coordinatograph are included in this result, there is little doubt that the planimetric accuracy of the B9 is very high.

The A9 and B9 are equipped with the same type of scales for the vertical coordinate readings. The grid measurements on the A9 proved that the reading precision of the Z scale is insufficient compared with the instrumental accuracy. Therefore, on the B9, the elevations of the grid points were measured only after a dial gauge with a smallest reading interval of 10μ had been mounted on the Z column.

The elevation measurements were performed for two different projection distances of 132 mm. and 88 mm., with the base ratio $b/h = 1.14$. The mean square errors of the measured elevations for the two projection distances were 10μ and 9μ respectively (see Table II). This means that the instrumental error in plotting at the scale of, e.g., 1:100,000 would amount to only 1 m. and 0.9 m., respectively.

Use of the A9 Autograph in Aerial Triangulation

The use of reduced diapositives on the A9 may lead to reduced accuracy in aerial triangulation. This is an important question since analytical methods are already available which are suitable also for use in connection with superwide-angle photography. To clarify this problem, aerial triangulation of several strips was carried out on both the A9 and by analytical methods, using reduced and contact diapositives from the same set of negatives. A more complete report on this work will be published later. At present mention is here made of some of the results obtained in this investigation.

The photographs used for this study were taken from an altitude of 13.0 km., with resulting photoscale of 1:150,000. The strip, 100 km. in length, was flown over the Ottawa test area. The photography was made with a 90% longitudinal overlap, so that three independent strips with a 60% longitudinal overlap could be formed, each consisting of 7 or 8 photographs. Each of these strips was triangulated once on the Nistri TA3 Stereocomparator and twice, by different operators, on the Wild A9 Autograph. The analytical triangulation was performed on the IBM 650 with a program developed at the National Research Council. This triangulation program includes corrections for radial lens distortion, earth curvature, and atmospheric refraction.

The reduced diapositives, which were measured in the A9 Autograph, were corrected for lens distortion during the reduction by a compensation plate. This plate compensated for earth curvature and atmospheric refraction for a flying height of 10 km. only, as no compensation plate was available for a flying height of 13.0 km. The remaining effect of earth curvature and atmospheric refraction was corrected numerically.

After positioning the strip in the terrestrial coordinate system, the computed ground elevations showed a parabolic deformation that was about the same for both the Nistri TA3 and the Wild A9 measurements. To correct for this deformation, the transformation was repeated with a second-degree height adjustment included. In order to study the accuracy of the results, the mean square errors in the photogrammetric coordinates of 41 horizontal and 63 elevation control points were computed and are listed in Table III.

A comparison between the results obtained from the Wild A9 and the Nistri TA3 measurements is expressed by the ratio of the mean square elevation and position errors of the ground control points. Taking the averages of these values for the three strips is obtained:

$$\frac{m_h\,(\text{A9})}{m_h\,(\text{TA3})} = \frac{4.1\text{ m.}}{2.4\text{ m.}} = 1.7$$

$$\frac{m_p\,(\text{A9})}{m_p\,(\text{TA3})} = \frac{6.9\text{ m.}}{5.4\text{ m.}} = 1.3$$

where:

$$m_p = \sqrt{m_x^2 + m_y^2}.$$

These ratios show that a higher accuracy is obtained by the analytical triangulation method, probably because in this procedure the contact diapositives, and not the reduced diapositives were used. The fact that the ratio of the accuracies in elevation is greater than the ratio of the accuracies in the horizontal position may be partly explained by the too large scale interval and the resulting limited reading accuracy for the Z-coordinates on the A9.

Table II

Stereoscopic Grid Measurements on the Wild B9 Using a 0.01 mm. Dial Gauge

Model No.	*Focal length*	*Projection distance*	*b/h ratio*	*Number of points*	m_h
1	44 mm.	132 mm.	1.14	66	10 μ
2	44 mm.	88 mm.	1.14	66	9 μ

TABLE III

MEAN SQUARE ERRORS FOR AERIAL TRIANGULATION
SCALE OF PHOTOGRAPHS 1:150,000

Strip	*Instrument*	*Operator*	*Mean square errors*						
			On the ground				*At negative scale*		m_h *in* ‰ h
			m_x	m_y	m_p	m_h	m_p	m_h	
1	Nistri TA3	2	4.0 m	3.9 m	5.6 m	2.7 m	37 μ	18 μ	0.21
2	Nistri TA3	2	3.1	3.5	4.7	2.3	31	15	0.18
3	Nistri TA3	2	4.4	4.0	6.0	2.1	40	14	0.16
1	Wild A9	2	5.6	4.8	7.4	4.6	49	31	0.35
2	Wild A9	2	4.1	5.1	6.5	4.3	43	29	0.33
3	Wild A9	2	4.2	4.3	6.0	4.5	40	30	0.35
1	Wild A9	1	4.6	4.8	6.6	3.7	44	25	0.28
2	Wild A9	1	5.1	6.4	8.2	4.0	55	27	0.31
3	Wild A9	1	5.0	4.7	6.9	3.3	46	22	0.25

TESTS ON SINGLE MODELS FROM SUPERWIDE-ANGLE PHOTOGRAPHS

In the first test carried out on a single stereo model of superwide-angle photographs, the results of the Nistri TA3 measurements were compared with the results obtained on the Wild A9. A test model with a 60% longitudinal overlap, containing 45 points with known elevations, was selected from the photographs at the scale of 1:150,000, which had been used for the investigation of aerial triangulation. The original-size diapositives of this model were measured on the Nistri TA3 and the reduced diapositives on the Wild A9. The analytical relative orientation of the Nistri measurements was performed by using 25 points uniformly distributed over the model. On the Wild A9, the relative orientation was performed empirically, using 6 points in the usual positions. The height deformation in the model was adjusted by second-degree polynomials, as in the adjustment of the strips. The mean square errors in the elevations were computed and are listed in Table IV.

The accuracy of the elevations that are obtained from the original photographs is higher by a factor of 1.5 than the accuracy of the elevations obtained when the reduced photographs on the Wild A9 are used. This factor was obtained from the following ratio:

$$\frac{m_h\,(\text{A9})}{m_h\,(\text{TA3})} = \frac{2.4\text{ m.}}{1.65\text{ m.}} = 1.5.$$

In a second test, original size and half-size diapositives from superwide-angle photographs and the diapositives from wide-angle photographs made over the Renfrew test area, at the same scale 1:50,000, were measured on the TA3 Stereocomparator. Thus the influence of the reduction of diapositives could be directly established and the comparison between the results from superwide-angle photographs and wide-angle photographs could be obtained.

TABLE IV

MEAN SQUARE ERRORS IN ELEVATION FOR A SINGLE MODEL
SCALE OF PHOTOGRAPHS 1:150,000

Instrument	*Operator*	*Mean square errors* (m_h)		
		On the ground	*At negative scale*	m_h *in* ‰ h
Nistri TA3	2	1.8 m.	12 μ	0.14
Nistri TA3	2	1.5	10	0.12
Wild A9	2	2.6	18	0.20
Wild A9	2	2.3	16	0.18
Wild A9	1	2.4	16	0.18

All of the measurements were processed analytically in the IBM 1620. For relative orientation of the models 24 points, evenly distributed in the overlaps, were used. The remaining parallaxes for all measured points were computed in order to determine the accuracy of the relative orientation.

Lens distortion, earth curvature and atmospheric refraction were compensated analytically for the original size diapositives. For the reduced RC9 diapositives, these corrections were applied optically during the reduction process.

The absolute orientations were carried out on 30 terrestrial control points and were repeated with second-degree terms included in the height adjustment in order to correct for small systematic model deformations. The mean square errors of the photogrammetrically determined elevations of about 70 control points were computed and they are listed in Table V.

A comparison between the results from the measurements of RC8 diapositives, the RC9 diapositives and the RC9 reduced diapositives is given by the ratios of the corresponding mean square errors in elevation:

$$\frac{m_h\,(\text{RC8})}{m_h\,(\text{RC9})} = \frac{0.74 \text{ m.}}{0.41 \text{ m.}} = 1.8$$

$$\frac{m_h\,(\text{RC9red})}{m_h\,(\text{RC9})} = \frac{0.63 \text{ m.}}{0.41 \text{ m.}} = 1.5$$

$$\frac{m_h\,(\text{RC8})}{m_h\,(\text{RC9red})} = \frac{0.74 \text{ m.}}{0.63 \text{ m.}} = 1.2.$$

These ratios indicate that the accuracy in elevation determined from the superwide-angle photographs is higher by a factor of 1.8 than the accuracy of the same coordinates determined from the wide-angle photographs. This no doubt is the consequence of the larger base ratio for the superwide-angle photographs.

The ratio of the errors obtained for the reduced and the original RC9 diapositives demonstrates the loss in accuracy which is caused by plate reduction. This ratio agrees with the ratio of the results from the Wild A9 and the Nistri TA3 measurements of the 1:150,000 test photographs earlier mentioned.

Despite the influence of plate reduction, however, the accuracy of the elevations obtained from reduced superwide-angle diapositives is higher by a factor of 1.2 than the accuracy obtained from wide-angle photographs made at the same scale as the original superwide-angle photographs.

This agrees approximately with the results given in the paper by J. W. Halbrook.[2] The writer compared the results from superwide-angle and wide-angle photographs made at the same flying height. He reached the conclusion that equal vertical accuracies from both types of photographs can be obtained if superwide-angle photographs are taken at about 5/6th the altitude from which the wide-angle photographs are taken. For superwide-angle and wide-angle photographs made at the same scale, this would mean that the vertical accuracy of superwide-angle photographs if about $1.7\times\frac{5}{6}=1.4$ times higher than the accuracy of wide-angle photographs.

Interpretability of Reduced Diapositives

Small-scale cartography of sparsely populated areas will be the most important application of superwide-angle photography. In these areas, small artificial details are few. If they occur, however, it is very important that they should be identified on the aerial photographs in order to be mapped. To provide some information on this important question, the plotting results of one test area from superwide-angle photographs at three different scales of 1:150,000, 1:100,000 and

Table V

Mean Square Errors for the Renfrew Test Model

Diapositives	*Photo-scale*	*b/h ratio*	*Over-lap*	*Number of points*	*Mean square errors* m_h *On the ground*	*Mean square errors* m_h *In ‰ h*	*Mean square errors* *Residual paral-laxes*
RC9; 23×23 cm.	1:50,000	1.06	60%	64	0.41 m.	0.09‰	12 μ
RC9; 12×12 cm.	1:100,000	1.06	60%	65	0.63	0.14	14
RC8; 23×23 cm.	1:50,000	0.60	60%	66	0.74	0.10	10

1:68,000 were compared. Reduced diapositives made from these photographs were plotted on the B9 Autograph. The test area, 6×9 km. in size, consisted mainly of farm land and some wooded areas. The highest altitude photographs were mapped first, the medium altitude photographs next, and the lowest altitude photographs last. This was done to prevent the operator from remembering information about the terrain details from lower altitude photographs while the higher altitude photographs still had to be mapped.

In order to compare the completeness of linear details, the roads and railway tracks were mapped from the three different sets of photographs. It was found by comparing the three plots that a decrease of the photoscale did not seriously affect the completeness in recording of linear details. No important omissions could be established.

A completeness comparison of the mapping of small, point-like details was restricted to the mapping of the houses in the test area. Here important differences for the three differerent photoscales occurred. On the plot made from the 1:300,000 scale diapositives, 26% of the total number of houses which were plotted from the 1:136,000 scale diapositives were missing, while 3% were plotted where no houses existed. In the plot made from the 1:200,000 scale diapositives, 20% were missing and 5% not correctly interpreted, as compared with the plot from the 1:136,000 scale diapositives. The percentages of the missing details point to the restriction that has to be placed on the flying height if a correct interpretation of the aerial photographs is required.

Conclusions

A stereo pair of superwide-angle photographs has a base ratio which is 1.7 times the base ratio of a stereo pair of wide-angle photographs if both types of photographs have the same overlap. Therefore, one may expect, from a purely geometrical point of of view, the vertical accuracy of the superwide-angle photographs to be 1.7 times higher than the vertical accuracy of the wide-angle photographs made at the same scale. However, as imperfections in the optical system and the photographic material will have a greater influence if the angular field of the camera is wider, the purely geometrical considerations may not be sufficient to determine the actual gain in accuracy.

The measurements of both types of photographs made at the same scale over the Renfrew test area have been performed to compare the relative vertical accuracies. It is remarkable that the ratio between the obtained vertical accuracies of superwide-angle and wide-angle photographs is about what was expected on the ground of purely geometrical considerations. As only one stereo model of each type of photographs was measured, it is too early to make any definite statements. However, the results indicate that the optical and photographic imperfections are too small to influence the accuracy of superwide-angle photographs more than wide-angle photographs.

Several experiments mentioned in this paper prove as it was expected, that there is a drop in accuracy caused by the reduction of the negatives to half-size diapositives required for the A9 and B9 plotters. The plotting accuracy depends upon the scale of photographs used, and one would expect that diapositives reduced to half-size of the original negatives would have a 50% decrease in accuracy. As demonstrated, however, the decrease in accuracy amounts to about 30% only. This is probably due to the fact that the optical reduction of original negatives reduces also all geometrical errors in the original photographs.

Because of this and the very favourable base ratio in superwide-angle photographs as well as the excellent accuracy of the A9, the use of reduced diapositives in the A9 offers better vertical accuracy by a factor of about 1.2 than the use of the original 23×23 cm. wide-angle photographs.

This result demonstrates a very attractive feature of superwide-angle photographs. It proves that there is a definite technical and economical advantage in using them particularly in the mapping of large less developed areas. There is also no doubt that the use of analytical triangulation methods will permit the exploitation of the favourable features of superwide-angle photographs in a more complete manner.

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