*Resolution of Vibration Isolated Cameras**

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INTRODUCTION

In aerial photography, to achieve results which will reflect the full capability of the camera system being used, it is important that the camera be isolated from disturbing motions and vibrations inherent in the moving aircraft. An opportunity for providing sufficient isolation from these disturbances occurs when choosing the method by which the camera is to be mounted to the aircraft. Whether a particular mounting will adequately limit camera movement to a permissible level will depend upon how effectively its isolating characteristics attenuate the input disturbances over the range of frequencies at which they occur. Depending upon the method selected for mounting, varying degrees of isolation will result.

MOUNTING METHODS

There are several ways in which the camera may be mounted. The simplest of course would be to rigidly bolt the camera onto the airframe. Simplicity, however, is about the only virtue of this method. The camera body can now be considered as a rigid extension of the airframe and hence experiences the same motions and vibrations. Nevertheless, in certain cases it may be determined that a particular aircraft's anticipated roll, pitch and yaw rates, as well as its vibration spectrum, may be sufficiently small to permit this type of mounting and yet still achieve an adequate degree of camera steadiness. In such cases, before a final decision to employ rigid mounting, the effects of landing shocks upon the camera and the possible resonant motions of optical elements within the camera should not be overlooked. In many instances these motions internal to the camera could prove highly detrimental to the results originally anticipated.

When it is necessary to achieve the highest resolution possible, the rigid type mounting would rarely ever be suitable. For these situations, the best camera mounting system available is that which provides a stabilized platform for the camera to operate from. Using gyroscopic references and electronic controls these platforms can keep a camera essentially motionless in spite of very severe aircraft perturbations and vibrations. In addition to providing excellent steadiness, stabilized platforms also provide a highly accurate vertical from which the camera may be very precisely referenced.

While the gyroscopically stabilized platform is the best available means for mounting aerial cameras, it is also the most costly. Nevertheless, when good photographic results must be guaranteed, it is the surest way, and perhaps in the long run the least expensive way to achieve them.

In between the two extremes of a rigid mounting and a stable platform, is the method using vibration isolator or spring type devices. The vibration isolator, while affording no compensation for aircraft roll, pitch and yaw motions, as does the stabilized mount, does offer excellent isolation against airframe vibrations, as well as pro-

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viding shock protection for the camera. For these reasons and because of its simplicity and inexpensiveness, the vibration isolator is widely used as a mounting device.

Although many photographic installations employ vibration isolators, they are not always installed with the attention they deserve. Improperly applied vibration isolators may amplify rather than reduce undesired motions imparted to the camera. As a result photographic quality is unnecessarily degraded producing results which are often disappointing.

Center of Gravity Considerations

The manner in which performance degradation occurs can be seen by referring to Figure 1. In Figure 1(a) the camera is properly mounted with its center of gravity (c.g.) exactly between the suspension points of identical isolators. Vertical vibrations (x_f) cause the camera to translate upward at a reduced amplitude (x_c). However, in Figure 1(b), the c.g. is displaced to the right of the center line causing the camera to rotate (θ_c) as well as translate when vertical vibrations are impressed. Image quality is extremely more sensitive to rotational motion than to translation, and hence the generation of this type motion (θ_c) as a result of mount unbalance deserves serious consideration.

Reasons for Unbalance

Although care may be taken to place the camera c.g. at the geometric center of the isolator suspension, an unbalance can still exist because of several reasons.

1. Manufacturing tolerances in the camera, lens cone, film, film spools and film cassettes can produce c.g. uncertainties in the range of $\pm\frac{1}{4}$ inch.
2. Film transfer from storage spool to take-up spool causes c.g. shifts up to an inch and most certainly should be compensated.
3. Uncertainty in the location of the isolators' elastic center both in the vertical and horizontal planes can affect misplacement of the c.g. These therefore must be accurately known and aligned. In addition, their spring and damping constants must be carefully matched so as to prevent any phase or amplitude differences in their movement from causing additional rotation.

Because of these practical tolerances it is quite likely that a c.g. displacement will exist in spite of careful attention to the above factors. It is therefore useful to develop an expression which would relate camera rotation to camera unbalance in a given isolator system. The degree of balance could then be evaluated for its adequacy in attaining the required system performance.

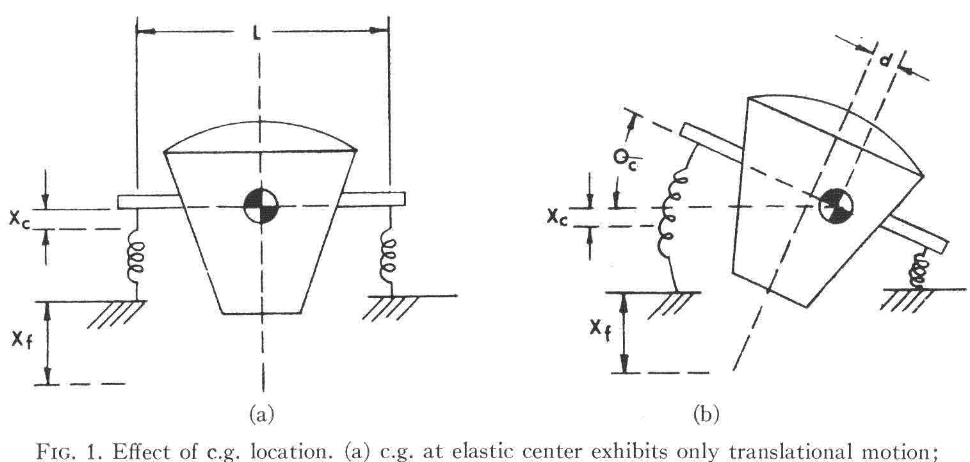

FIG. 1. Effect of c.g. location. (a) c.g. at elastic center exhibits only translational motion; (b) c.g. displaced from elastic center causes rotation to occur.

Physical Derivation

Camera rotation due to an unbalance will be expressed as a function of airframe vibration. A derivation of the desired relationship is best understood from the physical action involved.

1. Airframe translational vibrations (a_f) pass through the isolators causing the camera to vibrate at the same frequency but reduced amplitude (a_c)

$$a_c = a_f \cdot G_t \tag{1}$$

where:

a_c = camera vibration acceleration
a_f = aircraft vibration acceleration
G_t = isolator translational transfer function

2. The vibration line of action does not pass through the camera c.g. when the c.g. is not at the elastic center of the isolator. Hence a disturbance torque (T_d) is produced. This causes the camera to pivot about its uncentered c.g. The magnitude of the disturbance torque is determined by:

$$T_d = a_c \cdot m \cdot d \tag{2}$$

where:

T_d = disturbing torque
m = camera mass
d = c.g. displacement

3. The disturbing torque (T_d) produces a camera rotation (θ_c). This rotation is restrained by the isolators and is a function of the isolators' characteristic in rotation (G_r)

$$\theta_c = T_d \cdot G_r \tag{3}$$

where:

θ_c = camera rotation
G_r = isolator rotational transfer function

Combining equations (1), (2), and (3) camera rotation (θ_c) as a function of airframe vibrations (a_f) can be expressed as:

$$\theta_c = a_f \cdot m \cdot d \cdot G_t \cdot G_r \tag{4}$$

The above expression demonstrates the proportional relationship between camera rotation (θ_c) and c.g. displacement (d) from the elastic center. If the camera c.g. could be accurately located at the elastic center, presumably midway between isolators, no unbalance would exist and hence no rotation could occur. However, since neither the c.g. or the elastic center is ever precisely known, a perfect balance is rarely achieved.

Expanded Expression

Equation (4) will be further developed to demonstrate the manner by which camera rotation can be numerically evaluated. To do so, the airframe vibrations (a_f) and the isolator characteristics (G_t) and (G_r) will be further expanded.

Airframe vibratory acceleration (a_f) is the independent variable whose amplitude varies with frequency in accordance with some predetermined vibration spectrum. The peak value of a_f at any particular frequency is

$$a_f = \frac{X_f}{2} s^2 \tag{5}$$

where:

X_f = double amplitude displacement
$s = j\omega$ = vibration frequency

The translational characteristic (G_t) of a vibration isolator relates camera vibrations on one side of the isolator to the input vibrations on the other side. Expressed in operational form:

$$G_t = \frac{X_c}{X_f} = \frac{(2\zeta_t/\omega_{n_t})s + 1}{(1/\omega_{n_t})^2 s^2 + (2\zeta_t/\omega_{n_t})s + 1} \tag{6}$$

where:

X_c = double amplitude, camera displacement
X_f = double amplitude, airframe displacement
$\omega_{nt} = \sqrt{k_t/m}$ = natural frequency of isolator system in translation
k_t = spring constant of isolator
m = mass of camera
ζ_t = damping factor of isolator system in translation
$s = j\omega$ = vibration frequency

A plot of this type response is shown in Figure 2 for various damping factors.

The rotational characteristic (G_r) of a vibration isolator, such as shown in Figure 1, is related to the translational parameters of Equation (6) by the spacing (L) between isolator centers and the radius of gyration (r) of the camera. Analysis yields:

$$G_r = \frac{\theta_c}{T_d} = \frac{1}{k_r[(1/\omega_{n_r})^2 s^2 + (2\zeta_r/\omega_{n_r})s + 1]} \tag{7}$$

where:

θ = camera rotation (single amplitude)
T_d = disturbance torque

$k_r = k_t \dfrac{L^2}{2}$ = rotational spring constant of isolator

L = spacing between isolators

$\omega_{n_r} = \dfrac{L/2}{r}\,\omega_{nt}$ = natural frequency of isolator system in rotation

r = radius of gyration

$\zeta_r = \dfrac{L/2}{r}\,\zeta_t$ = damping factor of isolator system in rotation

Examination indicates that it would be advantageous to keep the isolator spacing (L) as large as practically possible. This increases k_r and ω_{n_r} in the direction which would tend to decrease rotation.

Substituting Equations (5), (6), and (7) into Equation (4) the following generalized expression for θ_c can be obtained:

$$\theta_c = \frac{2dX_f s^2}{\omega_{n_t}{}^2 L^2} \cdot \frac{(2\zeta_t/\omega_{n_t})s + 1}{(1/\omega_{n_t})^2 s^2 + (2\zeta_t/\omega_{n_t})s + 1} \cdot \frac{1}{(1/\omega_{n_r})^2 s^2 + (2\zeta_r/\omega_{n_r})s + 1} \tag{8}$$

In the above expression X_f is double amplitude (peak to peak displacement) while

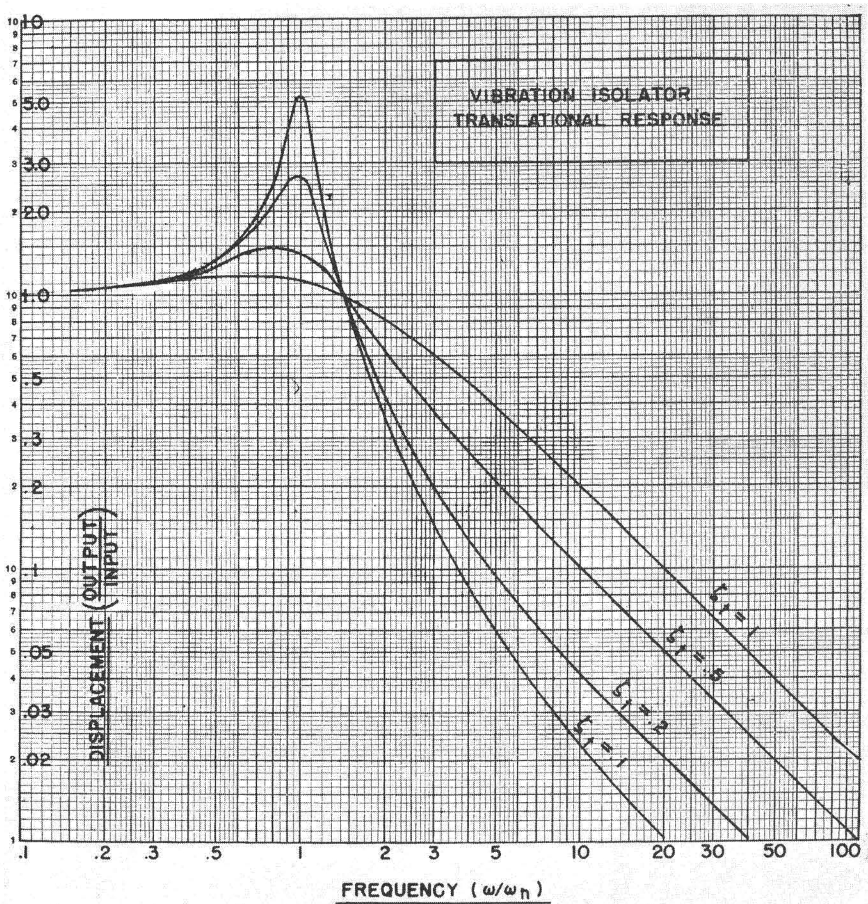

FIG. 2. Translational response of vibration isolators for various damping factors.

$$\frac{x_c}{x_f}=\frac{(2\zeta_t/\omega_{nt})s+1}{(1/\omega_{nt})^2s^2+(2\zeta_t/\omega_{nt})s+1}$$

θ_c, in radians, is single amplitude when dimensions X_f, d, and L are all in the same units. Normalizing the coefficient $2X_f ds^2/\omega_{n_t}{}^2L^2$ as unity (constant acceleration) and assuming a typical isolator damping factor of $\zeta_t=.1$, a plot of the above expression relating camera rotation to frequency of vibration is shown in Figure 3 for various ratios of $\omega_{n_r}/\omega_{n_t}$. Note that maximum rotation occurs when the peaks of the isolator's two resonant modes, translation and rotation, coincide ($\omega_{n_r}/\omega_{n_t}=1$).

EXAMPLE

For a practical application of this derivation, consider parameters similar to a typical mapping camera mounted by four isolators arranged in a square 18 inches apart. Weight (W) will be assumed as 100 lbs., moment of inertia (J) 1 slug ft.^{2} and a unit c.g. displacement (d) of one inch. For simplicity, the displacement will be assumed offset from the elastic center in a direction parallel to a line adjoining two adjacent isolators as in Figure 1(b). An isolator system for this application typically might have a natural frequency (ω_{n_t}) of 10 cps (62.8 rad./sec.) and a damping factor (ζ_t) of .1 producing an amplification at resonance of 5.

The rotational response of the isolator system can be determined from the above data.

The radius of gyration (r) must first be ascertained.

$$r = \sqrt{J/m} = \sqrt{Jg/W}$$
$$= \sqrt{1(32)/100} = .57 \text{ ft.} = 6.85 \text{ in.}$$

Recalling that:

$$\omega_{n_r} = \frac{L/2}{r} \omega_{n_t}$$

$$\zeta_r = \frac{L/2}{r} \zeta_t$$

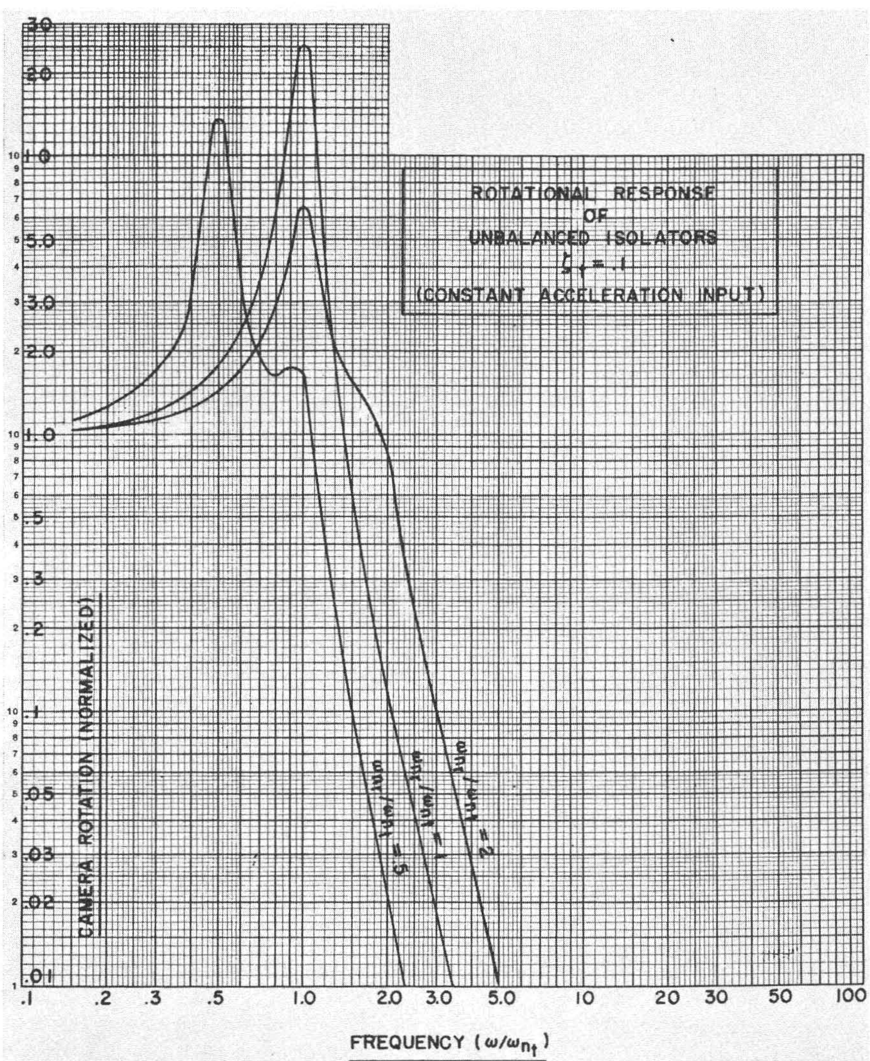

FIG. 3. Rotational response of unbalanced isolators.

$$\theta_c = \frac{(2\zeta_t/\omega_{n_t})s + 1}{(1/\omega_{nt})^2 s^2 + (2\zeta_t/\omega_{n_t})s + 1} \times \frac{1}{(1/\omega_{n_r})^2 s^2 + (2\zeta_r/\omega_{n_r})s + 1}$$

Then:

$$\omega_{n_r} = \frac{9}{6.85}(62.8) = 82.5\,\frac{\text{rad}}{\text{sec}} = 13 \text{ cps}$$

$$\zeta_r = \frac{9}{6.85}(.1) = .13$$

The rotation of the camera may now be expressed by evaluating equation (8).

$$\theta_c = 1.6(10^{-6})X_f \cdot s^2 \cdot \frac{3.2(10^{-3})s + 1}{2.53(10^{-4})s^2 + 3.2(10^{-3})s + 1} \cdot \frac{1}{1.47(10^{-4})s^2 + 3.2(10^{-3})s + 1}$$

where X_f is *double* amplitude vibratory displacement of the airframe in inches and θ_c is *single* amplitude camera rotation in radians.

To determine θ_c at every frequency one must first know what X_f is. Aircraft vibrations vary at different frequencies. These variations are specified for typical conditions in several aircraft and military publications. The document most frequently referenced for vibration spectrums is MIL-E-5272, a government publication covering various environmental conditions for many types of aircraft.

Figure 4 shows the values of X_f given in MIL-E-5272, under Procedure XII

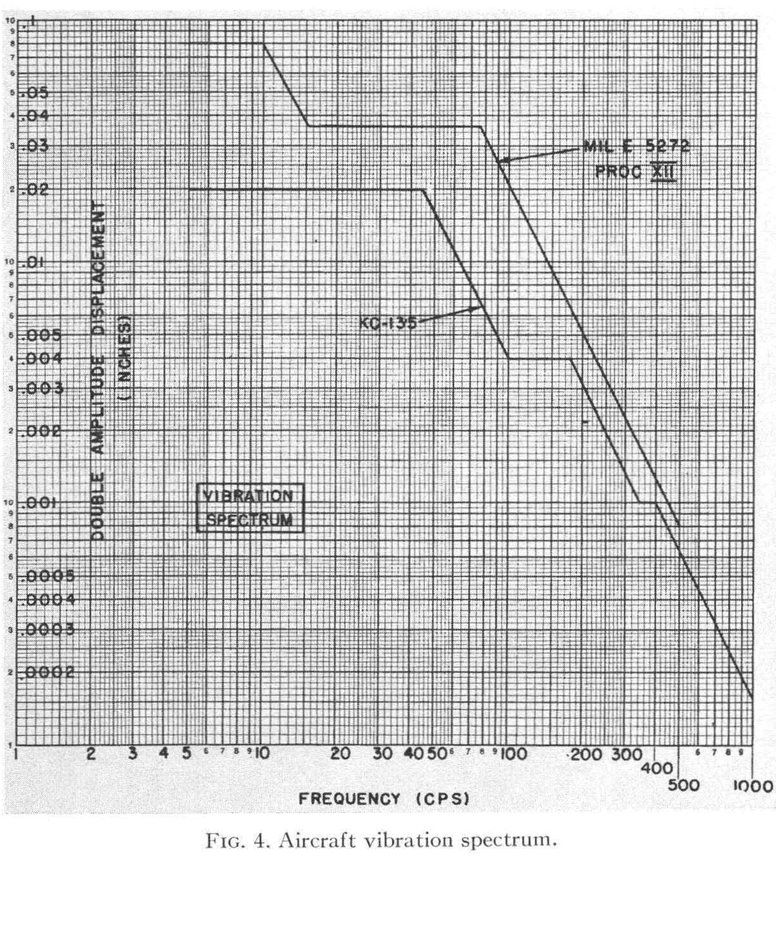

FIG. 4. Aircraft vibration spectrum.

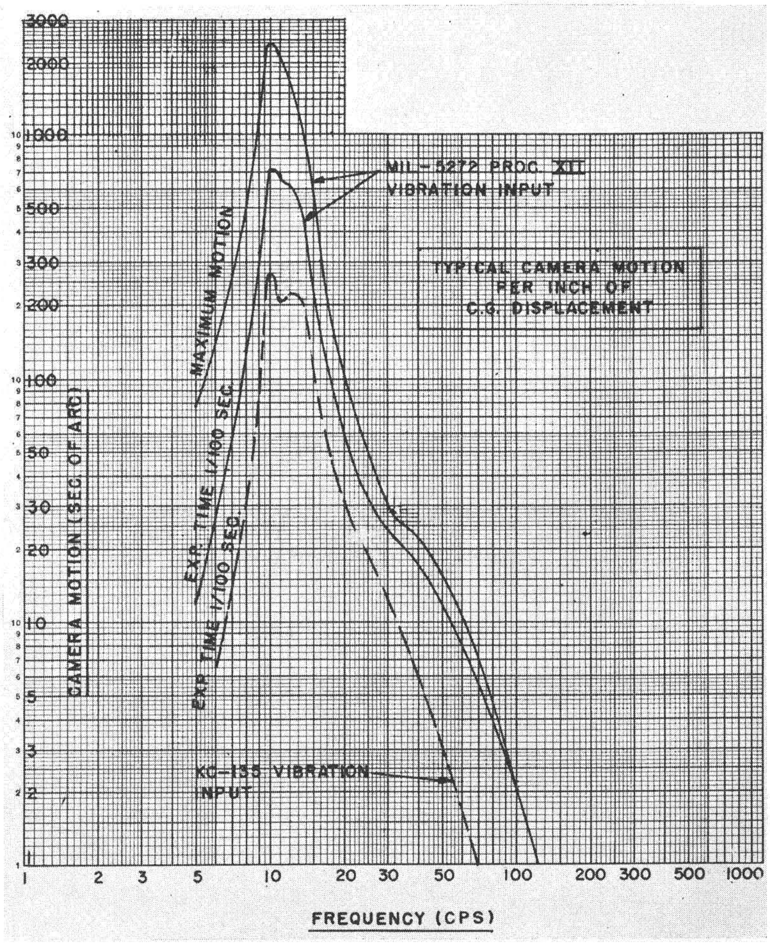

FIG. 5. Typical camera motion per inch of c.g. displacement.

therein, which are applicable to aerial cameras. Also shown in this figure is the vibration spectrum of a modern jet aircraft, the KC-135 (Boeing 707). Both these curves do not represent actual vibration but merely envelopes within which vibrations may be anticipated and hence are undoubtedly more severe than those encountered during normal cruising. Nevertheless, applying these as worst case conditions, the camera rotation that might result is shown in Figure 5.

Maximum motion occurs from disturbances close to the isolator natural frequency. The upper curve has been plotted showing the peak to peak motion at all frequencies. However, the amount of movement affecting the photograph is dependent upon that which occurs during exposure. Motion during a 1/100 sec. exposure has also been plotted in Figure 5. Notice that at frequencies above 50 cycles per second the entire peak to peak displacement can occur during a 1/100 sec. exposure period.

RESOLUTION

The effect of camera rotation upon resolution can be estimated by the expression:

$$R_r = \frac{R_c}{\theta_c R_c + 1} \cdot \frac{1}{f} \qquad \text{(Ref. 2)}$$

where:

R_r = Resultant resolution (lines/mm.)
R_c = Camera-film bench resolution (lines/mm.)
θ_c = Camera rotation during exposure (radians)
f = Camera focal length (mm.)

This expression has been plotted in Figure 6. Here the degradation of 100 lines/mm., 60 lines/mm., and 40 lines/mm. cameras, with various focal lengths, is plotted as a function of the camera motion. It would be somewhat pessimistic to use the values of camera motion shown in Figure 5 directly. They are based upon a full inch of c.g. displacement and vibration envelopes representing worst possible conditions. In actual practice this degree of unbalance should not exist and somewhat smaller motions may be anticipated. Nevertheless, assuming an overall reduction of 5 in the motions plotted in Figure 5, it is interesting to note the amount of degradation in resolution that still results. Referring to Figure 5, 1/5 of peak camera motions during a 1/100 sec. exposure would then be 130 seconds of arc in a MIL-E-5272 environment and 34 seconds of arc in a jet (KC-135) aircraft. Applying these values to Figure 6 indicates that degradation due to these motions would limit resolution, for a 12″ lens cone, to a maximum of 5 lines/mm. and 20 lines/mm. respectively, regardless of the degree of camera resolution which may have been selected.

Summary

It has been shown that the use of vibration isolators to protect aerial cameras from shock and vibration also deserves careful consideration of its effect on photographic results. Unless the c.g. is properly located at the elastic center, camera rotation will result which may seriously degrade resolution. Several factors exist which

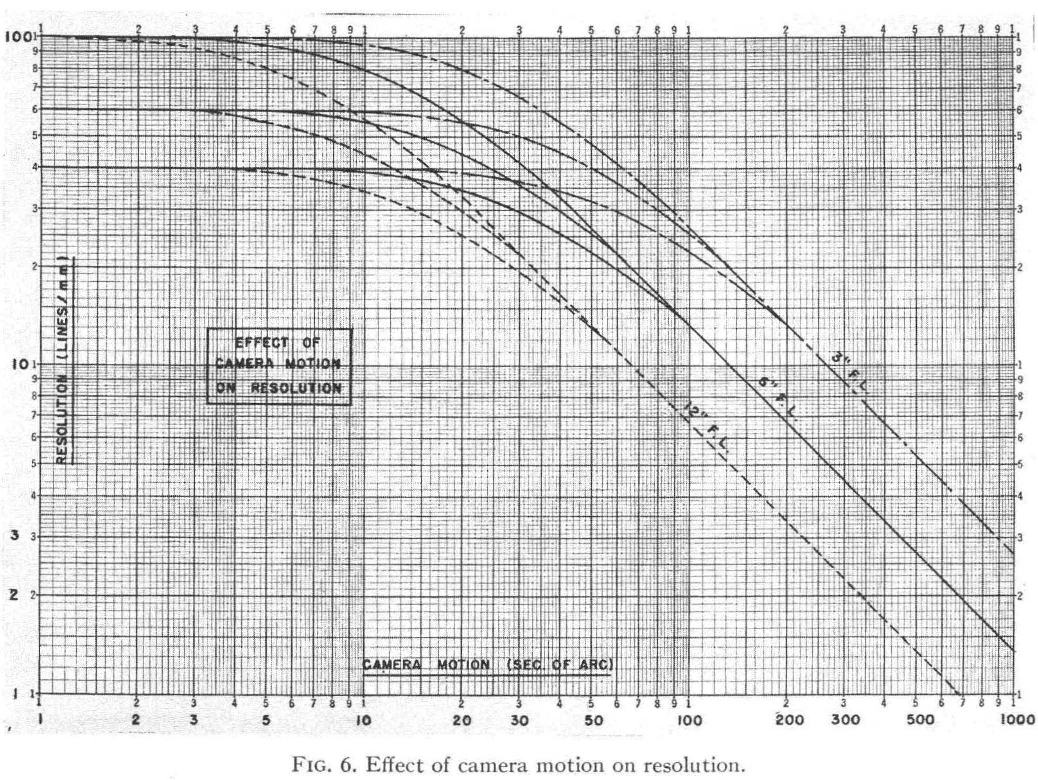

FIG. 6. Effect of camera motion on resolution.

contribute to the uncertainty of this location. Because of this, and the difficulty of statically determining when the c.g. is truly centered, displacements will certainly occur. To further minimize these displacements, it appears mandatory that in-flight film shifts must also be compensated to maintain a reasonable balance. From an examination of the derived equation of motion (8) it is also evident that the longer the isolator base, the smaller will be the rotation due to an unbalance. Structural rigidity, however, must be maintained. This assumption has been made throughout this discussion for both the camera and its mounting support.

It has also been assumed throughout this discussion that vibratory inputs to the camera platform are purely translational. This is the only type data usually available from airframe manufacturers. Aircraft structural flexure, however, must certainly have some rotational modes of vibration of which very little information is known.

Of further importance in the evaluation of the use of vibration isolators as a mounting method is the uniformity of each support. Identical isolators have been assumed here. Unless the spring constants and damping factors of these isolators are carefully matched to each other, additional rotation above that predicted by this analysis can result.

Although other factors, such as atmospherics, aircraft roll and pitch, and image motion exist which affect resolution, the discussion herein has been with reference to those effects caused only by vibration. In particular, the effect of c.g. location on resolution and the manner by which it may be evaluated has been outlined.

References

(1) Timoshenko, S., "Vibration Problems in Engineering," D. Van Nostrand Co., Inc., Princeton, N. J.. 1955.
(2) Timo. Trott, "The Effects of Motion on Resolution," PHOTOGRAMMETRIC ENGINEERING, December 1960.

*Simultaneous Three-Dimensional Transformation of Higher Degrees**

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ABSTRACT: *Transformation equations of second and higher degrees are mainly used to improve the fit on given control points by correcting for deformations incidental to strips or blocks. The common practice that is almost always applied at the present time is to perform the transformation for the horizontal position, X and Y, separate from that for height, Z. The purpose of this paper is to briefly present some of the equations currently used and then give a detailed development of new equations that would allow the simultaneous adjustment of all three coordinates, X, Y, and Z, in one operation.*

Introduction

TRANSFORMATION and adjustment of instrument coordinates to fit available ground-control became a problem with the development of instrumental triangulation on the first-order stereo-plotters. Owing to the absence of better techniques and/or facilities, the early solution to the problem was in the form of graphical

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