A Method Employing Star Backgrounds for Improving the Accuracy of the Location of Clouds or Objects in Space

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ABSTRACT: *The position and velocity of clouds or other objects moving through the upper atmosphere are often determined from a study of simultaneous photographs taken from two or more observing stations against a darkened sky with a background of stars. With correction for atmospheric refraction supplied by tables, the star background is utilized to increase the accuracy of camera orientation by correcting errors inherent in the camera system produced by: (1) film shrinkage, (2) light refraction in the glass fiducial grid plate, and (3) tilt of the camera with respect to the local horizontal. Techniques and empirical formulae are developed for use in analytic data processing with a digital computer to the order of a thousandth of a centimeter. The final procedure can produce angular position determinations of 0.3 milliradian. A method for accurately determining camera focal lengths is also presented.*

INTRODUCTION

GENERAL

THE general problem was that of determining the position (and time rate of change of position) of artificial clouds injected into the upper atmosphere at approximately 100 km. altitude. To accomplish this, several observing stations were set up approximately 100–200 km. apart and photographic data were taken. From these data the cloud position at any time was determined by triangulation techniques. The clouds were injected at times when the background sky was sufficiently dark that the stars in the field of view were photographable with exposure times on the order of a few seconds. This star background was used to determine the orientation in space of each of the several cameras involved in a triangulation study.

The purpose of this work was to develop a system of correction procedures designed to increase the accuracy with which camera orientation and object position in space may be determined. These procedures are especially suited for use with the K-24 camera system, but are also easily adaptable to other similar cameras.

THE K-24 CAMERA AND ITS FILM COORDINATE SYSTEM

The K-24 is a camera equipped with a multielement, 7 inch focal-length, f/2.5 lens. It has a quarter-inch glass plate near the focal plane on which a fiducial grid of fine lines has been accurately ruled 1.270 cm. apart. During exposure, the film is pressed against the rear surface of the fiducial plate, and a brief illumination records the grid lines on the film.

This grid system recorded on the film forms a natural x-y coordinate system in which measurements of the location of images on the film may be made. The center grid cross, near the intersection of the optical axis and the plane of the film, is taken as the origin of this system. Positive x and y are in the general directions of increasing azimuth and elevation, respectively. The origin of this system is referred to as the "center of frame."

TERRESTRIAL AND CELESTIAL COORDINATE SYSTEMS

From each of the observing stations the orientation of the cameras is determined in the azimuth-elevation, or *az-el*, coordinate system. Azimuth is measured east from north, and elevation is measured from the plane perpendicular to the earth radius passing through the site.

The location of stars is known in the celestial coordinate system, illustrated schematically in Figure 1. The right ascension, α, is measured eastward from γ, the vernal equinox (also called the first point of Aries). The declination, δ, is measured from the celestial equator.

The azimuth and elevation of a star from a given observation station are completely determined by knowing the right ascension and declination of the star, the time, and the latitude and longitude of the observing station. With these quantities known, the azimuth and elevation of a star can be found by transformation of coordinates (Albritton et al., 1962). In the following work it will be assumed that all of this information is known and that the necessary star azimuths and elevations have been determined.

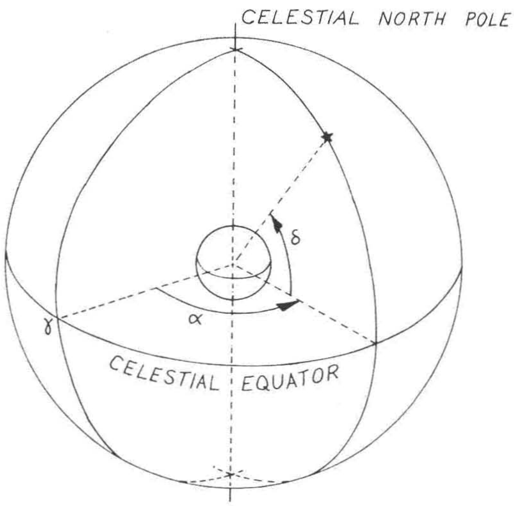

FIG. 1. Coordinates on the celestial sphere.

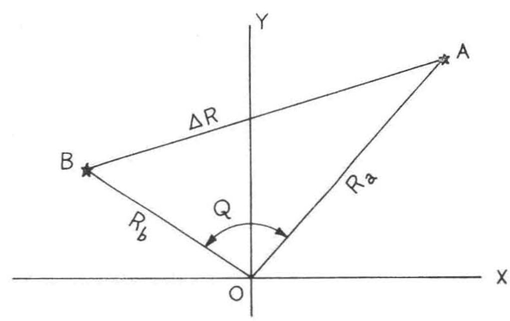

FIG. 2. Typical star image pair for determining focal length.

Camera Focal-Length Determination from the Stars

In order to utilize measurements of image position on the film, the focal-length of the camera lens must be accurately known. The focal-length of a camera lens may be determined from two star images appearing on the film. A typical pair of images, A and B, are shown in Figure 2. They are located at radial distances R_a and R_b from the center of frame, O, and are separated from each other by a distance ΔR. Angle AOB is called Q.

Figure 3 shows the same pair of stars (now labeled A' and B') as they appear on the *az-el* sphere. They form a spherical triangle $A'B'Z$ with the zenith. Sides c_a and c_b of this spherical triangle are the complements of the elevations of A' and B', respectively. The difference in the azimuths of the two stars is angle ΔA. The sides of the spherical triangle formed by the two stars and the projection of the center of frame onto the *az-el* sphere are q, a, and b, as shown. Angle $A'O'B'$ is called Q'.

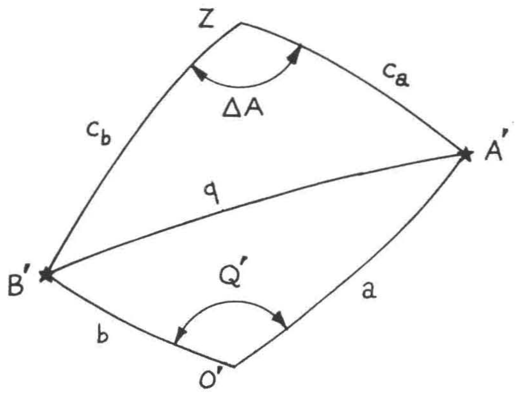

FIG. 3. Star pair used to determine focal length as they appear on the *az-el* sphere.

From the spherical triangle $A'B'Z$:

$$\cos(q) = \cos(c_a)\cos(c_b) + \sin(c_a)\sin(c_b)\cos(\Delta A) \tag{1}$$

Since all quantities on the right of (1) are known, cos (q) is known. The angles a and b are given by

$$a = \text{ARCTAN}(R_a/F) \quad \text{and} \quad b = \text{ARCTAN}(R_b/F), \tag{2}$$

where F is the focal-length of the lens (yet to be determined).

Since the center of frame may be considered to be on the optical axis, and since Q' is measured in a plane perpendicular to this axis

$$Q' = Q.$$

Employing the law of cosines on the plane triangle AOB provides a relation for cos (Q)

$$\cos(Q) = \frac{R_a^2 + R_b^2 - (\Delta R)^2}{2R_aR_b} \tag{3}$$

thus determining cos (Q) in terms of known quantities.

Applying the "law of cosines" for spherical trigonometry to the spherical triangle $A'O'B'$:

$$\cos(q) = \cos(a)\cos(b) + \sin(a)\sin(b)\cos(Q) \tag{4}$$

By employing Equations (1) through (4) and simplifying, one arrives at the relation

$$F = -\frac{V \pm (V^2 - 4UW)^{1/2}}{2U} \tag{5}$$

where

$$U = \cos^2(q) - 1 \tag{6}$$

$$V = (R_a^2 + R_b^2)\cos^2(q) - 2R_aR_b\cos(Q) \tag{7}$$

$$W = \cos^2(q) - \cos^2(Q)R_a^2R_b^2 \tag{8}$$

Equation (5) gives two values for F because of the $\pm$ in the relation. In practice both positive and negative roots may be considered and the value of F with the minimum deviation from the nominal focal length taken as the correct value.

Corrections for the K-24 or Similar Camera System

FILM SHRINKAGE

The film was found to shrink during the time between its exposure and analysis; consequently, this shrinkage must be taken into consideration when measurements taken from the film are used in calculations. Analytic correction is precluded because the shrinkage is not consistent between successive exposures on the same roll of film, nor does the shrinkage occur in any regular fashion over each individual exposure.

The fiducial grid superimposed on each exposure, besides its primary purpose of providing a film coordinate system, serves the secondary purpose of permitting the film shrinkage to be calculated and correction made.

Image displacement is assessed by counting 1.270 cm. increments, the nominal distance between fiducial grids, then measuring distances to the nearest fiducial lines in both x and y directions. If the procedure is carried no further, the effects of shrinkage will be confined to one fiducial square. Further reduction of the effects of film shrinkage may be obtained by measuring the dimensions of the fiducial square in which the image is located and by using a linear interpolation process to correct for the image location within the fiducial square.

To check the accuracy of the spacing of the ruled lines on the plates several measurements of their separation were made. The mean distance between fiducial lines was found to be 1.2697 ± 0.0004 (rms) cm.

Several measurements of the film shrinkage between consecutive fiducial lines showed a mean shrinkage of 0.003 cm., with a maximum of about 0.006 cm. and a minimum of zero. For the K-24 camera system 0.001 cm. amounts to 0.055 milliradian, which corresponds to 8 meters at 150 km. range.

LIGHT REFRACTION IN THE FIDUCIAL PLATE

Although the glass fiducial plate allows for the elimination of errors caused by film shrinkage, it introduces another error into the measurements taken from the film: light refraction in the glass plate. Unlike film shrinkage, this error can be evaluated analytically.

Figure 4 shows a light ray entering the camera with an angle of incidence β measured from the optical axis. If the plate were not present the ray would strike the film (which is pressed against the back of the plate) a distance ρ from the optical axis. Since the light ray is refracted in the plate, it actually strikes the film at a distance r from the optical axis. The angle of refraction is λ, shown in the expanded view of Figure 5, and $\phi = \beta - \lambda$. The focal-length, F, is the distance from the nodal point of the lens to the rear of the fiducial plate, t is the thickness of the glass plate, and n is the index of refraction in the glass plate. The refraction correction, C, would be given by $C = \rho - r$.

By Snell's law

$$n \sin(\lambda) = \sin(\beta) \tag{9}$$

From inspection of Figure 4

$$\sin(\beta) = \frac{\rho}{(\rho^2 + F^2)^{1/2}} \tag{10}$$

$$\tan(\beta) = \rho/F \tag{11}$$

Substituting (9) into (10) yields

$$\sin(\lambda) = \frac{\rho}{n(\rho^2 + F^2)^{1/2}} \tag{12}$$

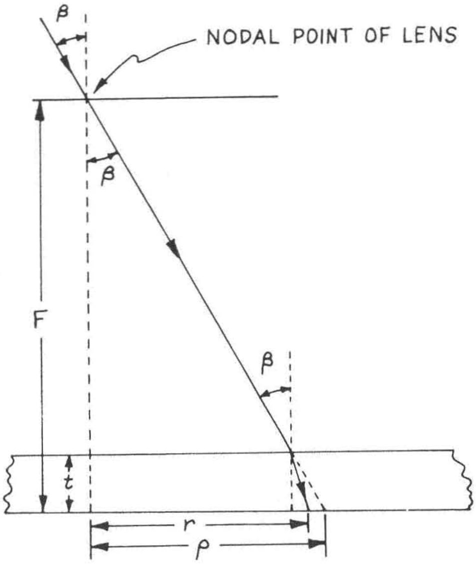

FIG. 4. The path of a light ray through the camera.

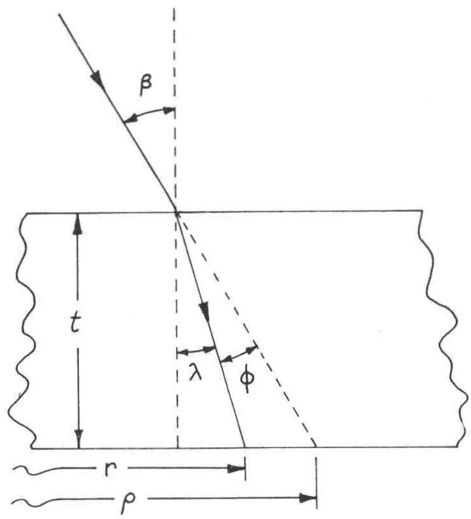

FIG. 5. Expanded view of the path of the light ray through the fiducial plate.

Noting that

$$\frac{r - t \tan(\lambda)}{F - t} = \tan(\beta) \tag{13}$$

and employing Equation (9) through (13), one can arrive at

$$r = \rho + \frac{t\rho}{F}\left(\frac{1}{n(1 + J^2\rho^2)^{1/2}} - 1\right) \tag{14}$$

where

$$J^2 = \frac{n^2 - 1}{n^2F^2} \tag{15}$$

Expanding the radical in (14) into a power series and making use of the method of reversion of series (cf. *CRC Standard Mathematical Tables*) one gets a power series expansion for ρ in terms of r

$$\rho = A_1r + A_3r^3 + A_5r^5 + A_7r^7 + \cdots \tag{16}$$

where

$$A_1 = 1/a_1 \qquad A_3 = -\frac{a_3}{a_1^4} \tag{17}$$

$$A_5 = (3a_3^3 - a_1a_5)/a_1^{10} \tag{18}$$

$$A_7 = (8a_1a_3a_5 - a_1^2a_7 - 12a_3^3)/a_1^{10} \tag{19}$$

and

$$a_1 = 1 - \frac{t}{F}\left(1 - \frac{1}{n}\right) \tag{20}$$

$$a_3 = 1/2 \frac{t}{Fn} J^2 \tag{21}$$

$$a_5 = - \frac{3t}{8Fn} J^4 \tag{22}$$

$$a_7 = - \frac{5}{16} \frac{t}{Fn} J^6 \tag{23}$$

The A coefficients depend only on n, t, and F. The index of refraction and thickness for each glass plate may be measured and used separately. However, measurements of n for several plates yielded a value of 1.524 indicating that this may be used as the value for all plates. The thickness of the plates was found to vary by about 10 per cent from plate to plate, so that separate plate thickness values are used. If the accurate focal-length has been determined by the method previously described, it may be used in the formulas for determining the A coefficients. However, since these formulas do not vary rapidly with F, if the focal-length is known to within about ± 2 mm. (for the K-24, 7 inch lens) the A coefficients determined from such a value will still be quite adequate. For example, if $r=6$ cm., the refraction correction C would be altered about one per cent for a 2 mm. change in the focal-length value used in the calculations.

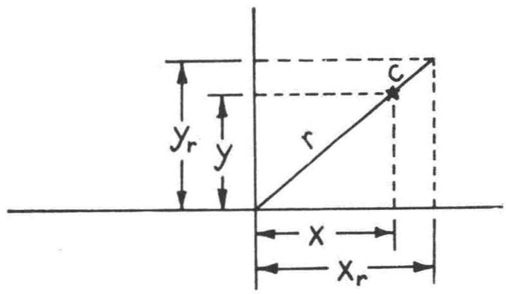

FIG. 6. Actual and corrected location of an image on the film.

Figure 6 illustrates an image as it appears on the film and its corrected location, at which it would appear if there were no plate refraction.

To find the corrected values of x and y(x_r and y_r) the relations

$$x_r/x = \rho/r \quad \text{and} \quad y_r/y = \rho/r$$

are used. Therefore

$$x_r = x(\rho/r) \tag{24}$$

and similarly,

$$y_r = y(\rho/r) \tag{25}$$

If Equation (16) is substituted into Equation (24) and (25)

$$x_r = x(A_1 + A_3r^2 + A_5r^4 + A_7r^6 + \cdots)$$
$$y_r = y(A_1 + A_3r^2 + A_5r^4 + A_7r^6 + \cdots) \tag{26}$$

Equations (26) are not exactly correct if the center of frame does not coincide with the intersection of the optical axis and the plane of the film. If the displacement of the center of frame from the optical axis is known, a new film coordinate system may be set up with the origin on the optical axis, and the values of x and y converted into the new system by translation. Equations (26) would then be correct with the values of x and y so altered. However, if the displacement of the center of frame from the optical axis is of the order of 2 mm., as indicated in the K-24 instruction manual (Vitro, 1954), the error caused by neglecting this coordinate translation is on the order of 0.001 cm. for the K-24 system. This assumes that the only error introduced by neglecting the translation is the error in evaluating the refraction C. That is, for a deviation, δ, of the center of frame from the optical axis $C(r+\delta)-C(r)$ is about 0.002 cm. for $\delta=2$ mm.

Although no direct measurements of the deviation of the center of frame from the optical axis have been made, tests have been made which show that no observable systematic radial error is still present after the plate refraction corrections have been applied, but without correcting for displacement of the center of frame from the optical axis.

Equations (26) may also be reduced in terms to give any desired degree of accuracy. In fact, if the terms in r^4 and higher powers of r are neglected, giving the simple formulas

$$\begin{aligned} x_r &= x(A_1 + A_3 r^2) \\ y_r &= y(A_1 + A_3 r^2) \end{aligned} \tag{27}$$

the corrected coordinates obtained from these formulas will be accurate to about 0.001 cm. at $r=9$ cm.

CAMERA TILT

Referring to Figure 3, it is apparent that a line $O'Z$, connecting the zenith with the projection of the center of frame onto the *az-el* sphere, should correspond to the positive y axis when projected onto the film. However, due to several causes which are either impossible or impractical to compensate for, this projection of $O'Z$ may not correspond to the y axis on the film. In such a case a new x'-y' film coordinate system, shown in Figure 7, could be formed by a rotation of the x-y coordinates through an angle θ, so that the projection of $O'Z$ would correspond to the y' axis.

It is advantageous to know the angle θ, called the camera tilt angle, so that the calculations of line of sight to the cloud or object being photographed may be carried out in the x'-y' coordinates. Note, however, that the camera tilt angle does not affect calculations which depend only on the radial distance from the center of frame, such as the fiducial plate refraction corrections and the focal-length calculations.

The camera tilt angle may be calculated by considering a pair of stars, A and B shown in Figure 8, as their images appear on the film. The images of A and B are located at radial distances R_a and R_b from the center of frame, O; they are separated by a distance ΔR. Point M is the intersection of ΔR and the y' axis. The angle between AM and the y' axis is η_a, and the angle between BM and the y' axis is η_b.

If the coordinates in the x-y system of stars A and B are x_a, y_a and x_b, y_b then the x'-y' coordinates of A and B are x_a', y_a' and x_b', y_b' given by

$$\begin{aligned} x_a' &= x_a \cos(\theta) + y_a \sin(\theta) \\ x_b' &= x_b \cos(\theta) + y_b \sin(\theta) \\ y_a' &= y_a \cos(\theta) - x_a \sin(\theta) \\ y_b' &= y_b \cos(\theta) - x_b \sin(\theta) \end{aligned}$$

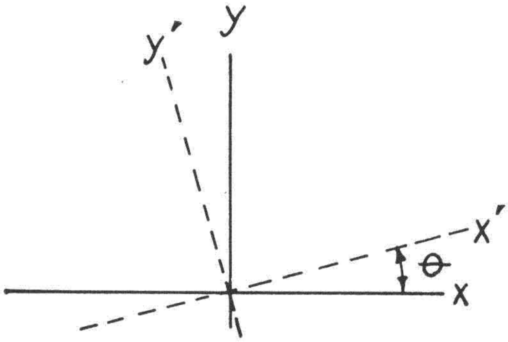

FIG. 7. The x'-y' axes and the camera tilt angle.

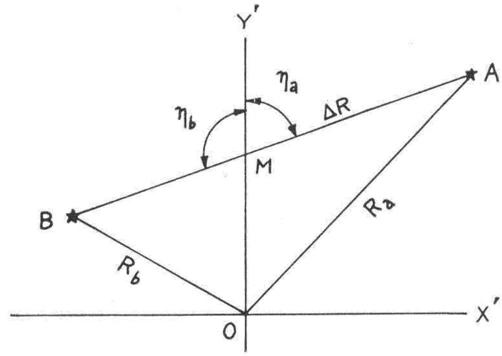

FIG. 8. Star pair used to calculate camera tilt angle, as they appear on the film.

Letting $\Delta x' = x_a' - x_b'$ and $\Delta y' = y_a' - y_b'$ one has

$$\Delta x' = \Delta x \cos(\theta) + \Delta y \sin(\theta)$$
$$\Delta y' = \Delta y \cos(\theta) - \Delta x \sin(\theta) \qquad (28)$$

where $\Delta x = x_a - x_b$ and $\Delta y = y_a - y_b$.

The slope, y_x', of the line joining A and B on the film is $\Delta y'/\Delta x'$ in the x'-y' coordinates. Therefore, using the values of $\Delta x'$ and $\Delta y'$ in (28), one obtains

$$y_x' = \frac{\Delta y \cos(\theta) - \Delta x \sin(\theta)}{\Delta x \cos(\theta) + \Delta y \sin(\theta)},$$

which, when solved for Θ, yields

$$y_x'[\Delta x \cos(\theta) + \Delta y \sin(\theta)] = \Delta y \cos(\theta) - \Delta x \sin(\theta)$$
$$\sin(\theta)[\Delta x + \Delta y(y_x')] = \cos(\theta)[\Delta y - \Delta x(y_x')]$$
$$\theta = \text{ARCTAN}\left(\frac{\Delta y - \Delta x(y_x')}{\Delta x + \Delta y(y_x')}\right). \qquad (29)$$

Relation (29) provides a determination of the camera tilt angle if y_x' is known. The slope y_x' is determined by the value of the angles η_a or η_b by the relations

$$\text{if } x_a' < 0 \qquad y_x' = \tan(\eta_a - \pi/2)$$
$$\text{if } x_a' > 0 \qquad y_x' = \tan(\pi/2 - \eta_a) \qquad (30)$$

or

$$\text{if } x_b' < 0 \qquad y_x' = \tan(\eta_b - \pi/2)$$
$$\text{if } x_b' > 0 \qquad y_x' = \tan(\pi/2 - \eta_b)$$

To determine the values of η_a and η_b, consider the stars A and B as seen on the *az-el* sphere, shown in Figure 9 as A' and B'. The coelevations of A' and B' are c_a and c_b. The coelevation of the projection of the center of frame onto the *az-el* sphere, O', is c_0. Point M' and the sides q, a, and b of the spherical triangle $A'B'O'$ are the projections onto the *az-el* sphere of M, ΔR, R_a, and R_b, respectively, from Figure 8. Several angles of the spherical triangles formed are also shown. Note that η_a' and η_b' correspond to, but are not equal to, η_a and η_b.

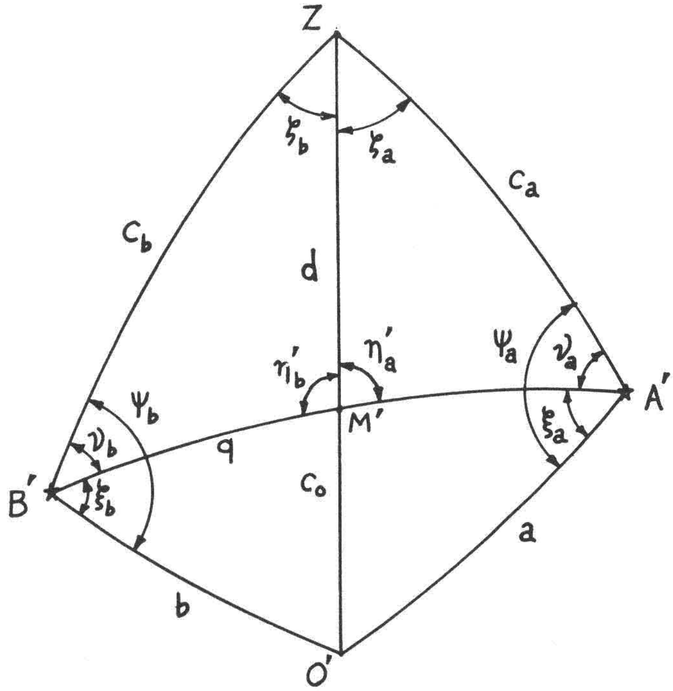

FIG. 9. Star pair used to determine camera tilt angle, as they appear on the *az-el* sphere.

Considering the spherical triangle $A'B'Z$, sides q, a, and b are given by formulas (1) and (2), previously employed. The various angles shown in Figure 9 may be solved for by spherical trigonometry.

From spherical triangle $A'B'Z$

$$\cos(\nu_a) = \frac{\cos(c_b) - \cos(q)\cos(c_a)}{\sin(q)\sin(c_a)} \tag{31}$$

From spherical triangle $A'B'O'$

$$\cos(\xi_a) = \frac{\cos(b) - \cos(a)\cos(q)}{\sin(q)\sin(a)} \tag{32}$$

From Figure 9 the relation of $\cos\psi_a$ is

$$\cos\psi_a = \cos(\nu_a + \xi_a) \tag{33}$$

In other cases the right side of Equation (33) should be replaced by $\cos(\nu_a - \xi_a)$. These cases can be determined from the geometry of the arrangement of the star images on the film.

With the value of $\cos\psi_a$ determined, the spherical triangle $A'Z'O'$ may be solved for side c_0 and angle ζ_a.

$$\cos(c_0) = \cos(c_a)\cos(a) + \sin(c_a)\sin(a)\cos(\psi_a) \tag{34}$$

$$\cos(\zeta_a) = \frac{\cos(a) - \cos(c_0)\cos(c_a)}{\sin(c_0)\sin(c_a)} \tag{35}$$

From spherical triangle $ZA'M'$

$$\cos(\eta_a') = -\cos(\nu_a')\cos(\zeta_a) + \sin(\nu_a')\sin(\zeta_a)\cos(c_a) \tag{36}$$

where ν_a' is angle $ZA'M'$. Angle ν_a is $ZA'B'$ so that in the case of Figure 9,

$$\nu_a' = \nu_a \tag{37}$$

in other arrangements the right side of (37) must be replaced by $\pi - \nu_a$. Geometrical considerations again allow the proper choice of the relations for ν_a'.

Side ZM' of the spherical triangle $ZA'M'$, called d in Figure 9 may be found.

$$\sin(d) = \frac{\sin(c_a)\sin(\nu_a')}{\sin(\eta_a')} \tag{38}$$

The angle ω between the lines from the observing site to M' and to O' (side $O'M'$ of the spherical triangle $O'M'A'$) is given by

$$\omega = c_0 - d \tag{39}$$

This angle ω relates the angle η_a' on the *az-el* sphere to the angle η_a on the film through the relation

$$\tan(\eta_a) = \tan(\eta_a')\cos(\omega) \tag{40}$$

which can be derived from geometrical considerations.

Equation (40) determines η_a, so that the slope y_x' is now determined by Equation (30) and the camera tilt angle θ is given by Equation (29).

Relations (31) through (40) have been a method of evaluating η_a from the spherical triangles and angles associated with star A in Figure 9. A similar procedure for evaluating η_b may be carried out with the spherical triangles and angles associated with star B', by substituting the appropriate counterpart angles and sides of spherical triangles into Equations (31) through (40).

With the camera tilt angle determined, coordinates x and y may be converted to x' and y' by the relations

$$x' = x\cos(\theta) + y\sin(\theta)$$
$$y' = y\cos(\theta) - x\sin(\theta) \tag{41}$$

Corrections for Atmospheric Refraction

CORRECTIONS OF STAR POSITION

The path of a light ray passing from free space to the surface of the earth is deviated from a straight line because of the index of refraction of the atmosphere. Because this refractive index varies not only with height above the earth but with temperature and pressure at the surface of the earth, this deviation is not a simply derived quantity.

If the assumption is made that the atmosphere has a spherically stratified index of refraction, the deviation will depend on the elevation of the line of sight to the light source (or equivalently on the zenith angle, z, the complement of the elevation), but will not depend on the azimuth. Figure 10 shows schematically the effects of atmospheric refraction on the path of a light ray coming from a star considered to be at infinity. Here z is the apparent zenith angle, ζ is the true zenith angle, and r is the angular change in the apparent position of the star due to atmospheric refraction.

Bessel's refraction table (Chauvenet, 1960) provides numerical evaluation of r

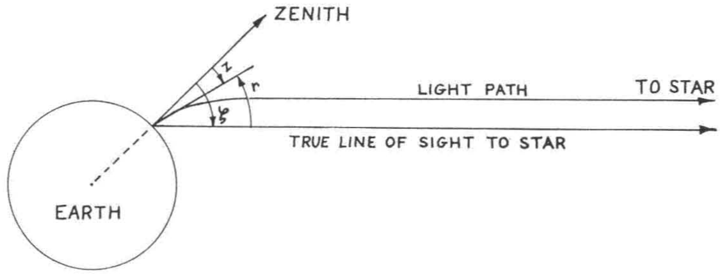

FIG. 10. Schematic representation of the effects of atmospheric refraction on a light ray from a star.

for various temperatures, pressures, and zenith angles. This table is based on the formula

$$r(z) = \alpha\beta^{A}\gamma^{\lambda} \tan (z) \tag{42}$$

$$\beta = BT$$

and gives the values of α, B, T, A, γ and λ. The quantities α, A and λ are functions of z; T and γ are functions of temperature; B is a function of pressure.

If the apparent zenith angle is known and one wishes to obtain the true zenith angle,

$$\zeta = z + r(z) \tag{44}$$

is used. If the true zenith angle is known it is more convenient to know r as a function of ζ so that the relation

$$z = \zeta - r(\zeta) \tag{45}$$

may be used. Based on the formula

$$r(\zeta) = \alpha'\beta^{A'}\gamma^{\lambda'} \tan (\zeta) \tag{46}$$

Bessel's table also gives values for α', A', and λ' as functions of ζ.

Bessel's table is in a convenient form for hand calculations of the effects of atmospheric refraction. But for the handling of large quantities of data, expressions for the various factors in terms of fairly simple functions which could be put into a computer program would be advantageous.

A least squares polynomial fit of the data from Bessel's table gave for γ, B, and T:

$$\gamma = 1.10553 - 2.3904 \times 10\theta + 4.987 \times 10\theta - 7.55 \times 10\theta \tag{47}$$

$$B = 00.337874p \tag{48}$$

$$T = 1.00288 - 8.944 \times 10^{-5}\,\theta \tag{49}$$

where θ is the temperature in degrees Fahrenheit and p is the atmospheric pressure at the surface of the earth in inches of mercury.

The form K_1 and $K_2 \tan^n (z)$ was assumed for α, A and λ, where K_1 and K_2 are constant and n is a function of z (or a constant). This assumption, and trial and error procedures to determine the constants and exponent, led to

$$\alpha = 57.751 - 0.07\,[\tan (z)]^{1.96} \tag{50}$$

$$A = 1 + 2.15 \times 10^{-4}[\tan (z)]^{1.7} \tag{51}$$

$$\lambda = 1 + 0.0018 \tan (z)^{[1-0.821 \sin^2 (9z/7.4)]} \tag{52}$$

The assumption of the form $K_1+K_2 \tan^n (\zeta)$ for α', A', and λ', led to

$$\alpha' = 57.734 - 0.0084[\tan (\zeta)]^{1.97} \tag{53}$$

$$A' = 1 - 2.15 \times 10^{-4}[\tan (\zeta)]^{1.7} \tag{54}$$

$$\lambda' = 1 + 0.0013\{\tan (\zeta)\}^{[1+0.4525(S+|S|)]} \tag{55}$$

where S is

$$\sin\left(\frac{9(\zeta - 45)}{2.9}\right).$$

Using the factors as given above in (42) or (46) gives r in seconds of arc. The expressions (47) through (55) reproduce the values in Bessel's refraction table over the range $0° \leq z < 75°$ to within an accuracy of about 0.01 second of arc. In the range $75° \leq z < 80°$ the accuracy is good to about 0.5 second of arc.

For stars it is the true zenith angle ζ which is known. But the stars are seen, and show up on the film at their apparent zenith angle z. Therefore (46) and (45) are used to correct for star position.

For bodies such as the artificial clouds mentioned in the introduction, the apparent elevation is known. However, for triangulation purposes one needs to know the true elevation, or zenith angle. Equations (42) and (44) are used for this, as will be explained more fully in the following section.

CORRECTIONS OF CLOUD POSITION

The formulas of the previous section apply to stars or other bodies which can be considered at an infinite distance from the observing site. The corrections for the effects of atmospheric refraction on the apparent zenith angle of an object close to the earth are actually dependent on the height of the body above the earth's surface.

Figure 11 shows that an object, such as the artificial cloud mentioned in the Introduction, which has the same apparent elevation as a star, would not have the same true elevation as the star.

The angle ρ shown in Figure 11 is the difference in the true zenith angles of the body and the star. The true zenith angle, ζ_b, of the body can be found from the apparent zenith angle, z_b, by the relation

$$\zeta_b = z_b + r(z) - \rho(z) \tag{56}$$

The function $r(z)$ is the correction found previously by relation (42) and its associated Equations (47) through (52).

Tabulated values (Jones, 1961) of $\rho(z)$ for various heights above the earth were found. These values are based on the same assumption as that for the star corrections, namely a spherically stratified index of refraction in the atmosphere. Also, it is assumed that the index of refraction between the body and the stars is unity. A

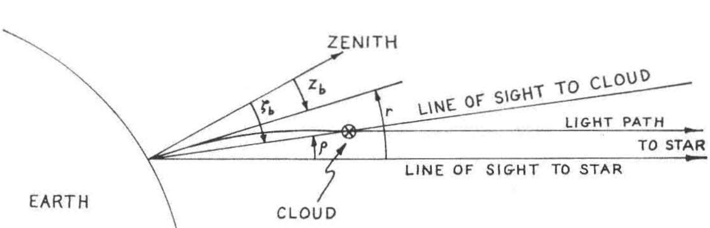

FIG. 11. Difference in true elevations for a star and a body at a finite altitude.

nominal value is assumed for the index of refraction of air at sea level and for the radius of the earth.

It is also desirable to have equations for the function $\rho(z)$ instead of the tabulated values. An approximation for finding $\rho(z)$ for any height h was found to be

$$\rho(z) = \frac{105.4}{h} \rho_{100}(z) \tag{57}$$

where $\rho_{100}(z)$ is the ρ function evaluated at a height of 100 km.

The assumption of the form $A \tan(z) + f(z)$ for ρ_{100} and trial and error evaluation of the constant A and the values of the function $f(z)$ led to the relation

$$\rho_{100}(z) = 5.59 \tan(z) + 4.23\left(1 - \frac{(z-36)^2}{1296}\right) + \Delta(z) \tag{58}$$

where

$$\Delta(z) = -0.29 \sin(3z) \qquad z \leq 60° \tag{59}$$

$$\Delta(z) = 0.28\left(\frac{z-60}{10}\right)^{2.87} \qquad 60° < z \leq 80° \tag{60}$$

All of these constants are in the necessary dimensions to calculate $\rho(z)$ in seconds of arc. Formulas (58) through (60) will reproduce the values of $\rho(z)$ in the tables to within 0.03 second for $h=100$ km. Formula (57) coupled with these formulas will reproduce the values for $\rho(z)$ at $h=200$ km. to within about 0.1 second $0° \leq z < 80°$.

Results and Conclusions

As an example of the accuracy obtainable by the analysis techniques discussed, the following sample results are given.

Data on rocket "Peggy," 1960 Project Firefly series, center of frame azimuth and elevation for the station at Fort Walton Beach, Florida, from the time of release of the cloud to release plus five minutes.

azimuth 152.925 ± 0.007 (rms) degrees

elevation 60.945 ± 0.009 (rms) degrees

tilt angle −0.29 ± 0.01 (rms) degree

The root mean square deviations in azimuth and elevation correspond to 0.12 and 0.16 milliradian respectively. Taking the square root of the sums of the squares of these deviations yields a total angular rms deviation of ±0.011 degree or 0.2 milliradian.

The release position of "Peggy" was determined to be

height (km.) 102.93 ± 0.03 (rms)

latitude (deg.) 30.0277 ± 0.0003 (rms) or ±0.03 km.

west longitude (deg.) 86.5213 ± 0.0002 (rms) or ±0.02 km.

Thus, the total rms error in position is ±0.05 km. Since the cloud is at an average distance of 154 km. from the observing stations, this amounts to an angular error of 0.3 milliradian, which corresponds to an error in position of the image on the film of about 0.005 cm. This order of accuracy is typical for cloud points which are sufficiently sharp and well defined so that point identification does not produce appreciable error.

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References

1. Albritton, D. L., L. C. Young, H. D. Edwards, and J. L. Brown, "Position Determination of Artificial Clouds in the Upper Atmosphere," PHOTOGRAMMETRIC ENGINEERING, September 1962, p. 608.
2. Chauvenet, W., *A Manual of Spherical and Practical Astronomy*, Vol. II (Dover, 1960), Table II, p. 572.
3. Jones, B. L., "Photogrammetric Refraction Angle: Satellite Viewed from Earth," *Journal of Geophysical Research*, April 1961, Vol. 66, No. 4, p. 1135.
4. Vitro, *Instruction Manual KLX 1639*, "Operation and Maintenance, K-24 Camera" (1954), p. 27.

The Optical Specification of Photographic Viewers

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ABSTRACT: *The modulation transfer function of optical instruments to view or enlarge photographic transparencies are determinable from a knowledge of the eye's modulation requirement and the amount of modulation and granularity on the transparency. Two hypothetical cases illustrate the specification of the required performance of a viewer or enlarger. It is concluded that the modulation transfer function of viewers and enlargers should be very high at all the spatial frequencies up to the limiting resolution contained in the transparency.*

Introduction

For photography which is viewed by means of an optical system or is optically enlarged, it is desirable to have a rational approach to specify the required optical performance. Any text on optics gives the basis on which magnification and field of view can be specified, but the resolution performance is not covered. In this discussion, it is shown how the modulation transfer function analysis commonly applied to the photographic acquisition process can be extended to viewers.[1–3] The benefit of this approach is that it is based on the physical processes involved, that it has been proven accurate for cameras, and that it is easy to apply to the viewing of any particular photography.

In what follows, the method will be explained and illustrated with hypothetical examples. While these are meant to be physically reasonable cases, other workers are cautioned to extrapolate with care, or preferably to apply the method exactly to any photography for which they wish to specify the performance of viewers.

Approach to Problem

General

Aerial photographs are obtained by cameras which, in airborne operation, have modulation transfer functions, $T(k)$, similar to that shown in Figure 1; these functions usually decrease steadily as the spatial frequency, k, increases. Such cameras will photograph objects, for which the Fourier components will have all possible modulations (contrasts), M_0, from high to very low. Consequently, the corresponding modulation in the exposure (aerial) image which impinges on the emulsion, M_A, will range from high to very low since

$$M_A = T(k)M_0$$

Even for high-contrast objects, for which $M_0=1$, M_A, decreases to very low values at high spatial frequencies because $T(k)$, decreases.

For the image of the Fourier component of an object to be resolvable on the film, M_A must equal or exceed a modulation detectability limit, M_D.[4–6] In general, M_D is a