*Optimum Field Angle for Aerial Cameras**

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ABSTRACT: *A new, positive criterion is introduced to assess the efficiency of any composition of significant elements of air survey photography: The ratio between height accuracy and neat model area. This efficiency ratio is a function of angular field, negative size, overlap and known error coefficients. From this function the optimum field angle for an aerial camera lens is determined; it is found to be in the region of 120°. A comparison of the area efficiency of different aerial cameras shows that the neat model area which can be plotted with equal height accuracy from ultra wide-angle photographs is nearly twice as great as if wide angle photographs were used. The area efficiency of wide-angle cameras at equal height accuracy is more than 3½ that of normal angle cameras. The results of these purely theoretical considerations show a surprising conformity with the results of correspondihg practical tests.*

THE production of a map by photogrammetric methods requires four major work stages:

1. PHOTOGRAPHY
2. FIXING CONTROL
3. ORIENTATION
4. COMPILATION

Generally in technology the aim of development is to improve the efficiency of the whole process. One strives to improve the quality of the product and at the same time to reduce the effort of labor, time and capital necessary for the production of a required quantity. Therefore the relationships between quality, quantity and effort must also be considered in Photogrammetry as the most important criteria when comparing different techniques. In this case the product is the map; the quality is the accuracy in each respect and the quantity the area to be mapped. Needless to say, the effort of labor, time and capital means money.

Since experience has shown that the coordinate errors in X and Y are smaller or less important than the errors in height, it has become customary to regard the height errors as a yardstick for the accuracy and quality of a photogrammetrically made map.

The other aspects of the quality of a map, such as completeness of interpretation and representation, have to be considered as a task of the compilation.

The amount of labor and time necessary for the compilation is determined by the area and the nature of the region to be mapped. the specifications (map-scale, accuracy, presentation), the method of compiling and the financial investment. Since area, nature and specifications are given, the requirements are for efficient plotting instruments, and recently, for attempts to automate the compilation process itself.

The effort for obtaining the photography, for fixing the control and for the orientation, depends mainly on the number of stereomodels required to cover the area concerned. If this number is to be small, the area covered by a stereopair must be as large as possible; so as small a photo-scale as possible is desirable. However this factor is opposed by the requirements for the specified accuracy and for identification of the details to be represented.

The plottable area per stereopair increases with the square of the flight height above ground. Because of this simple relation and because only the "normal-angle" disposition was available in the initial stages of Photogrammetry, it was convenient to describe the efficiency of the process in terms of the

* To reduce the length of this paper, deductions of formulas have been omitted as far as possible; sometimes only conclusions are given.

This paper is a condensation of a thesis which is to be published in German in the Austrian survey journal, *Oesterreichische Zeitschrift für Vermessungswesen,* under the title "Ueberlegungen zur Wahl von Format und Bildwinkel für die Luftbildmessung."

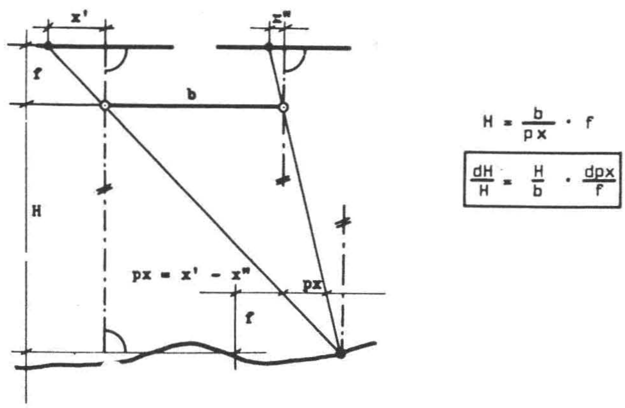

FIG. 1

"*relative height error*"—the ratio between height error and flight height—which is still in general use today. In English-speaking areas the "*C-factor*" definition is usual; this is also a ratio between flight height and height error, the latter being defined in a special way.

The relative height error is, however, suitable only for comparing the results obtained with one particular type of camera and photography, with equal overlap at various flight heights.

The relative height error does not provide a clear definition for judging the results obtained with different lens fields, different formats and different overlaps.

It follows from the foregoing that a relationship between the *height error* and the *plottable area per stereopair* must be regarded as a better criterion for the *efficiency of a given photogrammetric system.*

In order to determine how this relationship depends on the defining elements of a photogrammetric disposition, one starts with the well-known formula for the relative height error of vertical photography and eliminates the flight height with the neat model area.

Using the simple geometric relations and the denotations given in Figures 1, 2 and 3, one obtains:

$$\frac{dH}{\sqrt{A}} = \frac{1}{\sqrt{(1-p)^3 \cdot (1-q)}} \cdot \frac{1}{s} \cdot \frac{f}{s} \cdot dpx \qquad (1)$$

The ratio between the height error dH and the square root of the neat model area A depends therefore only on the longitudinal ($p = \bar{p}/100\%$) and the lateral $q = \bar{q}/100\%$) overlap, the format size s and the focal-length f of the camera and on the parallax error dpx.

The ratio $dh/\sqrt{A}$ is referred to in the following as the "*efficiency ratio.*" According to the foregoing and to the above equation the

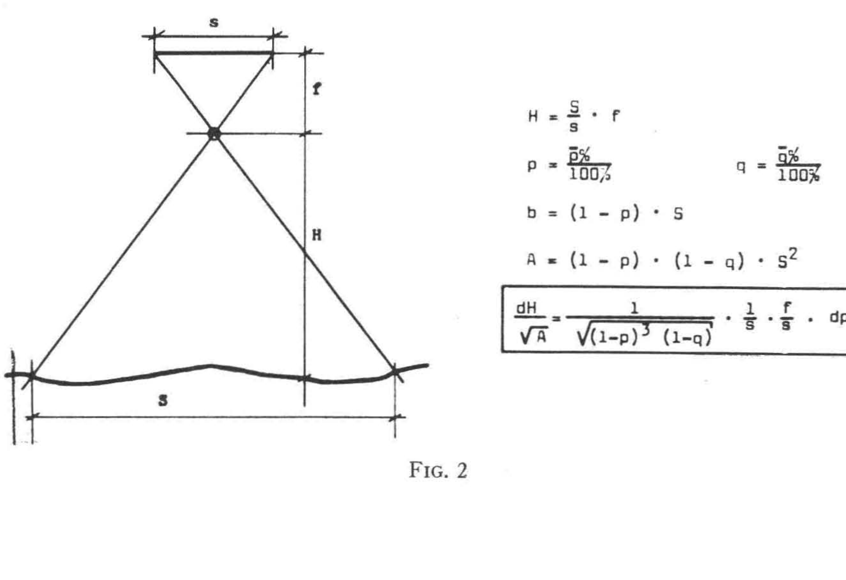

FIG. 2

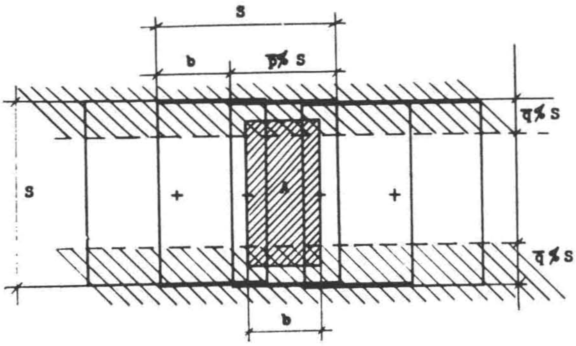

FIG. 3

overlap, the format and the focal-length may be considered as the only independent geometric variables of Photogrammetry.

The parallax error dpx takes into account the influences of all possible errors of the photographic and measuring process on the picture coordinate difference. It is now important to recognise that this influence of the errors on the picture coordinate difference is also a function of the overlap, the format and focal-length. Although this function may not be known exactly, some general statements on its nature and shape can be made.

The parallax error consists of the errors of the picture coordinates. Therefore the dependence of the picture coordinate error upon format and focal length has to be considered.

The photographic picture is influenced by the known optical errors (spherical aberration, astigmatism, coma, curvature of image surface, distortion and chromatic aberration), by the characteristics of the emulsion and its base, by the movement of the camera during the exposure and by atmospheric refraction. All of these influences result on the one hand in positional errors of the picture point, and on the other hand, the smallest recognizable and measurable details of the photograph have a certain magnitude. These positional geometric errors and the resolution-restricted measurability form the picture coordinate error. Therefore it is built up of several components. There must certainly exist constant elements (i.e. resolution of the emulsion); there are elements depending on the focal length and on the angle of incidence of the image-forming rays.

As there is no reason to believe that the absolute magnitude of the picture coordinate error decreases with increasing format size, one can assume for a first approximation proportionality and write for a first error component

$$dx_1 = F_0 + F_1 \cdot s \qquad (2)$$

Initially, the coefficients F_0 and F_1 need not be known. They represent all possible constant and variable influences which depend on the size of the format.

The unevennesses of the emulsion and its base result in radial displacements which are proportional to the tangent of the angle of incidence, as shown in Figure 4. Since the absolute magnitude of the unevenness may also increase with the format the component dx_2 of the error is

$$dx_2 = (U_0 + U_1 \cdot s) \cdot \frac{x}{f} \qquad (3)$$

where the picture coordinate of the point under observation is x and the constants for unevenness are U_0 and U_1.

An analysis shows that the effects of the optical errors are proportional to the smallest photographically resolved distance, provided

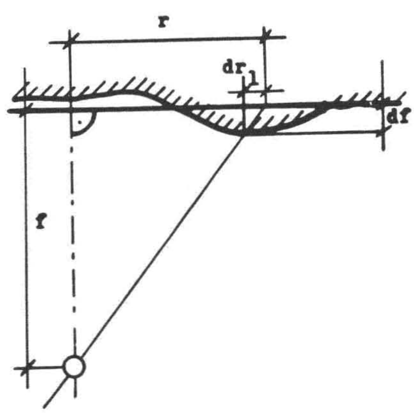

FIG. 4

TABLE I

Lens	*Type*	*Relative Aperture*	*Focal Length*	*Angular Field*	*Format*
Super-Aviogon	SAg 8,85	1:5,6	8,85 cm.	120°	23 cm.×23 cm.
Aviogon	Ag 10	1:5,6	10 cm	90°	14 cm.×14 cm.
Aviogon	Ag 11,5	1:5,6	11,5 cm.	90°	18 cm.×18 cm.
Aviogon	Ag 15	1:5,6	15,2 cm.	90°	23 cm.×23 cm.
Aviotar	At 17	1:4	17 cm.	60°	14 cm.×14 cm.
Aviotar	At 21	1:4	21 cm.	60°	18 cm.×18 cm.
Astrotar	As 30	1:2,8	30 cm.	46°	18 cm.×18 cm.

the regular lens distortion is measured, taken into account during the restitution and therefore eliminated. The smallest resolved distance is the reciprocal value of the well-known number of "Lines per millimeter."

In order to determine how this smallest resolved distance depends on the focal-length and on the angle of incidence, its value measured in microns has been plotted in Figure 5 as a function of the tangent of the angle of incidence for the whole range of aerial survey lenses which are manufactured by the WILD Company at Heerbrugg. The characteristic data of the lenses taken in consideration are listed in Table I.

The focal-lengths vary between 88.5 mm. ($3\frac{1}{2}''$) and 300 mm. ($12''$) and there are angles of field between 46° and 120°. The curves in Figure 5 are based on averages of radial and tangential resolution, computed from the results of the calibration of all cameras produced to date.

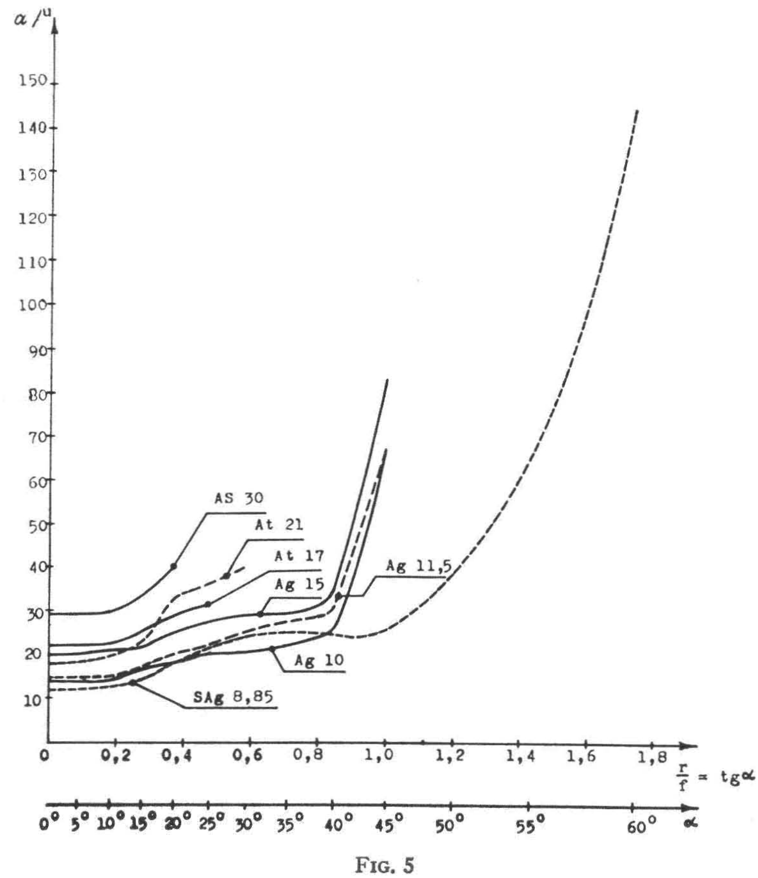

FIG. 5

It can be seen immediately that the smallest distance resolved in the photograph increases with increasing focal-length and increasing angle of incidence. A more detailed analysis shows that an equation of the following form

$$dr_1 = f\cdot[O_1 + O_2\cdot(r/f)^4] \tag{4}$$

represents with very good approximation all measured data. If it is assumed that the constants O_1 and O_2 also contain a coefficient of proportionality, the above equation gives immediately the dependence of this component of the picture coordinate error upon the focal-length and the angle of incidence.

Another influence which reduces the resolution is image movement. If the aircraft passes through a distance T during the exposure time, the image movement is

$$dv_1 = \sqrt{(1-p)\cdot(1-q)}\cdot\frac{1}{\sqrt{A}}\cdot T\cdot s, \tag{5}$$

the flight-height being eliminated by the neat model area.

Considering angular rotation during the exposure about an angle V (Vibration) one finds:

$$dv_2 = f\cdot[1 + (r/f)^2]\cdot V \tag{6}$$

The atmospheric refraction causes the following component of the picture coordinate error:

$$dr_2 = \frac{\sqrt{A}}{\sqrt{(1-r)\cdot(1-q)}}\cdot R\cdot f\cdot[1 + (r/f)^2]\cdot r/s \tag{7}$$

R represents a coefficient depending on the atmospheric refraction index. The flight height is again eliminated by the neat model area.

The combination of the error components described by the Equations (2), (3), (4), (5), (6) and (7) gives the picture coordinate error as a function of the format, the focal-length, the overlap coefficients, the distance from the principal point, the neat model area and of constant coefficients, which are the same for all cameras.

At this stage it is not necessary to make assumptions on the law governing the accumulation of the error components. It is sufficient if the error character of each term is designated in the summation by the $\pm$ sign.

The parallax error consists of the picture coordinate errors of the two pictures of the stereopair. For a preliminary estimate of the relationships it is now certainly permissible to assume that the parallax error cannot possibly be greater than twice the value of the picture coordinate error for the largest angle of incidence in the picture corners. In the corners, $r = s\cdot\sqrt{2}\cdot 1/2$. If this assumption is made in order to eliminate r the structure of the equations for the components remains unchanged. Only the constant coefficients get new values and one obtains the maximum possible parallax error. For simplicity the same capitals as before are used in the following for the new constant coefficients.

If the summation of the error components is substituted for the parallax error dpx in Equation (1), the efficiency ratio is obtained as follows as a function of the format, the focal-length and the various constants. It should be noted that the ratio s/f is directly related to the angular field.

$$\begin{aligned}\frac{dH}{\sqrt{A}} \leq \frac{1}{\sqrt{(1-p)^3\cdot(1-q)}}\cdot\Big\{ &\pm\left(F_0\cdot\frac{f}{s^2} + F_1\cdot\frac{f}{s}\right) \\ &\pm\left(U_0\cdot\frac{1}{s} + U_1\right) \pm\left(O_1\cdot\frac{f^2}{s^2} + O_2\cdot\frac{s^2}{f^2}\right) \\ &\pm\frac{\sqrt{(1-p)\cdot(1-q)}}{\sqrt{A}}\cdot T\cdot\frac{f}{s} \pm\left(\frac{1}{2} + \frac{f^2}{s^2}\right)\cdot V \\ &\pm\frac{\sqrt{A}}{\sqrt{(1-p)\cdot(1-q)}}\cdot\left(\frac{1}{2} + \frac{f^2}{s^2}\right)\cdot R\Big\}\end{aligned} \tag{8}$$

In interpreting this equation it is not necessary to know the values of the various coefficients. It is sufficient to consider the effect of the dominance of each one on the efficiency ratio. The following conclusions can be deduced concerning the format and the angular field:

1) If the coefficient F_0, representing the resolving power of the emulsion, were dominant, then both *image format and angular field* would have to be *as large as possible* for the efficiency ratio to become favourable.
2) The coefficients F_1, O_1, T, V and R, representing film shrinkage, resolution in field centre, translatory and vibration blur, and refraction, all indicate the desirability of *the largest possible angular field.* The format is without importance.
3) U_0: the unevennesses independent of the format indicate *a large format.* The angular field is unimportant.
4) U_1: the unevennesses dependent on the format make *no demands* on format or field.
5) O_2: the fall-off of the resolving power with increasing angle of incidence

indicates *a small angular field*. The format is unimportant.

6) There is no coefficient indicating a small format.

Summarizing, one finds that:

When independent of the values which the coefficients F_0, F_1, O_1, T, V and R might have, and independent of whether the individual errors accumulate at random or systematically, the efficiency ratio becomes more favourable the greater format and angular field are chosen. Only the coefficient O_2 speaks for a small angular field. Therefore the ratio between the coefficient O_2 on the one hand and all other coefficients on the other hand determines an optimum angular field.

With this it would seem that the more significant conclusions have been assembled, which can be made based on logic, on the question of which format and field are the most favorable for photogrammetric purposes.

In order to reach these conclusions the worst case—namely that the parallax error cannot be greater than twice the picture coordinate error in the corners—and all possible sources of error have been taken into account. For the determination of the optimum angle-of-field the following has to be considered:—The effects of image movement and vibration can be kept comparatively small by using appropriate shutter speeds and appropriately designed camera mounts. Since in addition the technique of photography could be improved in this respect, this influence can certainly be neglected when computing the optimum field. The same holds true for the atmospheric refraction. According to the few investigations available ([1] and [2]) this influence is small. Since further an average refraction value can be taken into account when evaluating the pictures, and since the irregularities must be smaller than the regular effects, the influence of refraction can certainly also be disregarded.

If the reasons given for neglecting the image movement and the refraction prove to be inapplicable, the value for the optimum angular field computed below would be *too small*.

Further it has to be taken into account that the elements of the picture coordinate error have to be considered as being accidental and independent of each other. The same

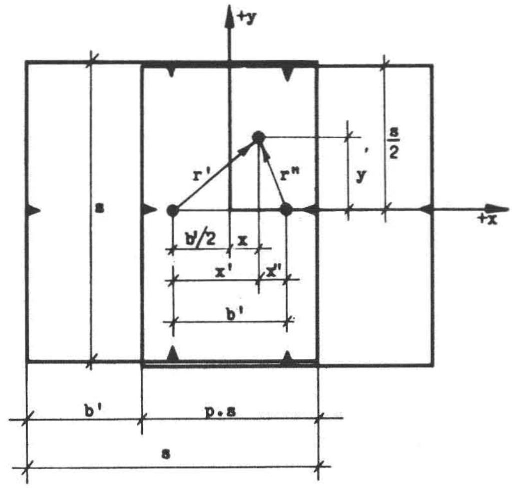

FIG. 6

holds true with respect to the errors of the left and of the right picture. Hence, the error elements may sum up or cancel each other accidentally. The known law of propagation of errors has been used therefore when calculating the parallax error for determination of the optimum. The parallax error obtained in this way is a function of both the radial distance r' in the left and r'' in the right picture.

The radial distances r' and r'' are dependent on each other and can be expressed both by the xy-coordinates of the model area of the pictures, the overlap p and the format s as shown in Figure 6. The parallax error can therefore be computed for any point of the model and for each disposition by means of the coordinates x and y.

The question arises of which point should be considered as being representative for the model. In practice it is the usual practice to judge the quality of a model, not by the errors in its extreme corners, but by the mean square height-error computed from the errors at as many as possible ground-control points distributed over the whole overlap. For determination of the optimum the mean square parallax-error calculated from as many points within the overlap as is possible has to be used therefore. The limit of the mean square parallax error m_{dpx} is obtained by the expression:

$$m_{dpx}^2 = \frac{4}{p \cdot s^2} \iint dpx_{(xy)}^2 dx dy. \qquad (9)$$

Because of symmetry it is sufficient to compute this double integral within the limits of one-quarter of the picture overlap area. The limits are then:

$$0 < x < \tfrac{1}{2} \cdot p \cdot s$$
$$0 < y < \tfrac{1}{2} \cdot s \qquad (10)$$

The integration presents no difficulties and the square root of the result is substituted into Equation (1) for the parallax error dpx. The new function can now be treated according to well-known rules in order to determine the minimum of the efficiency ratio $dH/\sqrt{A}$ with respect to the variable ratio f/s. If all constant coefficients representing the various error influences as before are combined in new constants g and h, one obtains the following equation:

$$(f/s)^8 + g \cdot (f/s)^6 - h = 0 \qquad (11)$$

This equation can be solved. Not considering the undefined sign there exists—given by the signs of g and h—only one real solution which represents the optimum ratio between format and focal-length for aerial cameras. The effective magnitude of the result depends obviously upon the values assumed for the error influence coefficients. If for shrinkage, unevenness, resolution etc. the values are used which can be found in textbooks, or have for example been published by Altman and Ball [3], Ahrend [4] and others, one obtains for the optimum angle of field:

$$\beta_0 = 123°$$

If it is assumed that the shrinkage becomes zero or that the coefficient of proportionality between picture coordinate error and resolution is very much greater than assumed, the coefficient g in the above equation becomes zero. In this case the optimum angle of field would be:

$$\beta_{\min} = 109°$$

If the opposite is assumed (namely that the shrinkage has double, and at the same time the coefficient of proportionality only half the values published in literature), the value of g becomes 8 times bigger. The optimum angle of field would then be:

$$\beta_{\max} = 138°$$

The constant h depends only upon the degree of the fall-off in resolution towards the corners. An improvement of the resolution in the corners would result in a bigger value for the optimum. Due to the eighth power of f/s in the above equation the change would however be completely insignificant, unless the improvement would change the value of h by orders of magnitude.

It can be concluded, that the optimum angular field must lie beteeen 109° and 138°.

Since the extremes chosen above greatly exceed the uncertainty of the values published for individual errors, the value 123° must be a very close approximation to the true magnitude of the optimum field. Therefore it can be stated that:

> The optimum angle of field
> is approximately 120°

The equation obtained for the efficiency ratio as a function of the mean square parallax error can not only be used for the determination but also for deciding if there is any practical significance of the optimum. If the numerical values of the error cofficients finally used for the optimum are inserted already in this equation, the efficiency ratio becomes a constant for each aerial camera, provided equal overlaps are assumed. The expression "efficiency ratio equal a constant" can be resolved in terms of the neat model area. One obtains that:

$$A = A_0 \cdot dh^2 \qquad (12)$$

In words:

The neat model area is proportional to the square of the height error. The value of the coefficient of proportionality depends on the camera used. It can therefore be called "*area efficiency factor*" of an aerial camera.

In Figure 7 the area efficiency factors for different types of cameras in practical use have been plotted as a function of their angular field.

The ratio s/f and the angle of field β are used as abscissae. The ordinate is the area efficiency factor in km.$^2/m^2$. The different types of cameras are denoted by P for plate and F for film cameras. The first figure is the format size, the second figure the focal-length, both in cms. Cameras of equal format and emulsion base are connected by the curves drawn in full.

From this diagram it can be seen how the neat model area increased with the angle of field and the format if a given height accuracy has to be obtained. In detail is to be seen that:

1) The efficiency of ultra wide-angle pho-

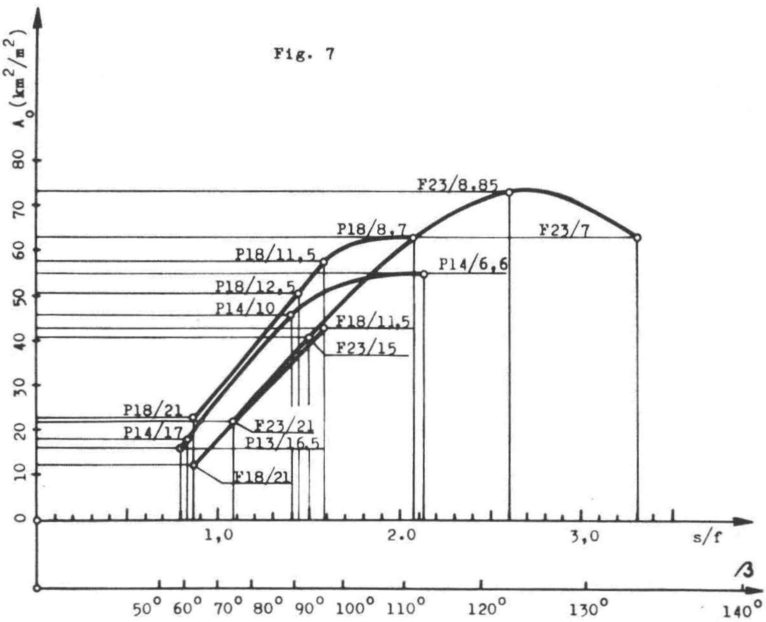

Fig. 7

tography on film is about 1.8 times greater than wide-angle photography of the same format.

This conclusion—based entirely on logic and the figures given in literature for the magnitude of the error influences—is in exact agreement with recently-published results of extensive practical tests [5].

2) Wide-angle photography is about 3.6 times more efficient than normal-angle photography.
3) The efficiency of the two usual film formats 18 cm.×18 cm. and 9″×9″ is practically equal. This is caused by the predominant influence of the film shrinkage (which increases proportional with the format) and the comparatively small unevennesses of the film.

Table II

Film Cameras

s/f	*23/7*	*23/8, 85*	*23/15*	*23/21*	*18/11, 5*	*18/21*
A_0km²/m²	63	73	41	22	43	12
$\frac{dh}{H}$ ‰ [1]	0, 23	0, 17	0, 13	0, 13	0, 14	0, 14

Plate Cameras

s/f	*14/6, 6*	*14/10*	*14/17*	*13/16, 5*	*18/11, 5*	*18/21*
A_0 km²/m²	55	46	18	16	58	23
$\frac{dh}{H}$ ‰ [1]	0, 16	0, 12	0, 11	0, 11	0, 12	0, 10

4) The efficiency of a plate camera is higher than that of a film camera of the same format and angular field.

In order to check the assumptions upon which the deduction of the area efficiency factors has been based, the well-known relative height error can be computed from the following expression:

$$\frac{dh}{H} = \frac{s}{f} \cdot \frac{0.566}{\sqrt{A_0}} \text{‰}^{1} \qquad (13)$$

These relative height errors calculated from the area efficiency factors agree with the values well-known from practical measurements. It can therefore be concluded that the assumptions made are correct.

[1] The denotation ‰ means 1/1000—thus 1‰=0.1%.

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Forest Photogrammetry at a Small Regional College

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STEPHEN F. Austin State College is a small college[1] located in the piney woods of East Texas, celebrating its fortieth anniversary during this school year. Originally, it was a state teacher's college, but it has since broadened its scope to include other degrees in the liberal arts, science, business administration, forestry, agriculture, home economics, fine arts and music, as well as pre-professional training in medicine and related fields, and in pre-engineering. The original orientation of the college was to serve the regional needs of central East Texas, but during the past few years the basic region served by the College expanded to all of East Texas, with a sizeable enrollment from Texas' metropolitan areas, notably Houston and Dallas.

Stephen F. Austin State College has the only four-year degree granting forestry department in Texas. As a result, the forestry enrollment is statewide. A four-year forestry curriculum at a small college is somewhat unique in this country. However, Stephen F. Austin State College has offered forestry since 1946. Total forestry enrollment has steadily climbed, reaching 120 this year.

Photogrammetry was first taught at Stephen F. Austin State College in 1949 with an enrollment of four students. It was handled by six different instructors during its first seven offerings. Since 1955, however, the Department of Forestry has become more stabilized within the College. The author has instructed forest photogrammetry during its last eight offerings and has developed system and continuity to it. The time period for discussion in this paper will, therefore, include only the last eight years.

Eight years ago, in 1956, the Department of Forestry budget was only beginning to assume reasonable proportions, and photogrammetry equipment and photographs were in short supply. The photogrammetry inventory then consisted of a few photographs

[1] Fall semster, 1963, enrollment was 3335 students.