Discussion Paper

WATER DEPTHS FROM AERIAL PHOTOGRAPHS

by G. C. Tewinkel

Discussed by M. C. VAN WIJK,
*National Research Council of Canada at Ottawa**

THE paper entitled "Water Depths from Aerial Photographs" published in the November, 1963 issue of PHOTOGRAMMETRIC ENGINEERING has been received with great interest inasmuch as investigations into the theory of multimedia photogrammetry were also carried out at the National Research Council some time ago.

I would like to make some remarks regarding differences between that work and ours. It was pointed out that in the case of equal heights of the camera station over an undisturbed water surface, the refracted light rays, when elongated, intersect at point b (Figure 1 of the reference paper). However, generally speaking this is not the case[1] as I shall try to demonstrate.

In the figure, I shall define the point of intersection of the perpendicular ab with the water surface as d. Instead of assuming that both refracted light rays L_1c_1 and L_2c_2 intersect the vertical line abd at the same point b, I shall define the intersection of L_1c_1 with ad as b_1 and the intersection of L_2c_2 with ad as b_2. The distance b_1b_2 can be expressed as a function of the distance h and the angles i_1 and i_2. Then from Figure 1,

$$b_1d = (h \tan i_1)/\tan r_1 \quad \text{and} \quad b_2d = (h \tan i_2)/\tan r_2.$$

The vertical distance b_1b_2 can be expressed by

$$\begin{aligned} b_1b_2 &= b_1d - b_2d \\ &= h[(\tan i_1/\tan r_1) - (\tan i_2/\tan r_2)]; \end{aligned}$$

and after applying Snellius' Law,

$$\begin{aligned} b_1b_2 = (h/n)[&(1 - n^2 \sin^2 i_1)^{1/2}/\cos i_1 \\ &- (1 - n^2 \sin^2 i_2)^{1/2}/\cos i_2]. \qquad (1) \end{aligned}$$

According to this expression, the distance b_1b_2 is equal to zero only if $i_1 = i_2$, such as for the cases when a is located at equal distances from L_1 and L_2.

[1] Rinner, K., "Abbildungsgesetz und Orientierungsaufgaben in der Zweimedienphotogrammetrie." Sonderheft 5 der Österreichischen Zeitschrift für Vermessungswesen, Vienna, 1948.

When point a is located in the vertical plane $L_1O_1L_2O_2$, both corresponding light rays L_1c_1a and L_2c_2a are in the same plane and the refracted light rays, extended, intersect in point b.

In addition to a vertical displacement of this point of intersection b with respect to a, a horizontal displacement also occurs. The amount of horizontal displacement can be expressed by the distance $a'b'$ where a' and b' are the vertical projections on the water surface of points a and b respectively. From Figure 1 it follows that

$$c_1a' = h \tan i_1$$

and also

$$\begin{aligned} c_1b' &= (k \tan r_1)/(\tan r_1 + \tan r_2) \\ &= (h \tan r_1)(\tan i_1 + \tan i_2)/(\tan r_1 + \tan r_2). \end{aligned}$$

The horizontal distance $a'b'$ can now be expressed as

$$\begin{aligned} a'b' &= c_1a' - c_1b' \\ &= \frac{h(\tan i_1 \tan r_2 - \tan r_1 \tan i_2)}{\tan r_1 + \tan r_2} \end{aligned}$$

and after applying Snellius' Law,

$$\begin{aligned} a'b' = h[&\tan i_1 \sin i_2(1 - n^2 \sin^2 i_1)^{1/2} \\ &- \tan i_2 \sin i_1(1 - n^2 \sin^2 i_2)^{1/2}] \\ &\div [\sin i_1(1 - n^2 \sin^2 i_2)^{1/2} \\ &+ \sin i_2(1 - n^2 \sin^2 i_1)^{1/2}]. \qquad (2) \end{aligned}$$

A numerical example might demonstrate the meaning of Formulas 1 and 2. For $i_1 = 10^g(9°)$, $i_2 = 40^g(36°)$ and $n = 1.5$, the following distances can be computed:

$$b_1b_2 = 0.27h \quad \text{and} \quad a'b' = 0.06h.$$

The fact that generally two corresponding light rays do not intersect after refraction makes the theoretical formulation more involved.

* Mr. Tewinkel has reviewed this discussion of his paper. He regards it as significant information on the subject and expressed hope that it would be published.—Editor