A Method for Correcting the Effect of Gravity Anomalies on Precision Inertial Navigation Mapping Systems

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ABSTRACT: *The errors caused by gravity anomalies limit the usefulness of inertial navigation systems, particularly when applied to aerial photographic mapping missions. The errors of inertial systems caused by uncorrected gravity anomalies are derived, and two methods of correcting these errors are presented: programming the anomalies in the guidance computer, and gravimetrically sensing the anomalies during flight, with subsequent correction during data reduction. The latter method, being self-contained and requiring no previous gravimetric survey, appears particularly promising.*

INTRODUCTION

AERIAL photographic mapping requires an accurate matching of the aerial photographs to known locations. This has been done in the past by accurately surveyed ground-control points. It is only through recent improvements in the accuracy of inertial navigation systems that a self-contained system may be used for aerial photographic mapping without a previous ground survey of the area to be mapped (Beauchemin, 1963).

Basically, an inertial navigation system uses accelerometers to measure translational motion, gyros to measure rotational motion, and a computer to process this information in computing present position. Since accelerometers cannot differentiate between the reaction force of acceleration and the force of gravity, it is necessary for the computer to compute the gravity vector as a function of the indicated position and to use it to correct the accelerometer output, leaving only that portion of the output corresponding to linear acceleration.

To some extent, then, the accuracy of an inertial navigator depends on the accuracy to which the gravity vector is known as a function of postion, or the accuracy to which it can be practically instrumented. Most present-day inertial systems instrument gravity on the basis of the International Gravity Formula, corrected for altitude by the free-air reduction (Handbook of Geophysics, 1961, pp. 12–68 and 12–69). This means that gravity anomalies will introduce errors into the system. Until recently, these errors have been insignificant compared to the errors caused by the inertial instruments. At this time, however, the accuracy of inertial instrumentation is at a stage where gravity anomalies constitute a major error source unless their effect is compensated.

It is possible to instrument the gravity anomalies in the system, to the extent that they are known. This, however, requires a gravimetric survey of the area to be mapped. A method of conducting this gravimetric survey simultaneously with the photographic mission is the major subject of this paper.

INERTIAL SYSTEMS THEORY

A pure inertial navigation system is a completely self-contained system continuously indicating the position of the vehicle in which it is mounted, here assumed to be an airplane. It does this by measuring linear motion with respect to a known reference coordinate system. Linear motion is measured by linear accelerometers; the reference coordinate system is defined by gyros.

A common instrumentation of an inertial system uses an inertial platform on which the sensing instruments (accelerometers and gyros) are mounted. The platform consists of an inner element containing the instruments, and a system of gimbals connecting the inner element to the airframe to allow the inner element to be stabilized independently of vehicle motion (Figure 1). For the present example, it will be assumed that the inner

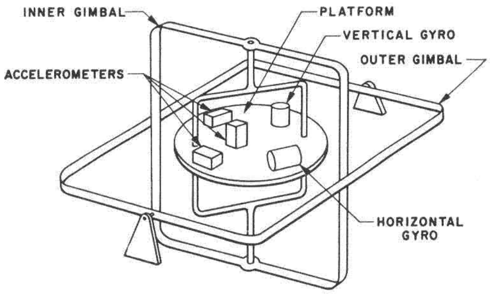

FIG. 1. Inertial platform.

element is stabilized with respect to the local vertical and the local meridian; in other words, the reference coordinate axes are oriented east, north, and up; the accelerometers measure along these axes; and the gyros measure rotations about these axes.

The basic angular reference is the set of gyros. If the inner element of the platform is perfectly aligned with the gyro reference system, the gyro pick-off outputs are nulled. If the inner element is not aligned with the gyros, electrical signals will appear at the gyro pick-offs; these are amplified and used to actuate motors at the gimbal bearings to drive the inner element into alignment with the gyro axes. This constitutes the platform stabilization system.

Gyros tend to keep their axes in a constant direction with respect to inertial space, i.e., the space defined by the fixed stars. In order to keep the system stationary with respect to the rotating earth, the gyros are torqued with a signal proportional to earth's rotation; since the platform is "slaved" to the gyros, as explained before, the platform itself will rotate at the earth's rate, and the system will remain aligned with the local vertical and local meridian as long as it is stationary. For a system in a moving airplane, additional signals are inserted to compensate for the rotation of the vertical and the rotation of the local meridian about the vertical.

The instrumentation of the rotation of the vertical is particularly important in inertial systems. It is clear that the rotation of the vertical is just the angular rotation of the airplane about the earth, and this is equal to the horizontal velocity of the airplane divided by the earth's radius (corrected for geographic location and altitude). The horizontal velocity component along one of the horizontal axes is obtained by integrating the output of the corresponding accelerometer. This is divided by the earth's radius, and applied to the appropriate gyro torquer so as to keep the platform level.

It is, of course, important to level and to align the platform accurately before starting, and to insert initial conditions. Leveling of the platform is achieved by connecting the outputs of the horizontal accelerometers to torque the gyros and thus rotate the platform until the accelerometer outputs are zero; this means that the gravity vector is perpendicular to both accelerometers, and that they are, therefore, both level. The platform may be aligned in azimuth by optical comparison with a surveyed directional reference, or, more conveniently, by gyrocompassing; the latter, however, requires gyros with very low drift rates. Initial velocity is automatically set to zero. Initial position information must be inserted in the computer.

EFFECT OF ANOMALOUS GRAVITY DEFLECTIONS

The effect of a sudden uncompensated gravity deflection on an inertial system will now be considered. The first small figure at the top of Figure 2 indicates schematically an accelerometer, in this case measuring in the x-direction, mounted on the platform. Immediately below it is shown the gravity vector g, displaced from the vertical defined by the International Gravity Formula by an angle δ_y about the y-axis. This causes an accelerometer output

$$\Delta a_x = g \sin \delta_y \cong g\delta_y \tag{1}$$

for small δ_y. This erroneous accelerometer output is integrated to form Δv_x and integrated again to form Δx. At the same time, Δv_x is divided by the earth's radius R and applied to the y-gyro torquer to rotate the gyro, and with it the platform, about the y-axis in accordance with the erroneous velocity indication. The second small figure of

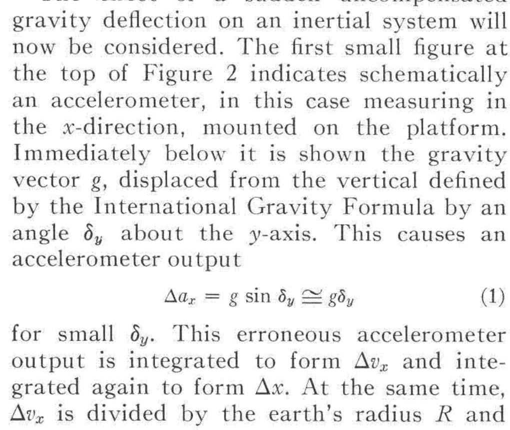

FIG. 2. Effects of step-function anomaly.

Figure 2 indicates the situation after the platform has been rotated through an angle δ_y; the accelerometer is now perpendicular to the gravity vector and its output Δa_x is zero, but Δv_x has reached a value which keeps rotating the platform, making Δa_x go negative. The negative Δa_x returns Δv_x to zero when the platform reaches an angle of $2\delta_y$, as shown in the third figure, with Δx reaching a maximum value of $2R\delta_y$. At this point, $\Delta a_x = -g\delta_y$, causing Δv_x to go negative, until finally the system returns to its original position, just to start the cycle again.

The total period of the cycle is 84.4 minutes at or near the earth's surface; in general, the square of the natural frequency is equal to the local value of gravity divided by the local radius of earth's curvature; the frequency is known as the Schuler frequency, and is characteristic of inertial systems. Figure 2 shows that the position error

$$\Delta x = R\delta_y(1 - \cos \omega_s t), \qquad (2)$$

where ω_s is the Schuler frequency:

$$\omega_s = \sqrt{g/R}.$$

In general, gravity anomalies will not, of course, occur as sudden step functions, but will be random in nature. Gravity anomalies may therefore be considered as random error inputs to a filter, the inertial system. The output of the filter, the position error, will then also be a random variable. The situation may be analyzed on the basis of the statistical theory of servo-mechanisms (James, Nichols, Phillips, 1947, pp. 262–291).

A filter may be characterized by its weighting function, defined as its response to a unit impulse. This is equal to the time derivative of its response to a unit step (James, Nichols, Phillips, 1947, p. 37). The response of the inertial system to a step function is given in Equation (2); the corresponding weighting function is, therefore,

$$W(t) = R\omega_s \sin \omega_s t \qquad (3)$$

The input may be statistically characterized in terms of its auto-correlation function. This is normally defined for a random time-varying signal $y(t)$ as

$$R(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} y(t)y(t+\tau)dt. \qquad (4)$$

A gravity anomaly is, of course, not a time-varying, but a position-varying phenomenon; it becomes a time-varying input to the inertial system, however, due to the speed of the airplane; it can be converted to a time-varying signal by dividing by the airplane speed.

It is possible to develop two-dimensional auto-correlation functions for gravity deflections over parts of the earth's surface (Hirvonen, 1962), perhaps even over the entire earth. However, for purposes of illustration, a one-dimensional auto-correlation function has been computed for a 550-mile stretch along the 35th parallel in the eastern United States, using data from 87 stations. The auto-correlation function over this moderately irregular area was found to be, as a function of distance,

$$R(d) = (4.5 \text{ arc-sec})^2 \exp\{-|d|/33 \text{ n.m.}\}, \qquad (5)$$

where d is distance measured in nautical miles. Equation (5) indicates that the root-mean-square (*rms*) value of the gravity deflection over this stretch is 4.5 arc-sec. For an airplane traveling at 330 knots, the auto-correlation function along this flight path would be

$$R(\tau) = \delta^2 e^{-\alpha|\tau|}, \qquad (6)$$

where

$$\alpha = 330 \text{ knots}/33 \text{ n.m.} = 10 \text{ rad/hr},$$
$$\delta = 4.5 \text{ arc-sec.}$$

The mean-square value of the filtered output is (James, Nichols, Phillips, 1947, p. 289).

$$(\Delta x)^2 = \int_0^t \int_0^t W(u)W(v)R(u-v)dvdu. \qquad (7)$$

Substituting Equations (3) and (6) in Equation (7), one obtains

$$\begin{aligned}(\Delta x)^2 = \delta^2 \frac{Rg}{\alpha^2+\omega_s^2}\Big\{&\alpha\left(t - \frac{\sin 2\omega_s t}{2\omega_s}\right) \\ &- 1/2(1-\cos 2\omega_s t) \\ &+ \frac{2\omega_s}{\alpha^2+\omega_s^2}\left[\omega_s(1-e^{-\alpha t}\cos\omega_s t)\right. \\ &\left.- \alpha e^{-\alpha t}\sin\omega_s t\right]\Big\}. \qquad (8)\end{aligned}$$

For flight times considerably greater than $1/\alpha = 6$ minutes, only the first term of Equation (8) is significant, and Equation (8) may be simplified to

$$(\Delta x) = \delta\sqrt{\frac{Rg\alpha t}{\alpha^2+\omega_s^2}}, \qquad (9)$$

showing the error increasing with the square-root of time. This is shown for this particular example in Figure 3.

Gravity Anomaly Programming

To the extent that they are known, gravity anomalies can be stored in the guidance computer and used to correct the system. Various methods are possible, demanding various

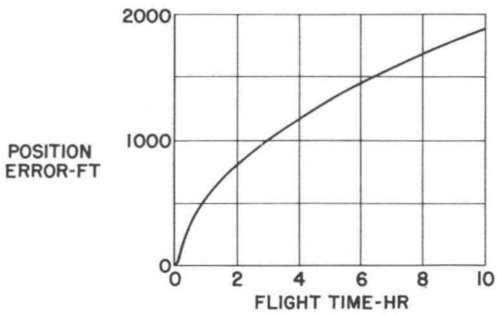

FIG. 3. Effect of random anomalies.

amounts of storage capacity and computation. Three of these will be described briefly.

POINT MASS SIMULATION

The geoid is simulated by the reference ellipsoid plus a finite number of discrete point masses, positive or negative, embedded in it. The gravity anomalies are computed on the basis of the distance, direction and magnitude of the point masses in the vicinity.

ANOMALY GRID

The magnitude and direction of the anomalous gravity deflection is obtained at the corners of a grid system, and inserted in the computer memory. Gravity deflections are computed by interpolating between the values at the corners of the square in which the airplane is flying at a given time.

CHANGING EARTH'S RADIUS OF CURVATURE

The radii of curvature of the geoid in the north-south and east-west direction are stored as a function of position. By using these values (corrected for altitude) in the guidance computer, the platform will always remain level with respect to the geoid, and no gravity correction is necessary.

The point mass simulation method appears to require the least storage, and would be very suitable for an area with many irregularities. The anomaly grid method requires considerable computer storage, but very little computation. The last method, changing earth's radius of curvature, would appear to be very useful in areas where few anomalies exist. It may be desirable for a given system to be capable of programming all three methods, only one to be used for a given mission.

AIRBORNE GRAVIMETER

It is often necessary to provide photographic mapping of areas which have not been gravimetrically surveyed. If inertial navigation systems are to be used for such missions, they must either be unaffected by gravitational anomalies, or they must be able to measure the anomalies and correct for them. It has already been seen that inertial systems are inherently sensitive to anomalies; fortunately, a method for measuring anomalies is possible. This method measures the variation in the magnitude of the gravity vector. The deflection of the gravity vector must be computed after the flight using, for instance, the Vening Meinesz equation (Vening Meinesz, 1928), and the flight data corrected accordingly.

The vertical accelerometer on the inertial platform measures the magnitude of the gravity vector, plus vertical acceleration, plus various Coriolis and centrifugal terms. The Coriolis and centrifugal terms are compensated by the guidance computer. The gravity vector is itself composed of the standard gravity at a reference altitude, the free-air correction from this altitude, and the gravity anomaly. If an accurate independent altitude reference is available, it is possible to separate the gravity anomaly from the other components.

Figure 4 is a block diagram of the airborne gravimeter. The output of the vertical accelerometer is shown to be

$$a_z = g_r - \frac{2g_r}{R_r}\Delta h + \Delta g + \ddot{h} + c_z, \qquad (10)$$

where the subscript r denotes the magnitude at the reference altitude, Δg is the value of the gravity anomaly, Δh is the altitude measured from the reference altitude, and c_z denotes the centrifugal and Coriolis terms. An independent altimeter provides Δh; this may be a hypsometer zeroed at the reference altitude and henceforth corrected for isobaric changes by geostrophic wind measurements.

Centrifugal and Coriolis correction terms and the standard gravity at the reference altitude are supplied to the output of the accelerometer by the guidance computer, the free-air correction is supplied by the altimeter, as shown in Figure 4. This leaves only a signal porportional to the sum of the value of the gravity anomaly and vertical acceleration. If, for the moment, it is assumed that the correction introduced in the next adding network is indeed equal to the gravity anomaly, the remaining vertical acceleration is then integrated twice, providing an inertial measurement of Δh. This is now compared with

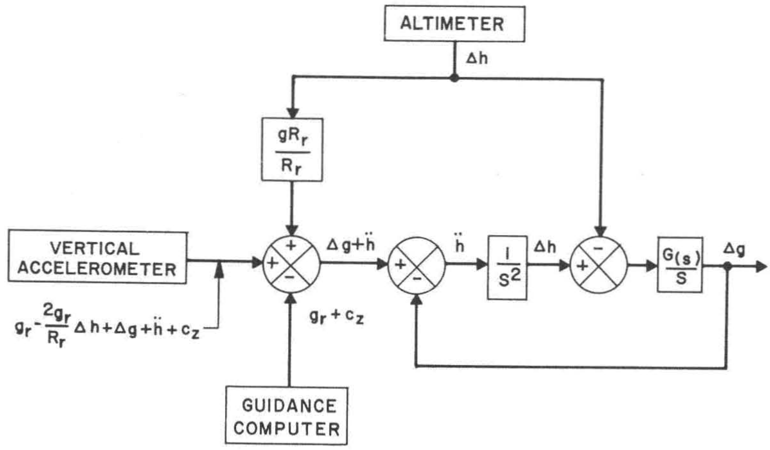

FIG. 4. Airborne gravimeter.

the value of Δh from the altimeter. If there is any difference, it must be due to an uncorrected gravity anomaly; this difference is then fed back through the transfer function $G(s)/s$ to provide the gravity anomaly correction. The dynamics of the loop are such that the output will, in the steady state, be equal to the gravity anomaly. By designing the loop such that its natural frequency is high compared to that of the aircraft, the output will give an almost perfect indication of the instantaneous gravity anomaly, assuming small instrument errors.

It is, of course, recognized that errors will be introduced by errors in the accelerometer, in the hypsometer, and in the geostrophic wind corrections. Recent studies at Bell Aerosystems Company indicate that the system errors can be reduced considerably by making a final measurement at the end of the mission at the point where the hypsometer was zeroed; this permits reduction of the average error over the mission to zero, and therefore reduces the instantaneous effective error over the entire mission.

CONCLUSION

Two basic methods of correcting inertial navigation systems for the effect of gravity anomalies have been presented: programming the effect in the computer, and measuring the anomalies during the mission. These methods are not mutually exclusive; it may be desirable to provide a rough correction by programming, a fine correction by measurement. A judicious combination may minimize the requirements on both the airborne guidance computer and the data reduction computer.

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See Announcement of International Congress on page 319.