ing. Similar success has been achieved over the years in the stocking of other nearby lakes. There appears to be no reason why the lake shown in figure 5, and many others like it, could not have been producing equally good catches of fish all these years.

It is, perhaps, fortunate that this paper has nearly reached its allowed length. Accusations of the type I have been making, and could continue to make, are not pleasing to hear, particularly at this meeting where we are assembled to honor the great Professor Earl Church. Yet I sense that he might have welcomed the presentation of at least one paper that sounded a sour note. Otherwise we might have been sweet-talked by some of the more optimistic papers into thinking that the value of photography, as an unbiased source of truth, is universally recognized.

Let us realize that there still is a big job to be done in selling photogrammetry and photo interpretation. To this end, let us appreciate the selling opportunity that is ours at meetings such as this; to this end let us also recognize the importance of our continuing, as a Society, to produce manuals and periodicals of the highest possible quality. In addition, let us not overlook the many opportunities for selling photogrammetry and photo interpretation that are afforded to each of us individually. Then let us proceed to do our selling job, both individually and corporately, with the same vigor, skill and enthusiasm as Professor Church himself exhibited in his lifetime of service to our profession. I suspect that he would regard our solemn resolve to do this as the highest tribute we could pay him on this commemorative occasion.

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*Adjustment of Elevations Derived from Instrumentally Bridged Aerial Photographs**

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Abstract: *Elevations derived from instrumentally bridged strips of aerial photographs exposed at a flight height of 9,000 feet were adjusted by three different methods: with a graphical method, with the ITC-Jerie analog computer, and with a mathematical method. The RMSE values of the adjusted elevations for the three methods were 4.6, 3.0, and 3.3 feet, respectively. Because of the accuracy and expediency found in the mathematical approach, the method will be evaluated under operational conditions.*

Introduction

In U. S. Geological Survey quadrangle mapping, supplemental elevations are determined for three principal purposes: to provide vertical control for individual stereomodels in photogrammetric compilation, to furnish map spot elevations, and to provide data for map accuracy evaluations. Because the required field operations are time-consuming and costly, the Topographic Division of the Geological Survey is investigating office methods of establishing supplemental elevations. In an earlier study, photogrammetri-

* Presented at the 1963 Semi-Annual Meeting of the American Society of Photogrammetry, Wellesley Island, Alexandria Bay, New York, September 13, 1963.

† Publication authorized by the Director, U. S. Geological Survey, Washington, D. C.

cally established elevations were adjusted by a graphical method and with an ITC-Jerie analog computer. A mathematical adjustment is now being appraised using the same input data as was used in testing the graphical and analog computer methods.

Although the primary purpose of this report is to describe the mathematical adjustment procedure and to present the results of a test of the method, the procedures and test data for the graphical method and the ITC-Jerie analog computer method are also briefly outlined.

Photogrammetric Procedure

An area of approximately 110 square miles near Marion, Ohio, was selected for testing all three methods. Photographic coverage consists of seven parallel east-west strips of vertical photographs taken at an altitude of 9,000 feet. The number of photographs in the strips varies from eight to nine.

Elevations were determined for 246 points by second-, third-, and fourth-order field survey procedures. All of the fourth-order survey lines closed within 0.9 foot, and only two failed to close within 0.6 foot. Therefore, it is presumed that errors in the field-established elevations do not seriously influence the adjustments, nor impair their value for use in evaluating the adjustments.

Thirty-five of the points with field-established elevations were designated as vertical control points (Figure 1). They were selected so that each bridge would contain eight vertical constraints. The remaining 211 points were designated test points for evaluating the results of the adjustments.

Vertical bridging for the tests was done on a long-bar with ER-55 projectors. Before beginning bridging operations, the vertical deformation, or sag, in the long-bar slate was determined. The z-deformation along lines parallel to the y-axis (transverse to the flight line) was found to be negligible; the deformation along lines parallel to the x-axis (parallel to the flight line) was appreciable (Figure 2). To compensate for this deformation, it was necessary to apply corrections to the tracing table readings amounting to as much as six feet at ground scale.

In constructing each bridge, the y- and z-scales of the projectors for a stereomodel near the center of the strip were set equal, and the photographs were relatively oriented by the two-projector or swing-swing method. The bridge was generated by relatively orienting successive projectors to the left and right of this base model. The scale of the bridge was adjusted in accordance with horizontal control at the ends of the strip so as to fix the scale of the stereomodels at 1:5,250.

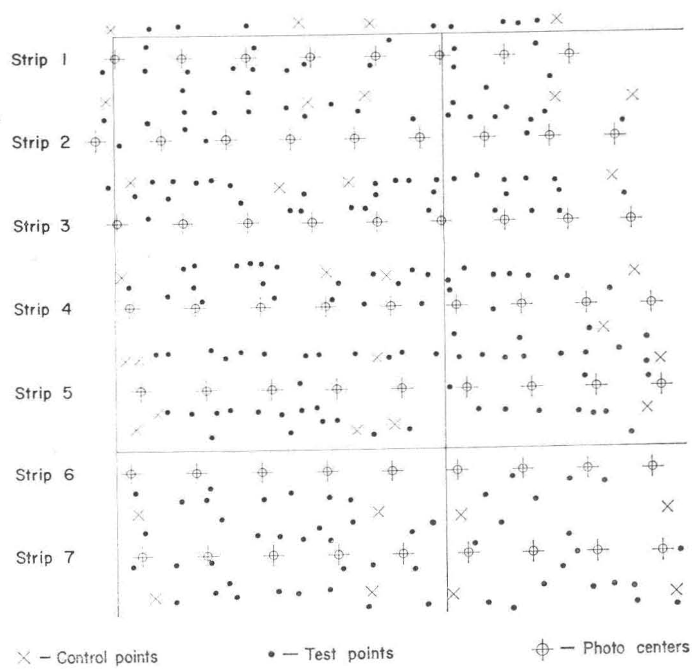

FIG. 1. Control and test point distribution for Marion, Ohio, test area.

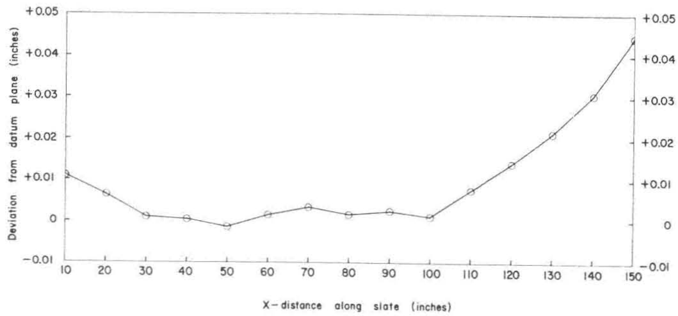

FIG. 2. Profile of surface of long-bar slate.

The bridge was then leveled from front to back based upon vertical control in the leftmost model of the strip. The dial of the tracing table was indexed on this vertical control and the z-coordinates of all control points and test points in the strip were observed and recorded. The z-coordinate for any point common to two adjoining models in the same strip was taken as the arithmetic mean of the observations from the two models. The horizontal position of each observed point was plotted on a base sheet.

The same input data has been used for testing the three methods, except for minor variations in the number of vertical constraints and test points.

Graphical Adjustment

In the graphical adjustment, vertical error curves based on errors in the observed coordinates of control points were constructed for each strip. To accommodate strips with cross-tilt, it was necessary to construct two curves for each strip—a front curve based upon errors at the control points along the near edge of the strip and a rear curve based upon errors at the control points along the far edge of the strip (Figure 3).

After the error curves were constructed, the observed z-coordinate of each point was adjusted in accordance with the error curve. The front curve was used to adjust the readings for points along the front of the strip, and the rear curve was used to correct readings along the rear. Adjustment values for points between the two were interpolated.

The root-mean-square error of the graphically adjusted elevations, based upon 35 control points and 211 test points, was 4.6 feet.

To ascertain the reliability of the results, bridging was repeated for strips 5, 6, and 7 and the observed z-coordinates were adjusted. The RMSE of the graphically adjusted elevations for these three strips was 4.9 feet. This value is identical to the RMSE of the same three strips for the first run.

Adjustment by the ITC-JAC Method

In the ITC-JAC method of adjustment, the horizontal positions of the vertical control points were circled on a base sheet taped to a base board. Metal posts representing the control used as constraints were centered in the circles and affixed to the sheet. In Figure 4 the north edge of the base sheet is to the left and the east-west flight lines are parallel to the left and right edges of the photograph.

The scale of the base sheet used for the ITC-JAC adjustment was 1:18,000. This fixed the distances on the base sheet between

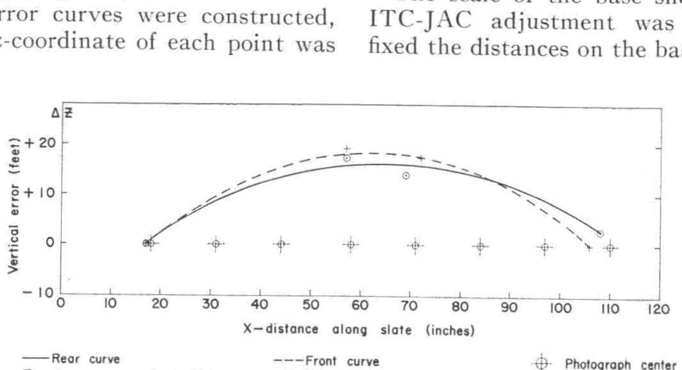

FIG. 3. Front and rear error curves for flight line 1.

TABLE 1

RESULTS OF GRAPHICAL ADJUSTMENT FOR 7 STRIPS AND A REPEATED ADJUSTMENT FOR 3 OF THE STRIPS

Strip Number	*RMSE of Initial Adjustment in Feet*	*RMSE of Repeatability Test in Feet*
1	3.0	
2	5.1	
3	5.6	
4	5.4	
5	5.1	5.9
6	4.1	4.1
7	5.5	5.2
RMSE, all strips	4.6	4.9
Number of control points	35	16
Number of test points	211	100

exposure stations at approximately 100 millimeters so that the mechanical limitations of the analog computer would not be exceeded. The vertical scale was exaggerated to make one millimeter when measured perpendicular to the base board represent one foot in elevation at ground scale.

Plastic axial rods were suspended from the brackets shown above the base sheet in Figure 4. Alternate brackets were offset eight millimeters in the direction of the flight lines to avoid mechanical interference between cross-ribs attached to rods. It was also necessary to separate the vertical datum of adjoining strips by 100 millimeters. The datum or zero error plane of strips 1, 3, 5, and 7 was fixed at 200 millimeters from the base board, and that of strips 2, 4, and 6 were set at 300 millimeters from the base board.

Strips were fitted to control by securing appropriate cross-ribs to the constraining posts at control points through spring linkages. The assemblies of adjoining flights were then linked together at corresponding tie points through spring equipped assemblies so as to form a single block comprising seven strips (Figure 5). The springs constraining the assembly to control were twice as strong as those constraining the strips to each other.

The apparent error in the observed z-coordinate of each point was accepted as the distance measured from the base board to the pertinent cross-rib, minus the distance from the base board to the zero error or datum plane (Figure 6). The apparent error was subtracted from the observed coordinate to obtain the adjusted elevation of a point.

The RMSE of the adjusted elevations based upon 33 control points and 212 test points was 3.0 feet.

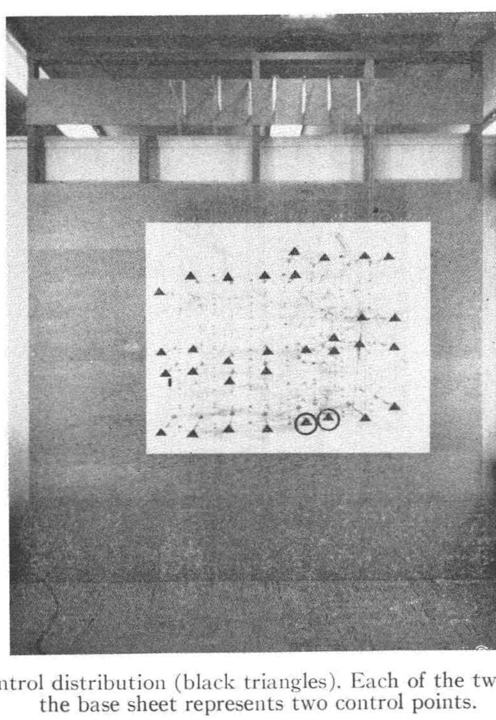

FIG. 4. Vertical control distribution (black triangles). Each of the two triangles circled on the base sheet represents two control points.

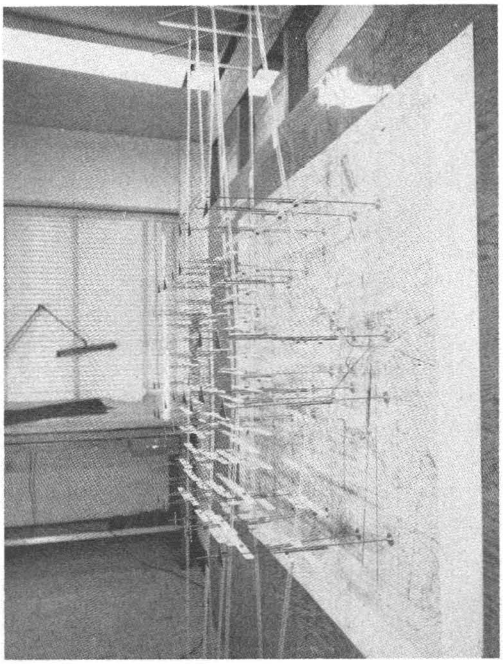

FIG. 5. The assembled ITC-Jerie analog computer.

Mathematical Adjustment

The mathematical adjustment is being investigated under a two-part research proposal. The objective of the first part is to determine whether the analytical approach has potential for achieving acceptable accuracy at reasonable cost. The continuance of the project through the second part is contingent upon the results of Part 1. Part 2 proposes that the analytical approach be refined by mathematically removing discontinuities in the error surfaces before entering the adjustment phase, and that this revised approach be tested with the same data as for the other tests. The first part of the project has been successfully completed.

The equation used to express the vertical error surface for a given strip is

$$A_s x^2 + B_s x + C_s + y(D_s x^2 + E_s x + F_s) = \Delta z, \quad (1)$$

where Δz is the apparent error in the z-coordinate value of an observation; x and y are strip coordinates based on a coordinate system with origin at the centroid of strip s; and A_s, B_s, C_s, D_s, E_s, and F_s are parameters determined for strip s [1].

The observation equation formed at a control or tie point i of strip s is

$$A_s x_i^2 + B_s x_i + C_s + y_i(D_s x_i^2 + E_s x_i + F_s) - \Delta z_i = v_i, \quad (2)$$

where Δz_i is the apparent error at point i (the observed elevation minus the true or accepted elevation); x_i and y_i are the strip coordinates of point i; and A_s, B_s, C_s, D_s, E_s, and F_s are unknown parameters to be determined for the condition in which the sum of the squares of the residuals, i.e., the v_i's, are minimized.

To solve for the unknown parameters for each strip, an observation equation (Equation 2) is formed at each vertical control point and, for a strip-to-strip fit, at each tie-point in the strip. With Δz_i and the x- and y-coordinates of each control point i of strip s known, the unknown parameters A_s, B_s, C_s, D_s, E_s, and F_s in the observation equations are determined by solving six normal equations formed according to the standard procedure for minimizing the sums of the squares of the residuals—the v_i's in the observation equations (Figure 7).

After the parameters are determined by solving the normal equations, they are used in Equation 1 to compute the value of Δz at each observed point. This value is subtracted algebraically from the observed z-coordinate of the respective point.

The computer program, written in machine code for the Burroughs 220 computer, is in double-precision, floating-point arithmetic. Data are brought into the memory of the

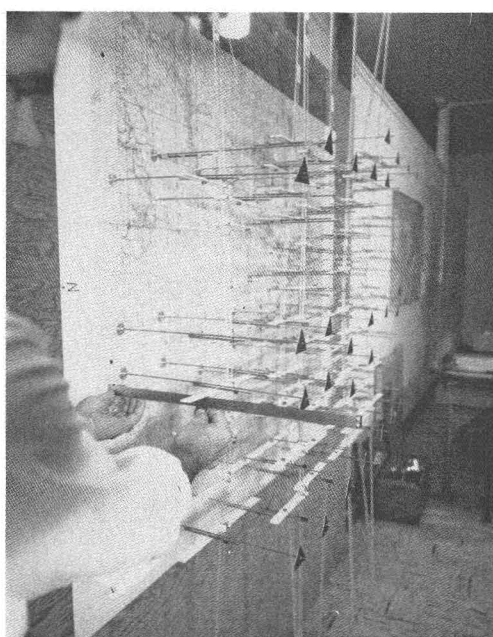

FIG. 6. Measuring the apparent error in an observed elevation.

$$
\begin{bmatrix}
\sum wx^4 & \sum wx^3 & \sum wx^2 & \sum wx^4y & \sum wx^3y & \sum wx^2y \\
\sum wx^3 & \sum wx^2 & \sum wx & \sum wx^3y & \sum wx^2y & \sum wxy \\
\sum wx^2 & \sum wx & \sum w & \sum wx^2y & \sum wxy & \sum wy \\
\sum wx^4y & \sum wx^3y & \sum wx^2y & \sum wx^4y^2 & \sum wx^3y^2 & \sum wx^2y^2 \\
\sum wx^3y & \sum wx^2y & \sum wxy & \sum wx^3y^2 & \sum wx^2y^2 & \sum wxy^2 \\
\sum wx^2y & \sum wxy & \sum wy & \sum wx^2y^2 & \sum wxy^2 & \sum wy^2
\end{bmatrix}
\begin{bmatrix} A \\ B \\ C \\ D \\ E \\ F \end{bmatrix}
=
\begin{bmatrix}
\sum wx^2\Delta z \\ \sum wx\Delta z \\ \sum w\Delta z \\ \sum wx^2y\Delta z \\ \sum wxy\Delta z \\ \sum wy\Delta z
\end{bmatrix}
$$

FIG. 7. Matrix representation of normal equations.

computer one strip at a time, initially processed, and put on magnetic tape. After all flight data have been processed, each strip in turn is fitted to ground-control. After this initial fit to the ground-control, the strips are fitted to both ground-control and adjoining strips in an iterative process. The join to adjacent strips is effected by using as the true elevations of strip tie-points the z-coordinates from the adjacent strips. In forming the normal equations, control points are given a weight of 2 and tie points are given a weight of 1.

The fit to control and adjacent strips is repeated back and forth across the project until convergence is reached. It can be noted from Figure 7 that the only elements in the normal equation matrices of each strip that need be updated after each iteration are those of the constant vector (the column matrix to the right of the equal sign). The coefficient matrix of the unknown parameters remains unaltered throughout the solution.

The final adjusted elevation of a point common to two strips is computed as the weighted mean of the two corresponding points. The weights used for this purpose are inversely proportional to the sums of the squares of all the residuals in the respective strips.

Forty-five minutes of computer time were required for a solution of 10 iterations. (Approximately 15 minutes would be required for a solution under operational conditions. The recorded time of 45 minutes included 3 unnecessary iterations and the printout of much information that would not normally be required.)

The RMSE of the mathematical adjustment based upon 34 control points and 211 test points was 3.3 feet. The results of the graphical, ITC-JAC, and mathematical adjustments are shown in Table 2.

The accuracies of the elevations derived by these methods are not within the accuracy tolerance of 0.1 contour interval, generally accepted as necessary for supplemental control. However, the mathematical approach offers a very expedient means of adjusting data. It is hoped that by using relatively low-flight-height photographs for a given contour interval, increased redundancy in the number of control points and photogrammetric observations, proficient personnel, and perhaps with some modifications in procedures as proposed for the second part of this research project, the mathematical method of adjustment can be used for office derivation of supplemental elevations of acceptable accuracy.

ACKNOWLEDGEMENTS

Tests of the graphical and ITC-JAC adjustments were very ably conducted by Messrs. Harry E. Matheny and Roy R. Mullen of the U. S. Geological Survey. The author has drawn heavily from their works in describing the graphical and ITC-JAC adjustment procedures.

TABLE 2

RESULTS OF THE GRAPHICAL, ITC–JAC, AND MATHEMATICAL ADJUSTMENTS

Strip Number	*Graphical Adjustment (RMSE, Feet)*	*ITC–JAC Adjustment (RMSE, Feet)*	*Mathematical Adjustment (RMSE, Feet)*
1	3.0	2.5	1.9
2	5.1	2.4	2.8
3	5.6	2.6	3.2
4	5.4	2.9	2.7
5	5.1	2.1	2.7
6	4.1	4.2	3.3
7	5.5	3.6	5.5
RMSE, all strips	4.6	3.0	3.3
Number of control points	35	33	34
Number of test points	211	212	211

NOTE: The error value for a tie-point is based on the mean of the adjusted elevations determined for the two strips upon which it appears.

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*Philippine—Numerical Photogrammetric Cadastre**

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Under the supervision of
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THE CADASTRAL SYSTEM IN THE PHILIPPINES

LAND registration in the Philippines is based on the Torrens System which originated in Australia and presently has been used in the State of Massachusetts, U.S.A. The title to the land rather than the evidence of ownership is registered in a registry of property. The underlying principle of the Torrens System is the indefeasibility of titles. Once a certificate of title is issued by offices of registry on order of our courts after due hearing, the technical descriptions of the land for which the title was issued become fixed and inalterable. This principle is necessary for the permanency of land ownership, The title issued contains among others: (1) the directions and lengths of boundary lines, (2) the adjoining lots or owners, (3) the direction and distance of corner one of each lot to a location or conservation monument, and (4) the area of the land. With these descriptions, the identity of the property—its location and orientation on the ground, its size and shape, and its relation to other lots or property—becomes definite.

The Philippines, though consisting of several thousands of islands, is subdivided into provinces. Each province in turn is subdivided into municipalities. All the unregistered land in a municipality may be the subject of mass registration. When in the opinion of the President of the Philippines the public interests require that the titles to the lands in that municipality be settled and adjudicated, he orders our Director of Lands to execute a cadastral survey. Upon the completion of the survey, the maps or plans of the cadastral survey and the technical descriptions of all lands subject to registration are forwarded to the Court for the settlement and adjudication of titles.

THE PRESENT METHOD OF CADASTRAL SURVEY

In the technical descriptions of a piece of land contained in our titles, the directions of boundary lines are given to the nearest minute of arc, the lengths to the nearest centimeter, and the area of each lot to the nearest square meter. These data are all derived by computation from ground corner coordinates. We also require in the position of a lot corner a precision of ± 30 cm. in our agricultural regions and ± 10 cm. in our urban areas. Towards these end results we execute our cadastral surveys. Ours, therefore, is a purely numerical cadastre.

Once an order of a cadastral survey is issued by the President of the Philippines, the following are the steps we take:

1. *Notification of the survey.*—The cadastral survey is published in our Official Ga-

* Presented in support of Agenda Item A, Natural Resources, A.1, Mapping and Surveying Practice Adapted to Use in Less Developed Areas: Accepted by the United Nations Conference on the Application of Science and Technology for the Benefit of the Less Developed Areas: February 4–20, 1963, Palais des Nations, Geneva, Switzerland.

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