

more accurate assessment of the sources of revenues for the government; this when channeled to the different development programs of the country leads to more improved facilities and conditions of living for all citizens; (2) the utilization of every available piece of land to production which together with the intense application of scientific methods in agriculture will lead to a greater agricultural production and thus, to national self-sufficiency.

PROPOSED LONG RANGE PROGRAM OF PHOTOGRAMMETRIC SURVEY

As a result of our experience in the Bulacan Pilot Photogrammetric Cadastral Project, our Director of Lands has drawn up a long range plan for a nationwide survey of the Philippines. He proposes a 15-year period within

which to finish the remaining two-thirds of the country's area. Under this plan we hope to achieve a unit cost of less than \$6.00 per hectare. To implement this program, we are recommending to our Congress the passage of a bill appropriating \$25 million for this purpose. We are also looking forward to possible aid from the UN Special Fund which we are negotiating through the local UNTAB representative.

Thus we have shown how we are utilizing science and technology in our cadastral surveys. And in proposing to do it in a nationwide scale we are striking at the heart of our country's socio-economic program of alleviating the living conditions of the masses of our people and securing continued prosperity for them, their children and their children's children.

*Development of Programs for Strip and Block Adjustment at the National Research Council of Canada**

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ABSTRACT: A description is given of recently developed programs for strip and block adjustment. The strip adjustment consists in three-dimensional transformation of strip coordinates by means of various polynomials. Conformal transformations are used systematically. Block adjustment consists in transformation of the coordinates of the individual strips by means of polynomials. It is performed separately for horizontal coordinates and for heights. Use is made of a direct solution of the complete system of normal equations for all strips. Results obtained with a block of RC9 super-wide angle photography are shown.

AT THE National Research Council, the use of analytical methods in photogrammetry has been a subject of research since 1953.

First, a method of analytical aerial triangulation was developed and programmed for electronic computation. In addition, a three-dimensional strip transformation was programmed for the conversion of strip coordinates to a geodetic coordinate system.

This transformation did not include corrections for strip deformation. Subsequently, methods were developed for the adjustment of the horizontal coordinates of single strips and of blocks of overlapping strips. Here, strip deformation was corrected by means of conformal transformations. Reports about these methods were published in PHOTOGRAMMETRIC ENGINEERING and in *The Canadian Surveyor* [1, 2, 3] in 1960 and 1961.

* Presented at 1953 Semi-Annual Meeting of the Society.

The present paper is a report on the work that has been done since then, and on plans for the near future. This work concerns a method for three-dimensional adjustment of single strips with corrections for various types of strip deformation, and further developments in block adjustment.

CONFORMAL TRANSFORMATIONS

1. For the adjustment of strip coordinates with an electronic computer, it is rather convenient to use transformation formulas that give the transformed coordinates as polynomials with respect to the strip coordinates.

Restricting ourselves for the moment to the adjustment of horizontal coordinates, it is found that various types of polynomial transformations are used. Of these, the formulas for conformal transformation are the most suitable since these transform any small area in the strip coordinate system without deformation. From this property, the transformation derives its name.

Because of the importance of the conformal transformation, a somewhat detailed description of it is given in this paper. In terms of complex numbers, this transformation can be written:

$$(X_{tr} + iY_{tr}) = (a_1 + ia_2) + (a_3 + ia_4)(X + iY) + (a_5 + ia_6)(X + iY)^2 + (a_7 + ia_8)(X + iY)^3 + \dots \quad (1)$$

Separating real and imaginary terms one obtains:

$$\begin{aligned} X_{tr} &= a_1 + a_3X - a_4Y + a_5(X^2 - Y^2) - a_6 2XY + a_7(X^3 - 3XY^2) - a_8(3X^2Y - Y^3) + \dots \\ Y_{tr} &= a_2 + a_4X + a_3Y + a_6(X^2 - Y^2) + a_5 2XY + a_8(X^3 - 3XY^2) + a_7(3X^2Y - Y^3) + \dots \end{aligned} \quad (2)$$

The effect of small changes in the strip coordinates X and Y upon the transformed coordinates X_{tr} and Y_{tr} is found by differentiation:

$$\begin{aligned} dX_{tr} &= \frac{\partial X_{tr}}{\partial X} dX + \frac{\partial X_{tr}}{\partial Y} dY \\ dY_{tr} &= \frac{\partial Y_{tr}}{\partial X} dX + \frac{\partial Y_{tr}}{\partial Y} dY \end{aligned} \quad (3)$$

For the conformal transformation, one finds that for any value of X and Y and for any degree of the equations

$$\frac{\partial X_{tr}}{\partial X} = \frac{\partial Y_{tr}}{\partial Y}, \quad \text{and} \quad \frac{\partial X_{tr}}{\partial Y} = -\frac{\partial Y_{tr}}{\partial X}$$

Therefore, Formulas (3) are of the same type as the linear parts of Formulas (2), which are the well-known formulas for transformation without deformation. This proves that Formulas (2) transform any small area without deformation. Consequently, any two lines that intersect each other at right angles before transformation will do the same after transformation. Also, the change in scale that a small area incurs through transformation is the same in any direction.

No other polynomial transformation is possible that has this property. If, for instance, the terms with Y^2 are omitted in the second-degree transformation, the transformation is not any more conformal. As a result, when the conformal transformation is not used, care must be taken that the strip coordinates are in a region of the X, Y coordinate system where the transformation changes the angle between any two lines by a negligible amount. For instance, if in Equations (2) the terms with Y^2 are omitted, the strip axis should either coincide with or be close to the X -axis.

THREE-DIMENSIONAL STRIP ADJUSTMENT

2. For three-dimensional strip adjustment by means of polynomials, one can attempt to develop formulas for conformal transformation as follows:

$$\begin{aligned} X_{tr} &= a_{10} + \lambda(a_{11}X + a_{12}Y + a_{13}Z) + \dots \\ Y_{tr} &= a_{20} + \lambda(a_{21}X + a_{22}Y + a_{23}Z) + \dots \\ Z_{tr} &= a_{30} + \lambda(a_{31}X + a_{32}Y + a_{33}Z) + \dots \end{aligned} \quad (4)$$

The linear terms in Formulas (4) produce a linear conformal transformation (change of scale, rotation, and shift) if and only if, disregarding the factor λ , the matrix of coefficients of $X, Y,$ and Z has the properties that the sum of squares of the elements in each column is equal to 1 and that the sum of products of corresponding elements in each two columns is equal to zero. A matrix that has these properties is called orthogonal.

If one includes in (4) terms of higher than the first degree with respect to $X, Y,$ and $Z,$ and differentiates $X_{tr}, Y_{tr},$ and $Z_{tr},$ with respect to $X, Y,$ and $Z,$ the coefficients of $dX, dY,$ and dZ are the nine partial derivatives. The differentiated formulas give the effect of small changes in $X, Y,$ and Z upon the transformed coordinates, as do Equations (3) for the two-dimensional transformation. Therefore, they define the character of the transformation of a small area.

In order that the transformation be conformal it is necessary that any small area be transformed without deformation. Consequently, disregarding a scale factor, the matrix of the nine differential quotients must be orthogonal for any values of X , Y , and Z . This leads to a number of conditions which the coefficients in Formulas (4) must satisfy.

Unfortunately, it turns out that the conditions can be satisfied only if the coefficients of all terms that are of higher than the first degree with respect to X , Y , and Z are equal to zero. This proves that a conformal transformation in three dimensions which includes strip deformation is not possible by means of polynomials.

3. The above discussion can be summarized in two statements:

- a) For three-dimensional strip adjustment, it is rather convenient to use formulas that give the transformed coordinates as polynomials with respect to the strip coordinates.
- b) Of the different types of polynomial transformations, conformal transformations are the most suitable. However, these are possible in two dimensions but not in three dimensions.

These statements lead directly to the following specifications for transformation formulas:

- i. The adjustment must be done in steps, in each of which a suitable section through the strip is subjected to a two-dimensional conformal transformation.
- ii. In each step where the conformal transformation includes an appreciable change of scale in the area of the strip, the third dimension must receive the corresponding scale correction.

In this way, each step can correct for one type of strip deformation, such as vertical curvature in the strip direction, torsion along the strip axis, vertical curvature across the strip, and gradual changes in scale and azimuth. The resulting three-dimensional transformation cannot be exactly conformal but it will be very close to it.

4. An alternative solution can be found in a geometric approach.

One of the requirements of a three-dimensional strip adjustment is that the strip must be corrected for tilt and for vertical curva-

ture. During this correction, differences in terrain elevation must be taken into account. For this purpose, it is possible to compute a reference surface in the strip coordinate system which after adjustment shall be a horizontal plane. Differences in terrain elevation can be taken into account by specifying that any perpendicular on the reference surface must after transformation be perpendicular to the horizontal plane.

Recently, papers have been published by Ackermann and Perks which describe methods based upon this principle [7, 5]. In both methods vertical sections through the strip are made, one along the strip axis and one across the strip. In these sections, the line of intersection with the reference surface and the perpendicular from a point on the line of intersection are drawn. The adjusted coordinates are computed as distances measured along the line of intersection and along the perpendicular.

Ackermann, in a method employed at the International Training Centre, uses a torus as reference surface: the strip is assumed to have constant curvature in the strip direction as well as across the strip. The method includes leveling of the strip and is intended as the first step in a more comprehensive strip or block adjustment.

Perks, in a method used at the Surveys and Mapping Branch of British Columbia, defines the reference surface by means of a polynomial with respect to the X - and Y -coordinates. Therefore, the lines of intersection with the reference surface are polynomials of a specified degree.

In Ackermann's procedure, the computation of lengths in the strip coordinate system requires the use of trigonometric functions. In Perks' procedure, the length along the line of intersection in the longitudinal section was computed first as an integral, later as a sum of chords.

When these methods are programmed for electronic computation, one will replace the above functions by their series development with respect to the strip coordinates. Since at any point in a strip the radius of curvature is very large compared with the length of the strip, this series development will require only a few terms. Thus both methods end up with a formulation in which the adjusted coordinates are expressed in polynomials with respect to the strip coordinates.

This raises the question whether it is not

simpler to define the adjusted coordinates directly as such.

5. The approach of successive two-dimensional conformal transformations has been followed in a three-dimensional strip adjustment developed at the National Research Council and programmed for the IBM 1620.

The computation, which is performed in one pass of the computer, contains the following steps:

The strip coordinates are first transformed to an axis-of-flight coordinate system (X, Y, Z) with origin in the centre of the strip.

The strip is then brought to terrain scale and leveled. At the present time, the leveling does not include strip deformation but a correction for vertical curvature in the strip direction by means of a second-degree conformal X, Z transformation can easily be included.

Vertical curvature in the strip direction and residual tip are corrected by means of a conformal X, Z transformation of specified degree.

Torsion and residual cross-tilt are corrected by means of a linear Y, Z transformation which is a rotation about the origin:

$$Y_{tr} = qY - pZ$$

$$Z_{tr} = pY + qZ$$

The coefficient p is defined as a polynomial of specified degree with respect to X . The coefficient q is found as $\sqrt{1-p^2}$.

Vertical curvature across the strip is corrected by means of a second-degree conformal Y, Z transformation. The coefficient of the second-degree terms may be computed either from a specified radius of curvature or from the height control.

Finally, the horizontal coordinates are adjusted by means of a conformal X, Y transformation of specified degree.

The formulas are given in more detail in reference [5].

6. In the three steps where a correction for height deformation is applied, vertical cross sections are subjected to conformal transformation. Therefore, a small area in a cross section is rotated but not deformed. In this way, the effects of tilt and height differences upon the horizontal coordinates are automatically taken into account.

Further, in the case of height corrections, those coefficients which do not contribute a height correction if $Z=0$ are assumed to be equal to zero. Thus, for instance, the correction formulas for vertical curvature in the strip direction are:

$$X_{tr} = X - 2b_6XZ - b_8(3X^2Z - Z^3) - \dots$$

$$Z_{tr} = Z + b_6(X^2 - Z^2) + b_8(X^3 - 3XZ^2) + \dots \quad (5)$$

Because, in addition, the remaining coefficients are computed from the Z equations only, while the origin has been shifted to the centre of the strip and the strip has first been leveled, the height corrections leave the scale practically unchanged. Therefore, no scale correction of the third coordinate is necessary.

However, in the the case of the correction for vertical curvature in the strip direction, any variation in scale has a cumulative effect upon the X coordinate. When the strip is 100 km long, has horizontal-control at both ends, and a curvature equal in size to the earth curvature, this curvature correction causes largest X residuals of 0.2 meter. For longer strips, the errors increase proportionally to the third power of the strip length.

If one wishes to avoid these errors, it is necessary to re-introduce the terms that were omitted in Equations (5) and to compute their coefficients. One can now specify that the curve which in the X, Z coordinate system has the equation $Z_{tr}=0$ must be transformed at true length. For this curve, the second of the Equations (5) gives Z as an implicit function of X . If Z is solved from this equation and the obtained polynomial in X is substituted into the first of the Equations (5), X_{tr} is obtained as a polynomial in X . If the polynomial for Z is differentiated with respect to X , distances S along the curve can be obtained from $dS = \sqrt{dX^2 + dZ^2}$ and can be written as a polynomial in X . Since the curve $Z_{tr}=0$ must be transformed at true length, the polynomial for X_{tr} and the polynomial for S must have identical coefficients. In this way, the coefficients with odd-numbered indices are found as functions of the coefficients with even-numbered indices. In the case of a second-degree correction, $b_7 = -\frac{2}{3}b_6^2$; in the case of a third-degree correction, in addition $b_9 = -\frac{7}{2}b_6b_8$.

7. The scale correction that is included in the adjustment of horizontal coordinates should be applied also to the Z -coordinate. For this purpose, the scale factor can be computed for each point with the help of Equations (3).

The scaled Z -coordinates should be used in the computation of the formulas for correction of height deformation. On the other hand, the strip should be corrected for height deformation before the horizontal adjustment is performed.

These two contradictory requirements seem to make necessary performing the adjustment for height deformation and the horizontal adjustment in turn, in an iterative procedure. However, if this scaling of the Z -coordinate is omitted, one height adjustment followed by one horizontal adjustment has proved to be sufficient. Appreciable height errors will then occur only in the case of large variations in terrain height combined with a poor scale propagation through the strip. Because in this case the accuracy requirements can be less strict, it has not been found worthwhile to make the adjustment an iterative procedure.

BLOCK ADJUSTMENT, 1962

8. In a block adjustment of strips, each individual strip is adjusted so as to agree as fully as possible with the existing ground control and with the other strips.

This computation can be performed in steps. First, each strip can be approximately leveled and positioned with respect to the geodetic coordinate system. As a rule, sufficient height-control will be available to include here a second-degree correction for vertical curvature in the strip direction. For practical purposes, this makes the effects of residual tilt and of height differences upon the horizontal coordinates negligible. After this, therefore, the adjustment can be subdivided in an adjustment of horizontal coordinates and an adjustment of heights.

9. The program for three-dimensional strip transformation is used for the approximate leveling and positioning. It provides the possibility of storing the transformed coordinates of tie-points of a strip in memory and of using these coordinates as additional control for a following strip. This feature makes possible transforming all strips of a block in one pass on the computer, even if only one strip has sufficient ground-control for independent positioning. Also, if one strip has sufficient control for second-degree transformation of horizontal coordinates or heights, all strips can be transformed in this way.

For horizontal block adjustment, an IBM 650 program in which second-degree conformal transformations of individual strips were used in an iterative procedure gave very favorable results. This was a reason to re-code this program without modification for the IBM 1620.

A program for block adjustment of heights was coded also, using the same iterative procedure. Here, the heights are adjusted by means of polynomials with respect to the strip coordinates.

These two programs were coded by Topographical Survey of the Department of Mines and Technical Surveys and by the Army Survey Establishment in close cooperation with the National Research Council.

REMARKS ON PROGRAMMING

10. When the program for analytical aerial triangulation was first coded for the Ferut electronic computer, everything that was reasonably possible was done in order to reduce the computation time to a minimum without relinquishing accuracy. A simple set of formulas was derived, fixed-point arithmetic was used throughout the computations rather than floating-point subroutines, and the shortest possible formulation in terms of computer instructions was sought. When the program had to be re-coded for the IBM 650, the same policy was adopted in the expectation that this computer would be available for a long time to come. That made the expenditure of the necessary time and effort worthwhile.

The results of this policy became visible in an investigation by GIMRADA, reported on by Matos [6], who stated that the NRC method of analytical triangulation proved to be about 5 to 10 times faster than the other methods tested.

However, two years after completion of the program, an IBM 1620 was installed at the N.R.C. laboratories. A year later, the last of the IBM 650 to which N.R.C. had access was withdrawn from the Ottawa area. At about the same time, N.R.C.'s IBM 1620 was provided with the optional floating-point hardware. Also, the ever increasing demand for computer time has raised the question whether this computer will not sooner or later be replaced by a faster one. These circumstances and the continuing activity in the field of computer development give rise to

the speculation that it may become necessary to re-code our programs periodically.

In view of this situation, it does not seem worthwhile anymore to spend an appreciable amount of time on the writing of programs in fixed-point arithmetic and on looking for other economies in terms of computer time. At the present time, the best policy appears to be to use floating-point arithmetic, to spend the necessary time on writing an economical set of general purpose computational subroutines for the current computer, and to spend a minimum of time on the framework of instructions that surrounds the actual computations. In this way, a better balance between programming time and computation time will be reached. As a result, the programs may become considerably slower than they could be, but they can become operational before the computer for which they were written is modified or even withdrawn.

BLOCK ADJUSTMENT, 1963

11. Having accepted the regular use of floating-point arithmetic, a new approach to block adjustment becomes possible.

An adjustment requires the solution of a system of normal equations with as many unknowns as there are coefficients in the transformation formulas that must be solved for simultaneously. The iterative adjustment was adopted partly because a fixed-point program for single strips had been written already and could be used as a subroutine in an iterative block adjustment; partly because it was felt that with fixed-point arithmetic it might not be possible to solve a great number of normal equations with sufficient accuracy. The use of floating-point arithmetic makes it feasible to solve large systems of linear equations and therefore to attempt a direct solution of the complete system of normal equations for all strips of a block.

In the case of block adjustment by strips, the solution is facilitated by the fact that the matrix of coefficients in the normal equations contains a relatively large number of zeros.

If in this matrix the unknowns, which are the coefficients in the transformation formulas, are grouped according to the strips, each ground-control point provides contributions only to one sub-matrix on the main diagonal. To each strip corresponds one such sub-matrix. If one type of transformation formula is used for all strips, these sub-

matrices are all of the same order. They can be used as a starting point for subdividing the matrix into square sub-matrices of the same order.

Each tie point between two strips provides contributions to the two corresponding sub-matrices on the main diagonal and to the two off-diagonal sub-matrices that lie with the diagonal ones on the corners of a square.

Therefore, in the case of a block of parallel and uninterrupted strips contributions are made only to the sub-matrices on the main diagonal and to those immediately adjoining it. All others are equal to zero.

If, instead of with sub-matrices, one had to do with real numbers, the solution of these normal equations through successive elimination and back substitution would be very simple. In the case of matrices, exactly the same procedure can be used. It is only necessary to replace each operation with real numbers by the analogous operation in matrix algebra.

12. As a check on the feasibility of the direct solution, a program for horizontal block adjustment of parallel and uninterrupted strips has been coded following this procedure. The formulas for second-degree conformal transformation have been used.

The computations are performed in floating-point arithmetic with 10-digit mantissas. In order to reduce the requirements on the number of significant digits in the computations, the origin of the coordinate system is temporarily shifted to a point inside the block, and the transformation formulas are set up to give, primarily, corrections to approximate coordinates.

Since the transformations are conformal, all computations can be performed with complex numbers. In this case, the sub-matrices are of order three and have complex elements. In the case of real numbers, the matrix of coefficients in the normal equations is symmetric. Here, this matrix and the sub-matrices on the main diagonal are hermitian. These properties have been used to economize on storage space for the normal equations and on computation time.

Rather than storing the ground-control points and tie-points in memory, the computation has been made a two-pass process.

During the first pass of the data, the normal equations are computed and solved, and the transformation equations are stored. During the second pass, all points are transformed, and transformed coordinates, residuals on ground-control points, and half-differences in tie-points are punched. In this way, sufficient storage space for the normal equations is available to adjust 60 strips simultaneously, and an unlimited number of ground-control points and tie-points can be used.

The data deck contains one card for each point with the input coordinates. The cards are sorted in groups according to strip and to type of point, and each group is preceded by a card that gives strip number, type of points, and their weight in the adjustment. No further preparations are required.

A test of the program with a theoretical example of 20 strips with the minimum of three ground-control points and three tie-points in each overlap, and covering an area of 200 by 200 km, gave an exact fit on ground-control points and tie-points. Two computations with opposite sequence of the strips in the formation of the normal equations gave results that differed by at the most 1 cm. The computation time for this adjustment, including program read-in, was 8 minutes.

13. Because of the excellent internal accuracy and speed of this program and its simplicity of operation, its usefulness has been increased by introducing a modification which allows the adjustment of blocks of strips with breaks in the triangulation. Now, each strip may have tie-points in common with the strips with the three next lower strip numbers and the strips with the three next higher strip numbers.

This affects the storage space that is needed for the normal equations. Space has been provided for the off-diagonal sub-matrices that are one, two, and three places removed from the main diagonal. As a result, not more than 28 strips can be adjusted simultaneously.

When an off-diagonal matrix has non-zero elements as a result of a tie-point condition, the elimination procedure that is used during the solution of the normal equations may produce non-zero elements in other off-diagonal sub-matrices down the same column, until the main diagonal. The resulting increase in the amount of computation, compared with the case of uninterrupted strips, makes the accuracy requirements more critical. For

instance, in an extreme case of three parallel strips with two cross strips on one side of the block and a distribution of tie-points and ground-control points that was designed to tax the capabilities of the program, slightly inaccurate results were obtained if during the computation the origin of the coordinate system was shifted to the centre of the block. For centimeter accuracy, the origin had to be nearer the centroid of the control points and tie-points.

14. A similar program for vertical block adjustment is planned also. Here, a complication is caused by the fact that one type of polynomial cannot be used for the adjustment of all blocks, and possibly not always for the adjustment of all strips in one block.

The program for analytical aerial triangulation has not been re-coded for the IBM 1620. Instead, the IBM 650 program is used with a simulator on the IBM 1620. This makes the program 4 or 5 times slower than it was on the 650. Although this is rather unsatisfactory, the limited use that is as yet made of the program does not warrant the effort of re-coding.

AN APPLICATION

The programs for strip and block adjustment were used in the adjustment of a block of aerial photography taken with an RC9 camera over an area in Southern Rhodesia. The block contains 5 strips with 22 photographs each and measures 140×65 km. The focal-length of the camera is 88.48 mm, the photograph-scale is about 1:80,000, and the average forward-overlap is 63%. Ilford HRA film was used.

The distribution of ground control points and of tie points between strips is shown in Figure 1. The ground-control points were not targetted.

The photographs were measured by the Rhodesian Department of Trigonometrical and Aerial Surveys in a Nistri TA3 stereocomparator and the obtained coordinates were corrected by that Department for film distortion, asymmetric and symmetric lens distortion, refraction, and earth curvature. The correction for film distortion was based upon measurements of the four corner fiducials, and was performed with a second-degree non-conformal polynomial transformation. For relative orientation and scale transfer, eight points were measured per

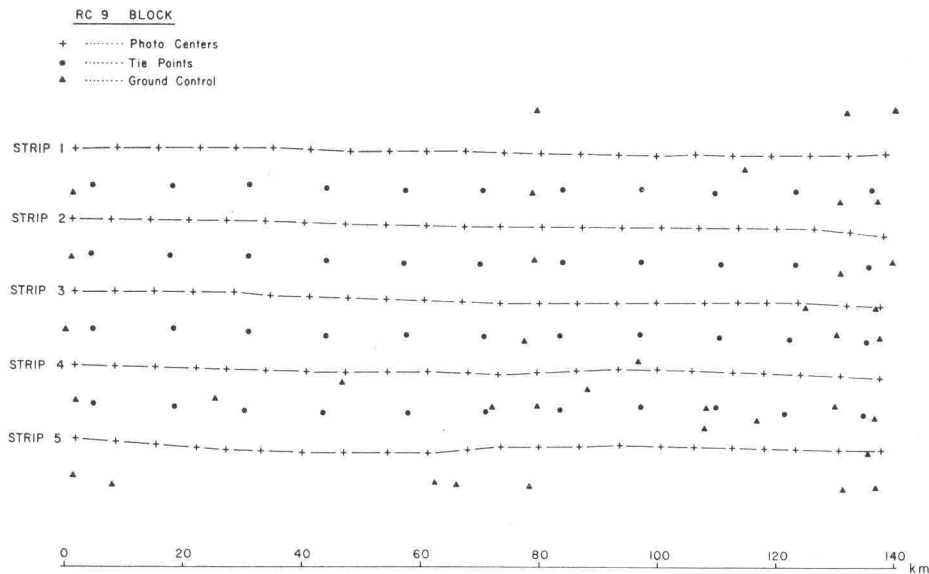


FIG. 1

model: six points in the usual positions and two near the centres of the two squares formed by the first six points.

The analytical triangulations gave satisfactory results except for one model in strip 1 which showed exceptionally large residual parallaxes and height deformation. The mean square value of the residual parallaxes in the points used for relative orientation is 4.8 micron at photograph scale.

The strips were first transformed independently, using second-degree transformations in order to correct vertical curvature in the strip direction, scale, and azimuth. Figure 2 shows the discrepancies in tie points after these transformations. They give a clear indication of third-degree deformation in X and Z . The strips had little or no torsion.

Subsequent third-degree transformations showed systematic height errors of a few meters in ground-control points and systematic discrepancies on tie-points of the same size which could be reduced by a fourth-degree correction of vertical curvature. Finally, therefore, strips 4 with the most control, and strips 5, 3, and 2 were transformed in this sequence by means of third-degree X, Y transformations and fourth-degree Z transformations. For the latter three strips, a few tie-points with an already transformed strip were used as additional control. Strip 1 was adjusted in two parts by means of second-

degree transformations. This procedure made a subsequent block adjustment unnecessary.

Because in this block the third-degree X, Y

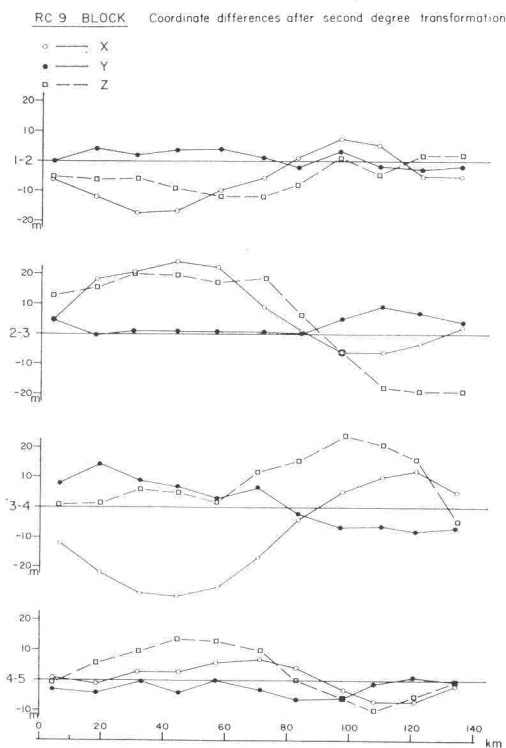


FIG. 2

transformations give significantly better results than the second-degree transformations, and the horizontal block adjustment applies second-degree transformations only, it is of interest to see what can be achieved with this block adjustment. For that purpose, each strip has been divided into two parts of about equal length and a block adjustment has been performed using the same ground-control points and tie-points as in the final strip transformations. In order to obtain a strong connection between strip halves, two "wing points" at the break were used as tie-points and given a ten times greater weight than the tie-points between strips and the ground-control points.

The results of both adjustments are listed in Table 1. The largest value in the table, 2.1 m, equals 26μ at photograph scale. Evidently, the result of the block adjustments is somewhat better than that of the strip adjustments. Both are quite satisfactory for the required mapping at scale 1:50,000.

TABLE 1

MEAN SQUARE VALUES OF RESIDUALS IN GROUND CONTROL POINTS, AND HALF-DIFFERENCES IN TIE POINTS, IN METERS

	<i>Strip transformations</i>			<i>Block adjustment</i>	
	m_x	m_y	m_z	m_x	m_y
All ground control	1.9	2.1	1.9	1.6	2.1
All tie points	1.4	1.2	1.7	1.0	1.2

The time required for analytical triangulation, using the program to simulate the IBM 650 on the IBM 1620, was between 55' and 60' per strip. The final strip adjustments took altogether about 30' with the latest program which employs the floating-point hardware. The block adjustment required 20'. The times for these two adjustments include the transformation not only of the used control points but also of about a thousand additional points. These points are the eight points used for relative orientation in each model, the projection centres, one of each two ground control points that occurred in pairs, and unused tie points.

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ANNOUNCEMENT

TENTH CONGRESS OF THE
INTERNATIONAL SOCIETY FOR PHOTOGRAMMETRY

LISBON, PORTUGAL

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