Model Flatness—A Guide for Stereo-Operators

G. P. KATIBAH, *Photogrammetric Engineer,*
California Division of Highways,
Sacramento, California

(*Abstract on preceding page*)

INTRODUCTION

THE experienced stereo-operator of photogrammetric mapping instruments is well aware that a certain amount of deformation of the model surface can be expected. If an abundance of vertical control is available, he may comment that he cannot read all the points, or that "there are too many." Such a comment is simply an acknowledgement of the problem of surface deformation but is not a statement of the cause of the problem.

The sources of error in the photogrammetric system are extremely difficult to isolate. Perfect restitution of the stereomodel, point for point, is an ideal situation which is not attained in everyday practice. Every stereomodel is deformed to some extent, and it is the degree of deformation which determines whether or not it will be detected, considering that sufficient vertical control is available.

Two broad sources of error can be accounted for to some extent by the operator, namely, instrument calibration, and the characteristics of the lens components in the total system. The scope of this paper emphasizes the latter considerations, especially those associated with the direct projection plotter of the Kelsh-type since these plotters are so widely scattered among the small private firms that concentrate more on large scale engineering mapping than on the smaller reconnaissance scales.

GENERAL DISCUSSION

The perfect restitution of the model depends entirely on whether or not the cone of rays emerging from the projection lens is angularly identical with the cone of rays received by the camera lens. Any deviation whatsoever of the projected rays from their original entrance paths will contribute to model deformation. Causes for deviations may be divided into three broad independent groups, as follows:

1. Mechanical——(a) Imperfections in camera fabrication, calibration, or operation. Operation would include vacuum failures, excessive aircraft velocity, aircraft vibrations transmitted to the camera, and sudden rotations of the aircraft in turbulent air while the shutter is open during exposure of the negative. (b) Imperfections in plotter fabrication, calibration, or operation. Operation would include sticking of guide rods and cams (if used), causing in turn an unwanted movement of the projection lens or projector.
2. Photographic——Any shift of the image position on the aerial film or on the diapositive after exposure of either one. Examples are possible emulsion creep, and film-base dimension changes due to differentials in stresses, temperature, and humidity.
3. Optical——Lack of data pertaining to lenses, or failure to compensate for radial distortion.

Atmospheric conditions could be listed as a fourth category. It is the author's opinion, however, that atmospheric conditions play a relatively minor role in low altitude, large scale photography, and are not considered a significant cause of light-ray deviations compared with high altitude, small scale photography. Before dropping this topic there is one very important thought which should be mentioned. Hot exhaust gases from the aircraft engine should not be allowed to pass in front of the camera lens. While this may seem obvious, even elementary, it is frequently overlooked by new flying firms unfamiliar with precision aerial photography. It is easily prevented by exhausting a single engine aircraft at the sides rather than at the bottom of the engine.

Since this paper is mainly concerned with optical causes of stereomodel deformation, this topic is discussed in some detail in the following sections.

ANALYSIS OF MODEL ERRORS

A mathematical analysis of a stereomodel provides a method of predicting model

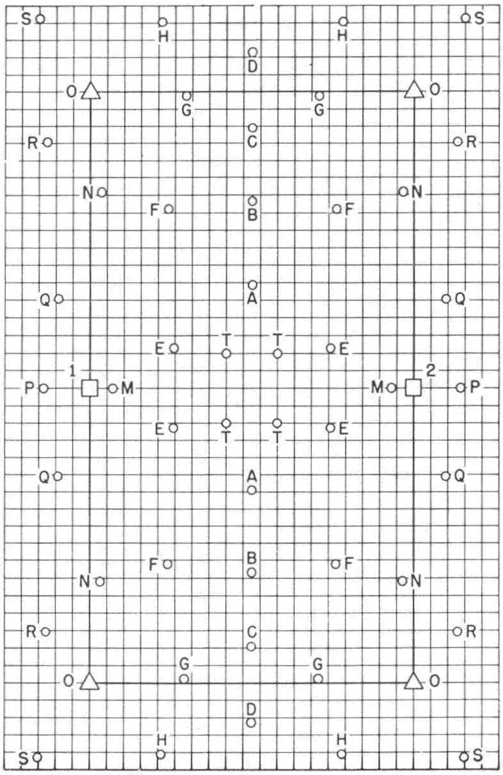

FIG. 1. Distribution of points in relation to 5 mm. grid interval, Lewis method.

deformation in terms of vertical error. The usual assumptions are that the photographs are truly vertical, and that any two exposures comprising a stereo-pair are identical in scale. In addition, a base-height ratio and a width-height ratio must be assigned to determine the size of the neat model. The data for analysis are the distortion values of any lens component in the system. These are customarily given in a calibration report.

The distortions in a system accumulate algebraically. One may begin an analysis with the algebraic sum of all the known distortions to determine the resulting vertical error in the model, or the known distortions can be separated according to lens component, analyzed individually, and the separate results at each point added to arrive at the final vertical error. Either way will yield the same answers. However, if calibration data are available for the camera only, the components will have to be separated for analysis, and the final results in the model determined by adding the separate errors. This makes it necessary to resort to methods of analysis other than mathematical to determine errors by the projection lens of the plotting instrument. A calibration report is not always available for a projection lens.

Probably the most effective method for testing performance of a projection lens is the "grid model" method. Precise grids on glass are used as diapositives in the formation of a grid model with a base-height ratio equivalent to the value used in mathematical analysis. Vertical errors are determined by reading the model at the optimum projection distance. A perfectly restituted model would read as a truly plane surface, whereas any deformation would show as a vertical departure from this criterion. Grid model deformations are a result of all errors associated with the plotter, and serve as a final test of the overall performance of the instrument. The instrument therefore must be carefully calibrated and tested to minimize the influence of mechanical sources of error. Assuming that all other sources of errors have been accounted for, the resulting vertical errors in the grid model are attributable to the distortion in the projection lens.

Of the various methods of computation, the one devised by J. G. Lewis (1956) has been found to be especially useful. Lewis' method analyzes vertical errors of 16 points which are well distributed in 52 locations in the total model area, with 32 of them being in the neat model area. The distribution of the points in relation to a projected grid model is shown in Figure 1 and similar figures referred to in the text. The neat model in the figure is the rectangular area with the corners marked by triangles. The base-height ratio is 0.62 and the width-height ratio is 1.12, which corresponds to a neat model size of 3.72 by 6.72 inches at photograph scale. This is a realistic size for large scale design mapping.

CAMERA CALIBRATION

Most mapping projects require the use of nominal 6-inch photography, with several designs of lenses available in cameras of different manufacture. The calibration report which accompanies a cartographic camera assures the user that the camera was designed and manufactured to certain standards, and further provides detailed data pertaining to focal length, radial distortion pattern, and resolving power of the lens as mounted in the camera. Cameras which have not been certified by a qualified testing agency should not be used for carotographic photography.

While calibration reports contain basic data important to the user, there is a disconcerting lack of uniformity among reporting agencies in the manner of presentation, which may cause doubt regarding the consistency of results. To illustrate, the following four agencies present data in varying ways and to different tolerances:

U. S. Bureau of Standards: Lists both equivalent and calibrated focal lengths to a stated tolerance of ±0.10 mm. Six distortion values are given to a stated tolerance of ±0.02 mm., based on both E.F.L. and C.F.L. A recent report indicates this tolerance has been modified to ±10 microns.

Fairchild Camera and Instrument Corporation: Lists both equivalent and calibrated focal lengths to a stated tolerance of ±0.10 mm. Eleven distortion values are given to a stated tolerance of ±0.01 mm. based on both E.F.L. and C.F.L.

Zeiss-Aerotopograph: Lists calibrated focal length to a stated tolerance of ±0.02 mm. Fourteen distortion values are given to a stated tolerance of ±0.002 mm., presumably based on C.F.L.

Wild-Heerbrugg Instruments, Inc.: Lists calibrated focal length with no stated tolerance. Eight distortion values are given with no stated tolerance, presumably based on C.F.L.

The apparent confusion in data presentation is noted here because it is a situation with which the practicing photogrammetrist must cope. It does not necessarily mean the data are not useable: it does mean, however, that data are not transferable from the terms of one agency into the terms of another agency, so that two cameras reported individually by two agencies cannot be compared on a uniform basis. This is particularly annoying because the photogrammetrist is forced to regard any camera report as absolute, unless of course evidence to the contrary exists.

PROCEDURE FOR TESTING

The first step in testing the model flatness to be expected by a given camera with a given plotter begins with the plotter. It is imperative that the performance of the instrument be known to the operator.

Plotter instrument performance starts with good calibration. Calibration in this sense means that the internal geometry (interior orientation) of projectors is made to fit a standard; that is, the principal distance is precisely known for any setting of the principal distance adjustment ring or micrometers, and that the intersection of the fiducial axes accurately locates the principal point. We can now assume that the first node of the projector lens describes the correct perspective center for projection of the diapositive. (It is recommended that a thorough review of calibration be made before it is attempted: Refer to references, such as (1) and (7)).

The next step is to find out how well the projectors will project precise grids to form a grid model. 5-mm. grid diapositives were used in the examples as subsequently reported in this paper, because Lewis' calculation procedure for checking cameras utilizes a 5-mm. grid interval. This eliminates the inconvenience of interpolating values for points common to both plotter and camera. The grids are used as diapositives and are placed in the plate holders "emulsion-side" down to preclude any influence of projecting through the glass. With the center grid intersection as the principal point, the base-height ratio will be 0.62 when the base is 95 mm. and the principal distance is set at 153 mm.

Referring to Figure No. 1, the y-parallax is cleared and the model scaled between the principal points projected at the optimum projection distance. For the typical 5-diameter plotter, this base distance is 18.6 in. when the projection distance is 30 in. The neat model is leveled at its corners, denoted by the triangle symbol and labeled zero.

After scaling and leveling, the nearest grid intersections to the lettered points are read and the elevation differences from the zero datum are recorded. These vertical departures from datum define the deformation of the grid model caused essentially by lens distortion. However, if the slate or granite reference surface is uneven, this will also contribute to the deformation pattern as read by the operator. The deformation readings are therefore a measure of the performance of the plotter, *without* influence of the aerial camera. We are now ready to verify the influence of the aerial camera to be used with the plotter.

Analysis of the aerial camera is based on its calibration report, especially upon the values for radial distortion. The net result in the stereomodel is calculated according to the Lewis method for each of the lettered points previously read in the grid model. The plotter values and the camera values for each point are added algebraically to derive the combined performance of the two units, *without* influence of operational conditions or aerial film. If the results show greater vertical departures than desired, the camera should be rejected or compensation devices, such as cams or correction plates, should be employed

TABLE 1

DIAPOSITIVE GLASS DISTORTION[a]

Angle Off Axis (*deg.*)	*Distortion* (*mm.*)
5	0.000
10	0.002
15	0.005
20	0.013
25	0.026
30	0.048
35	0.081
40	0.130
45	0.202

[a] Emulsion surface up on 0.06-in. thick glass.

to offset the combined plotter-camera model deformation (4).

TEST EXAMPLES

DIAPOSITIVE GLASS

Kelsh plotter procedure normally requires only two lens components in completing the optical path from the exposure of a ground area to the projection of it onto the platen. The lenses involved are the camera lens and the projector lens. It is currently normal procedure to make the diapositives by contact printing through the film-base, using a point-source light, in order to register a reverse image. This produces the same results as a one-to-one ratio projection printer, but eliminates an optical step. This permits the diapositives to be placed emulsion-surface down in the projectors.

If the diapositive is made emulsion to emulsion in a contact printer the photo-image will have to be projected through the glass. (A reverse model will be formed if these diapositives are placed emulsion-surface down, and the resultant compilation will also be reversed.) This procedure introduces an optical step because the glass is actually a lens, each surface being of infinite radius. The light rays transmitting the image through the glass will be refracted, causing a distortion which will result in model deformation. Distortion values can be readily determined for glass of any particular thickness considering the angular distance from the axis of the lens system according to the tabular values on page 47 of the MANUAL OF PHOTOGRAMMETRY (2).

Printing diapositives emulsion-side up on 0.06-in. thick glass is still in practice, mostly because of the lower costs of materials and the facility of conventional printing methods. The 0.06-in. thick glass does not measure up to the quality of thicker glass currently available for diapositive materials, as indicated in the brochures of the commercial outlets. Experience shows that it requires support in the middle to prevent sag, and there is possibility of wedge effect caused by lack of parallelism between the two planar surfaces.

The distortion values for 0.06-in. thick glass are listed in Table 1. Utilizing these distortion values, 16 points distributed in 52 locations in the stereomodel can be computed by the Lewis method, yielding results in terms of vertical errors, or deviations from a truly plane surface. Computational results are given in Table 2: Line 1 gives the computed vertical errors in millimeters at model scale which is five times the scale of the diapositives; Line 2 gives the computed vertical errors in feet at a model scale of 1 in. = 50 ft.; Line 3 gives actual average readings in feet at 1 in. = 50 ft. in a grid model as set up in a Nistri-Photomapper. (The scale of 1 in. = 50 ft. is the usual design mapping scale required by the California Division of Highways.) The location of the points in relation to a projected grid model (5 mm. diapositive grids enlarged 5 diameters) is shown in Figure 2. The contours have been interpolated between

TABLE 2

VERTICAL ERRORS IN MODEL[a]

Point	*Comp. Vert. Error*		*Actual Average Reading*
	(*mm.*)[b]	(*ft.*)[c]	(*ft.*)[d]
A	−0.415	−0.82	−0.85
B	−0.320	−0.63	−0.75
C	−0.180	−0.36	−0.55
D	−0.015	−0.03	0.00
E	−0.465	−0.92	−0.90
F	−0.330	−0.65	−0.60
G	−0.145	−0.29	−0.10
H	−0.040	+0.08	+0.55
M	−0.360	−0.71	−0.70
N	−0.195	−0.38	−0.45
O	0	0.00	0.00
P	−0.170	−0.34	—
Q	−0.175	−0.35	−0.45
R	+0.015	+0.03	+0.05
S	+0.230	+0.45	+0.75
T	−0.485	−0.95	−0.95

[a] Caused by 0.06-in. thick glass, emulsion surface up.

[b] At model scale 5 times scale of diapositive.

[c] At model scale 1 in. = 50 ft.

[d] At 1 in. = 50 ft in grid model as set up in Nistri-Photomapper.

computed values to depict the expected model deformation, which is a "dished" effect approaching one foot in equivalent value at the model center.

The close agreement between computed and actual values demonstrates the validity of the computational approach, and of course is a tribute to the skillfulness of the instrument operator since he had no prior knowledge of the computed values. The biggest spread between computed and actual values occurred at points *H* and *S*, both far outside the neat model area within about $\frac{1}{2}$ in. from the margin of a corresponding photograph. Point *P* was so far outside the neat model area that it was beyond the physical limitation of the instrument, and therefore could not be read. Within the neat model area the biggest spread in readings is 0.2 ft., which is actually 0.1 mm., probably attributable to some extent to the projection lenses and to the quality of diapositive glass.

While it may be physically possible to compensate for the large distortion values exhibited by 0.06-in. thick glass, the use of thicker plates with emulsion down is generally considered to produce improved results. The latter plates eliminate an optical step in the projection system as well as remove for all practical purposes the possibility of variations caused by sag, wedge, and glass quality.

PROJECTION LENSES

Projection lenses should be tested as mounted in the projectors. While some designs of projectors permit the testing and calibration by optical laboratory procedures similar to that for aerial cameras, the most common type is that in which the projectors can be disassembled into three components, namely, the projector unit, projector cone, and plate holder. The latter type is tested best by the grid model method as it is impractical to test it by optical laboratory procedures.

There is considerable variation in lenses used in older instruments. Figure 3 shows the deformation pattern characteristic of some of the older instruments in use. Projection lens design has improved and better performance should now be expected.

The California Division of Highways has adopted a standard for model flatness produced by plotter equipment, as follows:

> The projector lenses shall provide for the projection of a stereoscopic grid model, using precise rectangular grid diapositives, not to exceed plus or minus 0.05 mm. from plane of flatness within the neat model area. This will include any deviation from flatness attributable to the reference plotting surface.

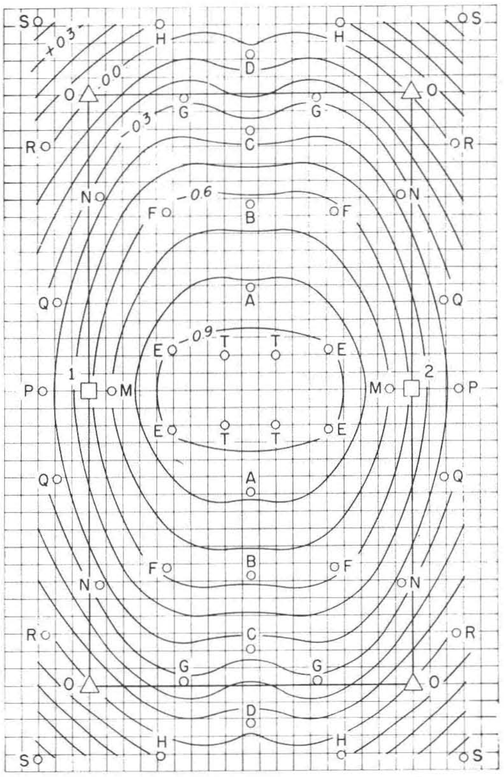

FIG. 2. Model deformation, 0.06-inch thick glass. Assumptions: camera and projection lenses distortion-free.
Instrument: 5× projection plotter
Model Scale: 1 inch = 50 feet
Contour Interval: 0.1 foot
B/H = 0.62; W/H = 1.12
25 mm. grid at model scale

An instrument capable of forming a model within these tolerances can be used directly with virtually distortion-free photography without employing correction devices.

CAMERA LENSES

The four camera lenses commonly used in this country for mapping photography are all of nominal 6-in. focal length, and are considered to be wide angle lenses for use with the 9 by 9 in. format size.

Figure 4 illustrates the distortion curve characteristic of each of these lens designs.

Figure 5 illustrates typical model deformation patterns computed from the respective distortion curves.

The need for distortion compensation devices can be determined from these diagrams. For instance, the amount of deformation in

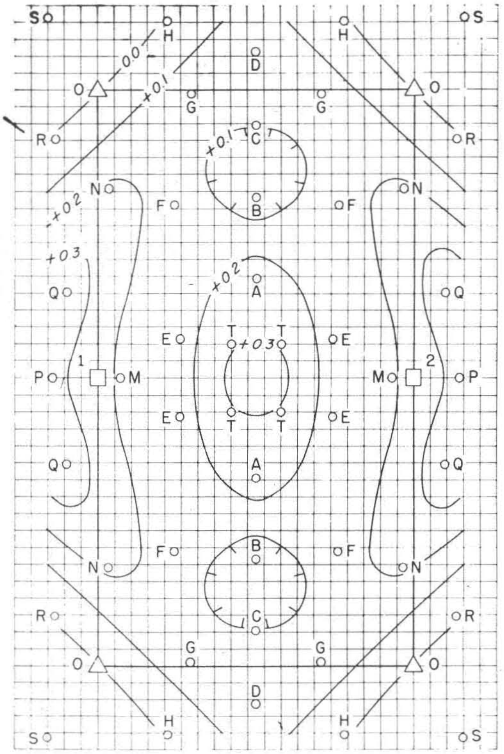

FIG. 3. Model deformation caused by possible Hypergon distortion. Assumptions: camera lens distortion free; diapositives emulsion down.
Instrument: 5× projection plotter
Model Scale: 1 inch = 50 feet
Contour Interval: 0.1 foot
B/H = 0.62; W/H = 1.12
25 mm. grid at model scale

the Metrogon model could not be tolerated under any condition. Distortion correction cams or plates are absolutely necessary to flatten the model within useable limits.

Planigon photography produces a much flatter model than does Metrogon, as shown by the deformation pattern. However, Planigon lenses, especially those of early manufacture, vary considerably in their distortion values, and this in turn makes it particularly difficult to design a correction device to compensate "average" Planigon distortion (6).

Aviogon photography produces fairly consistent patterns characterized by a camber or "hump" in the middle of the model. It is possible to design and manufacture correction devices to compensate for this deformation. However, many operators prefer to use this type of photography without compensation, providing vertical control is arranged to locate the model deformation. The most effective arrangement is one point in each of the four corners and one point in the middle, thus providing for local indexing during compilation.

Pleogon photography produces fairly consistent patterns which are usually too flat to be detected in the average instrument.

OPERATIONAL TESTING

Almost every organization has attempted a test under operational conditions to check the performance of the camera-plotter combination. The results of such a test are shown in Figure 6. The premarked points shown as crosses had been established by the mapping contractor in conformance with contract specifications. The premarked points shown as dots had been established by Division of Highways personnel, with the elevation of each point determined by spirit levels.

The four corner points, *A*, *B*, *C*, and *D* were used to level the model, and the elevations of all other points read accordingly. The model was actually set in four different instruments,

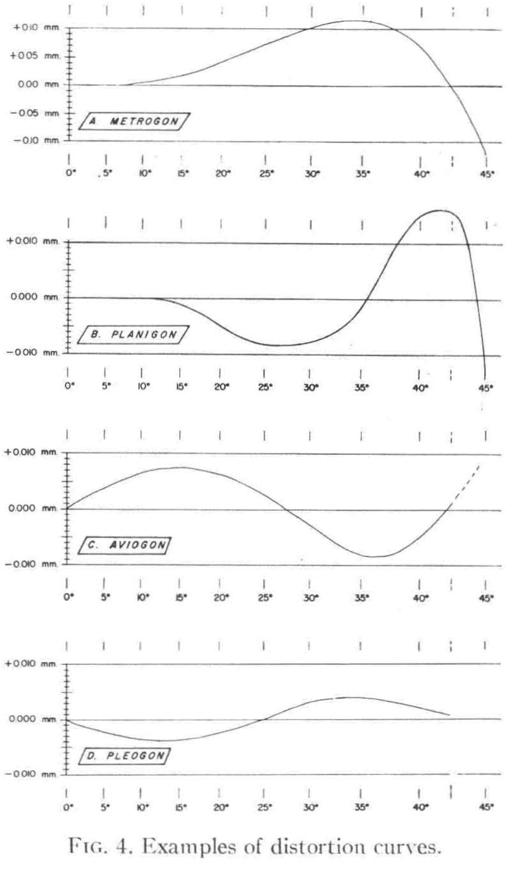

FIG. 4. Examples of distortion curves.

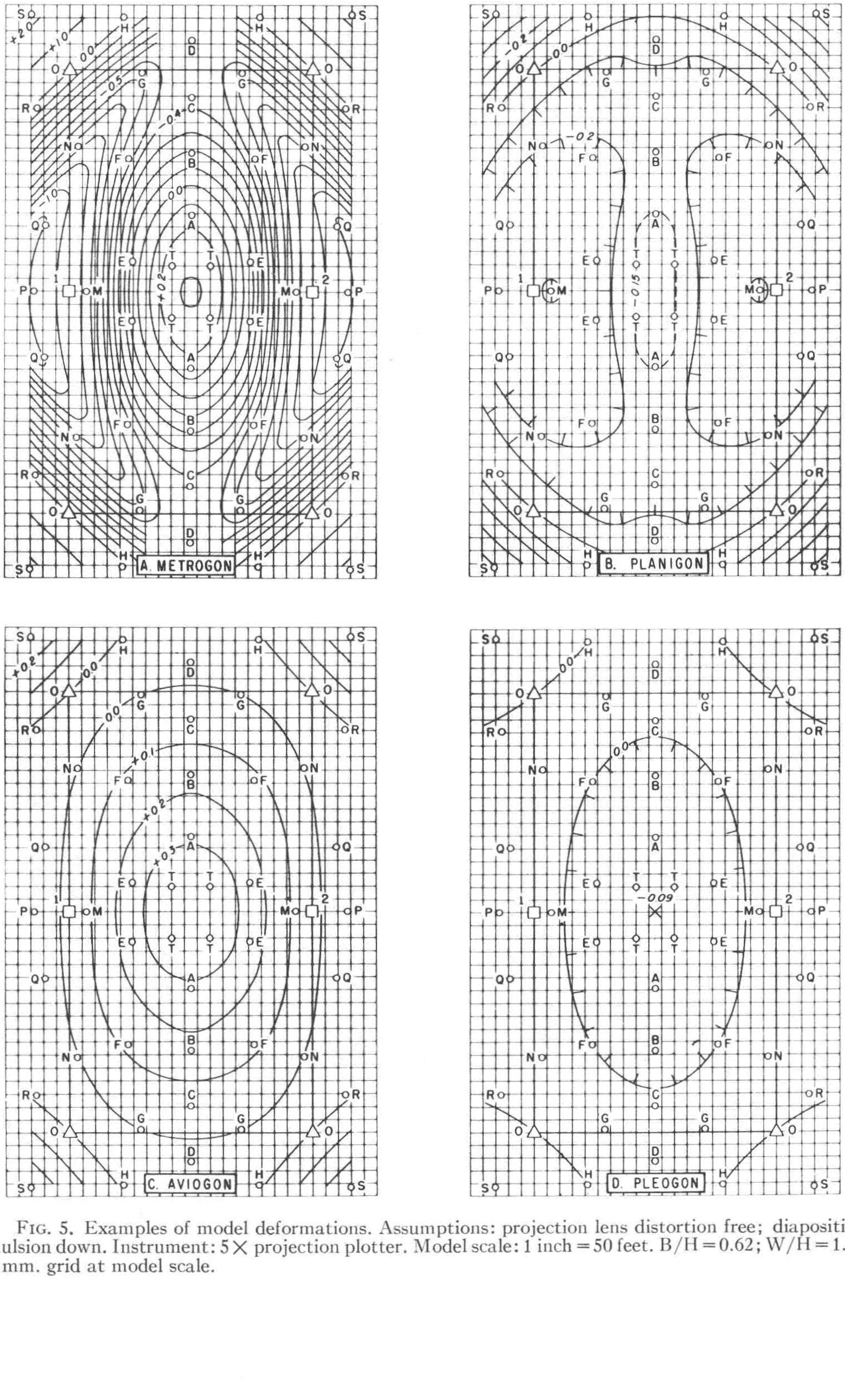

FIG. 5. Examples of model deformations. Assumptions: projection lens distortion free; diapositives emulsion down. Instrument: 5× projection plotter. Model scale: 1 inch = 50 feet. B/H = 0.62; W/H = 1.12. 25 mm. grid at model scale.

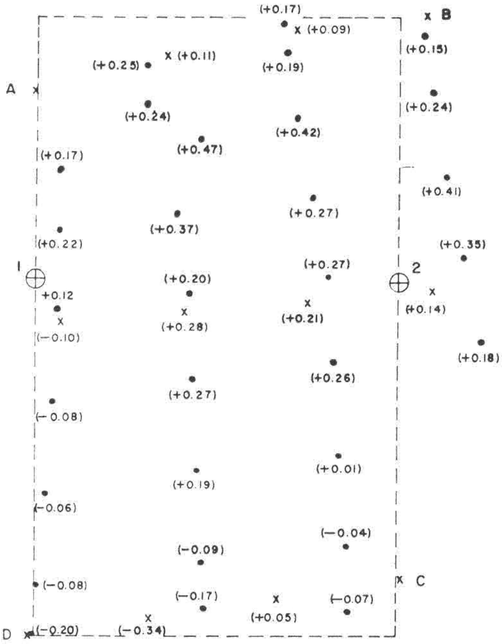

FIG. 6. Stereo-performance test results under operational conditions. Readings in feet at model scale of 1 inch = 50 feet.

with each instrument operated by a different individual. The resulting errors were averaged and compared with the known field elevations. These average errors are noted along side the individual points.

The camera used in this test was a Wild RC8 with an Aviogon lens, in fact the same camera analyzed in Figure 5c. The observed errors in the operational test do not duplicate the computed values taken from the calibration data, but do show the trend of deformation. Also to be noted is the asymmetric distribution of errors, undoubtedly derived in part from the fact that the camera lens distortion values are not symmetrical about the principal point.

Some unexplainable variations exist in the operational model. For instance, a test point happened to fall adjacent to the lower left-hand corner point *D*, but an error of 0.2 ft. was observed in the photogrammetric elevation. Other examples of this anomaly are evident. Photogrammetric elevations of premarked points are frequently difficult to determine, probably due to variations in image quality. Among the possible reasons for variations are: premarked images tend to halate; the premarking may be on sloping ground; surrounding ground cover may obscure part of the premarking.

Observed errors under operational conditions may not agree with computed errors based on distortion data for reasons other than image quality. It was previously pointed out that certain assumptions were made relative to the geometry of the overlapping photographs, as follows: the base-height ratio and width-height ratio was assigned to determine the size of the neat model; both photographs comprising the stereo-pair had identical scale values; both photographs were tilt-free. It is obvious that these specifications cannot be applied to operational photography, and it therefore follows that the geometry of actual exposure probably will differ from the assumed geometry used in mathematical analysis. Because varying geometric conditions are bound to occur, comparison of observed results with computed results will only verify the trend of model deformation. One cannot hope to definitely repeat point for point the identical model deformation values.

CONCLUSION

The performance of the camera-plotter combination can be verified without the expense, and subsequent doubt, of operational testing. The compatibility of the two instruments should be tested without influence of operational variables. It is recommended that plotting equipment be tested by the grid model method any time the operator suspects its performance is substandard.

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