# *An Analysis of Super-Long Aerial Triangulation and Its Applicability in Practice**

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ABSTRACT: *The use of high-altitude strip triangulation where the flight altitude exceeds 10 km. (33,000 feet) and which extends over several hundred kilometers is discussed in this paper. It has been found that this method permits the determination of the necessary ground control for medium and small-scale mapping in a minimum of time. A practical test was performed using the Wild Autograph A-7 and a photo strip flown from an altitude of 12 km. (39,000 feet) and having a length of 650 km. (400 miles). The accuracy attained is suitable for mapping at a scale of 1:100,000 and smaller.*

## GENERAL CONSIDERATIONS

TOPOGRAPHIC maps, in scales ranging from 1:50,000 to 1:200,000 represent important tools for the technical and economical development of many countries. According to reports from the United Nations Cartographic Section only a few per cent of the continental area of the earth are adequately mapped at large scales. For this reason it appears to be mandatory that the map compilation in these scales be highly intensified in the future.

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For the achievement of this task it is necessary to have available a sufficiently dense network of ground control, such as geodetic triangulation, spirit leveling and trigonometric elevations. A glance at the "World Triangulation Chart" printed by the U. S. Army Map Service reveals that only about half of the continental area of the earth is covered by principal arcs of triangulation and Shoran or Hiran networks. Gaps between the arcs of triangulation and between Shoran or Hiran stations vary from approximately 100 km. to 1,500 km. (60 to 900 miles). In many countries these networks are being completed by first- and lower-order geodetic triangulation, trilateration, and leveling. However, it will still take many years until the continental area of the earth is covered by an homogeneous density of higher-order horizontal and vertical control. It is obviously inconceivable that the economically and militarily important operation of mapping the continental area of the earth at medium scales can be postponed or delayed until the geodetic networks have reached a sufficient density.

For this reason the photogrammetrist who is in charge of large mapping projects is compelled to develop and to use fast and efficient control extension methods. Such a method—and probably the most efficient one—is stereotriangulation either in the form of strip triangulation, or of block triangulation, the

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latter consisting of a number of adjacent strip triangulations.

Although stereotriangulation is widely practiced, using primarily first-order stereoplotting instruments, and to a limited extent analytical triangulation, problems relating to methods are still subjects of current research. Such a problem is an analysis of the practical significance of strip triangulation with photography flown at altitudes in excess of 10 km. (39,000 feet). These strips currently have a length of several hundred km., and it is anticipated that strips will be flown with lengths up to several thousand km. in areas of sparse cloud coverage. The study of this problem, i.e. super-long strip aerial triangulation, is considered of practical interest in Canada. Here, first-order Shoran stations, spaced several hundred km., may be connected by stereotriangulation.

Furthermore, it seems feasible to study the application of independent geodetic base lines to long strip triangulation since this method is independent of the deflection of the vertical and therefore, might provide a means of yielding relative positions over long distances in areas with difficult access, with a higher accuracy than can be produced by astronomical positioning where the deflection of the vertical is unknown.

The study of super-long strip aerial triangulation is also of interest in connection with NASA's lunar exploration program. One of the missions being considered for this program is the establishment of a control network on the moon. This might be done by taking very long photo strips from a space vehicle orbiting around the moon at distances up to 150 km. from the lunar surface, returning the film to the earth, and performing the strip triangulation of the resulting photography.

## 2. Experimental Tests

The problem of super-long strip aerial triangulation is presently under study at the Department of Geodetic Science, the Ohio State University. The purpose is to determine the attainable accuracy of such strip triangulation for various spacings and distributions of ground control and for the various methods of strip adjustments. For this purpose a 650 km. (400 mile) strip of vertical photography, flown at an altitude of 12 km. (39,000 feet) above mean terrain, was available. It consisted of 81 exposures taken with a camera whose format was $9'' \times 9''$ and contained a 6 inch focal-length metrogon lens. This strip covers a ground area of approximately 12,000 square km. (4,300 square miles) located between the Arizona Test Area near Phoenix, and Los Angeles, California. The aerial photography was provided by Rome Air Development Center.

The strip triangulation covering 80 models was performed on the Wild Autograph A-7 at the Department of Geodetic Science, the Ohio State University; it required two weeks on instrumental work using the $bz=0$ method. In this method the $bz$-movements are not used, i.e. they are kept on zero and the relative orientation is performed by means of both longitudinal tilts $\phi'$ and $\phi''$; this requires a special adjustment of the recorded ground elevations according to the variations $\Delta\phi$ of the "fixed camera." For the recording of the instrument coordinates $x$ and $y$ (instrument scale 1:40,000), the elevations $H$ of the transfer points between adjacent models (3 points) and of the given ground control points used for the strip adjustment as well as of the available test points, the electric coordinate printer of the A-7 was used.

The flight-line deviated considerably from a straight line up to an amount which is equivalent to the entire width of the strip on the ground. There were also excessive amounts of tilt, both $\omega$ and $\phi$ as well as excessive swing $\kappa$. For these reasons a refined numerical model connection had to be performed which would not have been necessary for a well-flown strip when the $bz=0$ method is used. Various strip adjustment procedures were applied to the strip triangulation using desk calculators and graphical procedures. An electronic computer was not considered feasible since the various adjustment procedures would have required an excessive amount of programming and the funds were insufficient to support this phase.

To determine the accuracy of the strip triangulation and the various strip adjustments performed, only 1:24,000 and 1:62,500 scale topographic maps were available. From these maps 120 test points were selected whose machine coordinates $x$ and $y$ and the elevation $H$ were recorded during the performance of the strip triangulation on the *A-7*. The true positions and elevations of the test points were taken from the maps in terms of geographic coordinates and map elevations. The geographic coordinates of the test points were

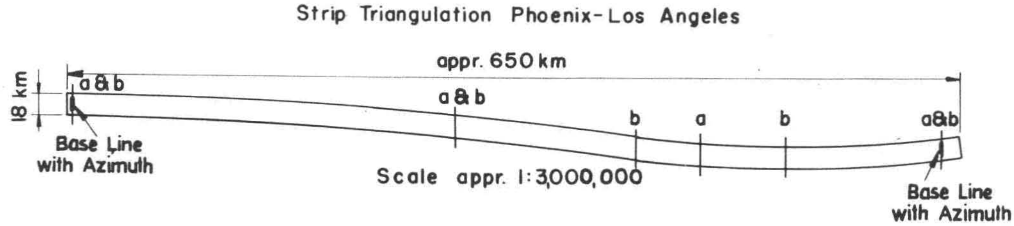


FIG. 1. Ground coverage.

then converted into local plane rectangular coordinates $X$ and $Y$ with the $X$-axis coinciding with the strip axis. This transformation was accomplished by means of a conformal Gauss projection from the ellipsoid to the sphere and by means of a conformal oblique cylindric projection from the sphere to the plane.

The closure errors of the 650 km. long strip triangulation before strip adjustment amounted to: $\Delta X = 2{,}200$ m., $\Delta Y = 300$ m., and $\Delta H = 10$ m. The extremely small closure error in elevation which represents only 1:65,000 of the strip length is explained by the nature of the $bz = 0$ method. It eliminates, as in the case of the aeroleveling method, the earth's curvature effect and also to a great extent the systematic error propagation; thus, resulting in a very minor error accumulation.

For the strip adjustment of the $X$ and $Y$ coordinates absolute ground control as well as independent ground control (independent base lines and azimuths) were used. The strip was adjusted using two different distributions of given absolute ground control. In each case the adjustments were performed using three equidistant longitudinal sections. In the first adjustment, four cross sections which were determined from given ground control at $X = 0$, $X = 270$ km., $X = 440$ km., and $X = 610$ km. were used in connection with third-order adjustment polynomials (Method $a$). For the second adjustment, cross sections determined from given ground control at $X = 0$, $X = 270$ km., $X = 390$ km., $X = 500$ km., and $X = 610$ km., were taken using fourth-order adjustment polynomials (Method $b$). The uneven distribution of given ground control used for the strip adjustment is explained by the fact that there was no given ground control available between the beginning of the strip and $X = 270$ km. Figures 1 and 2 show the location of the cross sections and the groups of given ground control points used for Method $a$ and $b$. The residual $\Delta X$- and $\Delta Y$-errors in the rear, center and front longitudinal sections of the strip, after strip adjustment according to Method $a$ and $b$ respectively, are illustrated in Figure 3.

For the computation of the residual standard errors in $X$ and $Y$ the standard accuracy specifications of the maps, from which the geographic coordinates of the test points were taken, have to be taken into account. For instance, for the maps at scale 1:24,000, 90 per cent of the points should be within a circular error of 40 feet; for a scale 1:62,500, the figure is 100 feet. After this reduction was properly applied, the following residual standard errors were obtained:

Method a: $\bar{S} = 200$ km.:

$$M_P = \sqrt{M_X^2 + M_Y^2} = \pm\ 60.5 \text{ m.} \quad L = 650 \text{ km.}$$

$$h = 12 \text{ km.}$$

Method b: $\bar{S} = 150$ km.:

$$M_P = \sqrt{M_X^2 + M_Y^2} = \pm\ 15.9 \text{ m.}$$

$$\text{Area} = 12{,}000 \text{ km.}^2$$

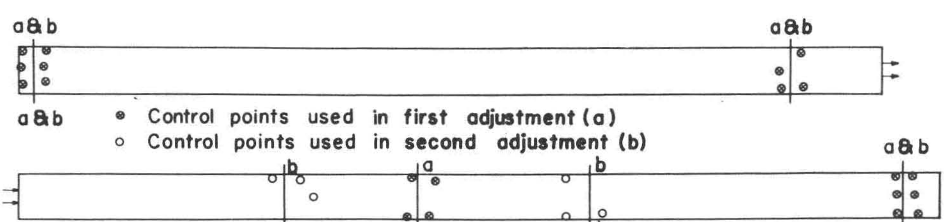


FIG. 2. Scheme showing ground control points and cross sections used for the strip adjustment.

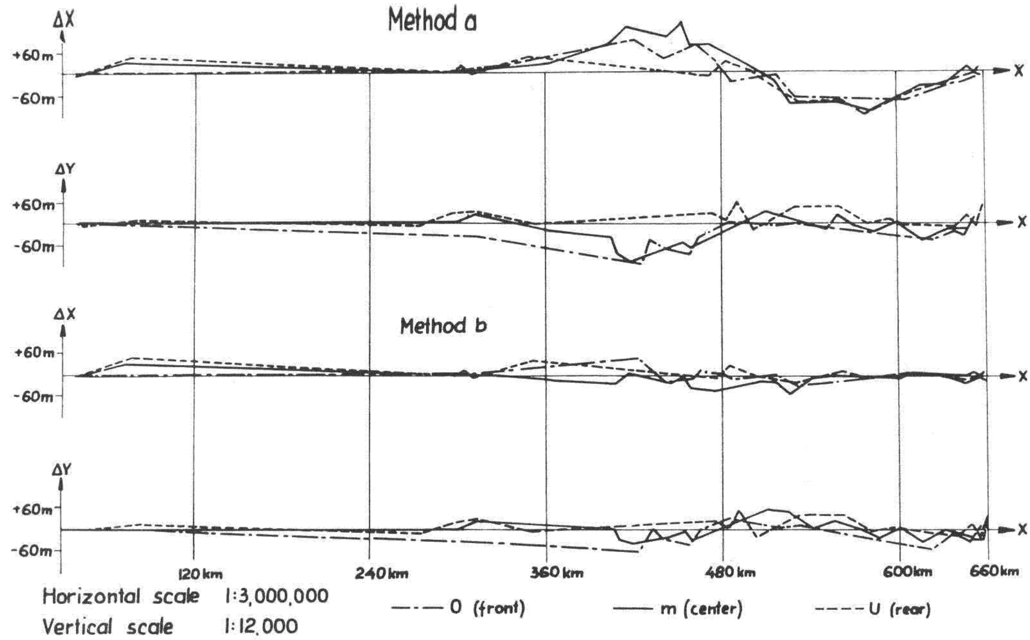


FIG. 3. Residual errors after strip adjustment in rear, center and front longitudinal section for methods *a*, and *b*.

From these results the conclusion can be drawn that a long strip triangulation flown from an altitude of 12 km. (39,000 feet) above ground with average spacing $\overline{S}$ of 200 km. (120 miles) between given absolute ground control will yield a sufficient horizontal accuracy of ground control for mapping at scales of 1:100,000 and smaller. If the average spacing $\overline{S}$ between given ground control is reduced to approximately 150 km. (90 miles), the horizontal accuracy of ground control is sufficient for mapping at scales of 1:50,000 and smaller. With regard to these favorable results, it is desirable to perform more such tests. Thus statistical material would be available from which valuable accuracy predictions could be applied to practical cases.

The strip was also adjusted *by using only two independent base lines and azimuths* (no absolute control), one at each end of the strip (see Figure 1). The following horizontal closure error was found:

$$\Delta S = \sqrt{(\Delta X)^2 + (\Delta Y)^2} = \pm 70 \text{ m.} = \text{appr. } 1:10{,}000 \text{ of strip length,}$$

or appr. 2″ in terms of geographic coordinates.

Such a small closure error is of practical significance when compared with the distance which would be obtained solely from astronomic fixes at both ends of the strip with unknown deflection of the vertical. In this case the change of the deflection of the vertical over distances of hundreds of km. is very often much greater than 2″. This means that the method with independent base lines and azimuths can be considered as a potential method to establish in a short time new fixes in areas without any geodetic triangulation. This is of particular importance when in such areas medium and small-scale mapping projects have to be completed in a minimum of time.

The *vertical adjustment* of the strip using different procedures is still in process and a report on this phase is anticipated at a later date.

The above described tests were performed by the following staff members of the Department of Geodetic Science, the Ohio State University: Dr. A. J. Brandenberger, Dr. Gunter Mulert, Mr. Robert B. Forrest, Mr. Kurt Bretterbauer, and Mr. Afifi H. Soliman.

## Closing Remarks

It is conceivable that the accuracy of high-altitude super-long strip aerial triangulation can be further increased when compared with the accuracy obtained in the test described above, since several unfavorable factors affected this test. Such factors were: large deviations of the photo strip from a straight line i.e. a Great Circle, excessive tilts and swing, low and heterogeneous optical resolution of the camera objective, and excessive irregular lens distortion. The use of high performance aerial camera with a minimum objective resolution of at least 20 lines per mm., and a maximum systematic and asymmetric lens distortion not to exceed 10 microns coupled with better navigation should produce a higher accuracy. This statement is also of importance if such triangulation is performed by analytical methods in combination with comparator measurements, which theoretically should produce a somewhat higher accuracy when compared with stereotriangulation. This increased accuracy, however, is not realizable as long as aerial photography is used whose inherent errors are several times larger than the inherent errors of a comparator or of a well-adjusted first-order stereoplotting instrument.

It is anticipated that, if the above outlined specifications are observed, even strip triangulations considerable longer than 650 km. (400 miles) and flown from an altitude higher than 12 km. (39,000 feet) would still produce interesting results; it also would permit an increase in the spacing between given ground control. This is of decisive economical significance since such high-altitude super-long aerial triangulation extensions cover tremendous ground areas, for which the necessary ground control for mapping in medium and small scales could be provided in a very short time and in a very inexpensive way, as is evident from using first-order stereoplotting instruments. To what extent this statement is also true for analytical aerial triangulation cannot be formulated since, up to the present time, insufficient information is available on the required time and costs for this method.

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