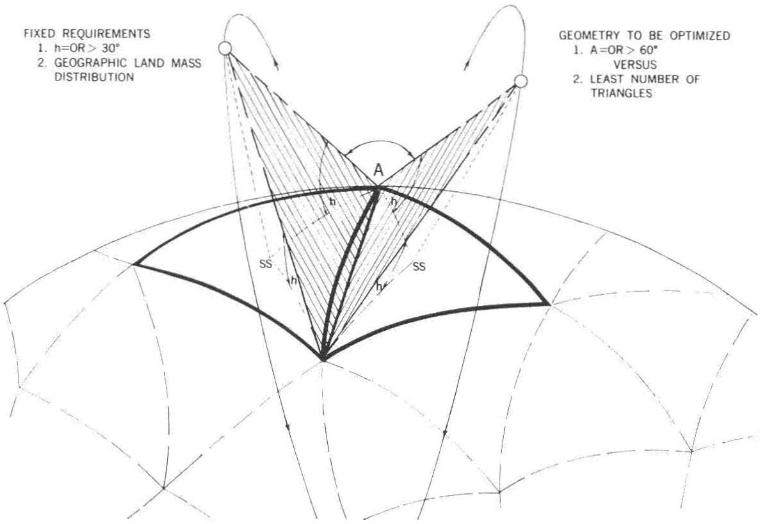


FIG. 4

(*Text on page 109*)

*Accuracy Aspects of a World-Wide Passive Satellite Triangulation System**

DR. HELLMUT H. SCHMID
U. S. Coast and Geodetic Survey
Rockville, Md. 20852

(*Abstract on page 107*)

A DISCUSSION about the accuracy aspects of a world-wide geodetic triangulation using passive satellites requires a few preliminary words concerning basic characteristics of geodetic satellite triangulation.

There is now a serious general interest developing for the practical execution of photogrammetric satellite triangulation. This interest is brought about by the realization that a geometric solution, will not only be a desirable approach for checking continent-wide classic triangulation systems, but will, if executed on a world-wide basis, allow, for the first time, the determination of three-dimensional positions of a selected number of points on the physical surface of the earth without the necessity of introducing geophysical hypotheses.

That is to say, without reference to gravity. Such a system will provide an absolute reference frame of stations around the globe to which can be tied both the predominantly geometrically-oriented mapping programs and the evaluation of satellite orbits for determining gravitational and related geophysical parameters.

Such a program can be accomplished by the use of a passive, sun-illuminated satellite, using either precision geodimeter traverse base lines or an electronic satellite ranging system, such as the Army-developed SECOR, for scale determination.

* Presented at 30th Annual Meeting of the Society, Hotel Shoreham, Washington, D. C., March 17–20, 1964.

The specific purposes of such a three-dimensional triangulation are:

a. To provide a world-wide reference net to which all geodetic topographic end products can be related.
b. To replace, for the determination of the shape and size of the earth, the classic, time-consuming, long-arc triangulation methods by a much more economical and theoretically superior approach.
c. To produce a world-wide reference frame of control, to which continental, three-dimensional reference nets on all accessible land masses can be tied, by again using photogrammetric triangulation with passive satellites.

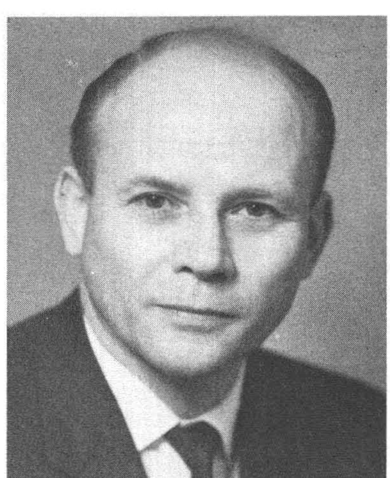
   These continental nets will provide the necessary control for numerical photogrammetric aerial triangulation executed from extremely high-flying aircraft or satellites. In this way, the geodetic control pattern will be further intensified in local areas, in order to provide control density in accordance with specific topographic mapping requirements. The creation of such a control pattern will provide an economical approach to world-wide mapping programs, and at the same time provide the geodetic community with a data library for a fast-response mapping capability.
d. To establish the necessary geometric fidelity for a system of world-wide tracking stations for observing orbiting satellites. The data collected from these satellite observations will be ideally suited for analyzing gravimetric and related geophysical parameters. These parameters, in turn, can then be used for determining the position of the center of mass and the overall shape of the gravitational field of the earth associated with its mass distribution.

Every known technique is applied in an effort to exploit the maximum accuracy potential of the system.

If there is a single, outstanding characteristic typifying the field of "Satellite Photogrammetry," it must be seen in the necessity of developing the classic photogrammetric method from its present character of a self-sustained interpolation method, providing predominantly precision, to a hybrid triangulation procedure whose results can be characterized by statistically significant expressions of accuracy.

Corresponding data evaluation methods require the mathematical simulation of numerous perturbations acting upon the idealized principle of central perspective. These perturbations are due in part to the design characteristics of specific photogrammetric components, and to imperfections encountered during the corresponding manufacturing processes. Further perturbations, which must be reckoned with, are caused by the physical nature of the environmental conditions affecting the various photogrammetric operations.

The processes of data adjustment and analysis in satellite photogrammetry will depend on the acceptance of modern mathematical statistics leading to a generalization of the classic least squares method. It is no longer sufficient to consider the purpose of an adjustment as a means of reducing observational errors, but the ultimate goal is to make a set of measured quantities compatible with an "economized" mathematical model. The basic idea underlying this approach is that all quantities used in the construction of a specific mathematical model are assumed to be measured quantities, which in turn are considered to be samples of stochastic variables. By assigning to these variables in advance relative variances and co-variances between the limits of zero and infinity, the least squares method will provide unbiased esti-



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mates for all variables, even if these are not normally distributed.

The interpretation of the discrepancies between the observed quantities and the chosen mathematical model, however, will provide the information necessary for analyzing the soundness of the theory on which, in the first place, the economized model was based.

It is the potential of the high-speed electronic computer which allows, in practice, the application of these sophisticated methods of numerical analysis.

Ultimately it will become necessary to elaborate even the most advanced ideas in computational photogrammetry in order to provide the means for including physical-dynamical constraints expressed by the geometry of satellite orbits.

The revolutionary development in geodesy brought about by the advent of satellites, consequently, challenges photogrammetry to become a tool of precision geodesy, by racking satellites from stations on the earth and by acquiring metric photography from satellite-borne camera systems. We are here only concerned with the tracking application.

The need for satellite triangulation is created by the shortcomings of classic first-order triangulations.

Even if we assume that bias errors, resulting from the physical characteristics of the classic angle-measuring instruments, are sufficiently eliminated, and even if we pretend that the propagation effects, often referred to by such terms as refraction, scintillation, etc., are well enough understood so as not to cause intolerable biases on the correspondingly corrected measured angles, classic geodetic triangulation is still confronted with a situation requiring rather complex theoretical considerations, the geometrical contents of which generally necessitate the a priori acceptance of certain hypotheses.

Aside from these physical problems, there is the unavoidable limitation of the length of the line of sight between specific points on or close to the physical surface of the earth. Practical geodesy not only suffers from the inability to establish intercontinental ties, but is forced to establish its basic so-called first-order reference systems as a mesh of all too numerous triangles. The undesirability of this situation is not in the number of points so determined, as clearly evidenced by the necessity to create even more control by filling in with second and third-order nets, but in the basically unfavorable laws of error propagation existing in any extended triangulation scheme which is based solely on angular measurements.

Satellite triangulation by photogrammetric means gives promise of overcoming some of the basic difficulties encountered in classic geodetic triangulation. Foremost is the fact, as already mentioned, that such a triangulation is independent of the direction of gravity, and, therefore, free of any a priori geophysical hypothesis. Because of the exceedingly large dimensions of its individual figures, satellite triangulation encompasses the whole earth with a relatively small number of triangles and closes its spatial triangulation in all three dimensions. Thus there is provided sufficient geometrical strength to prevent unfavorable error propagation.

The geometrical triangulation of a selected number of nonintervisible points on the physical surface of the earth can be accomplished by a process of spatial triangulation using auxiliary target points sufficiently elevated above the surface of the earth.

The creation of light signals, which can be seen over long distances, is possible with the help of artificial satellites. Because of the unavoidable motion of such targets, precise direction measurements can at present only be carried out by photogrammetric means. The photogrammetric method, because of the physical and chemical nature of its numerous components and operations, lacks in absolute metric accuracy and in reproducibility of its procedures. If measuring results of extremely high absolute accuracy are to be obtained by photogrammetric techniques, the only possible approach is to use the photogrammetric method as a tool of interpolation.

The reference system into which the direction to a sufficiently elevated target point can be interpolated is obviously the right ascension-declination system.

This reference system is attractive from the geodetic standpoint because one of its axes is by definition parallel to the rotation axis of the earth. From a quantitative standpoint, a practically unlimited number of fixed stars are available as reference points. From the fact that these stars are for all practical purposes at infinity, it follows that their direction coordinates are insensitive to translations. Therefore they cannot be used for any scale determination.

It is now of interest to consider the problem of the geometric feasibility of a three-dimensional triangulation system based on incomplete direction measurements; incomplete because of the non-intervisibility of the ground stations.

two such planes (AS_1B and AS_2B) containing the base line are observed, the direction of the base line can be computed as the line in which the two planes intersect.

It is now readily seen from Figure 2 that two base lines originating from a specific station ($\overline{BA}$ and $\overline{BC}$) will form a spatially oriented triangle if the two lines under consideration are intersected by a plane which contains

ABSTRACT: *Recent results indicate that eventually the error in right ascension and declination of an artificial passive satellite will be within 0.4 and 0.5 second of arc, or 1 part in 400,000. Although this error may be double that for classic geodetic triangulation within continental limits, it can well serve a world-wide system for tying the continental systems together and for determining improved values for the earth's geometric parameters with unsurpassed accuracy. The discussion is based on the principle that a sum-illuminated (passive; non flashing) satellite offers an accuracy superior to any other type of system. The system is also entirely independent from local variations in the earth's gravity field.*

The land areas of the earth lend themselves to the practical consideration for the establishment of 36 satellite camera stations where the height of the satellite is 3,600 kilometers. The low error figure can be obtained by taking precautionary measures in each and every important operational step in the system and by including a very large redundancy of observations of both satellite images and background stars as dictated by the classic principles of statistics. The data are based on preliminary results obtained from many photographs of Echo satellite made at first-order geodetic control stations.

The use of the star background for orienting cameras in connection with the triangulation of additionally recorded target points was applied for the first time during the 1930's by Hoppman and Lohman for trajectory measurements of shells and small rockets.

Prof. Väisälä suggested the same principle in the Sitzungsberichte der Finnischen Akademie der Wissenschaften 1946 under the title "An Astronomical Method of Triangulation." There are various ways to demonstrate the geometric feasibility of the method. Väisälä's approach allows a very elegant presentation of the basic geometric principles, but leads to a cumbersome data reduction method, especially if the corresponding triangulation consists of an appreciable number of triangles and redundant information has to be treated (Figure 1). The reasoning in Väisälä's paper is: two conjugate rays emerging from the endpoints of a base line ($\overline{AB}$) define a plane in space whose spatial orientation can be computed from the measured direction cosines of the two rays (e.g. AS_1 and BS_1). If neither of the two lines and whose orientation is known. As outlined before, each direction is determined from the intersection of two planes. In our case, the planes AS_1B and AS_3B determine the direction of base line BA and analogously the planes BS_2C and BS_5C the direction of base line BC. Consequently, five planes are necessary and sufficient for establishing a unique solution for a spatially-

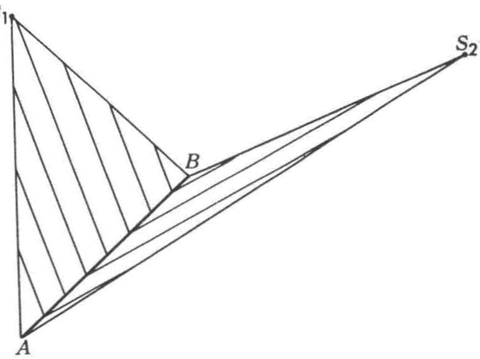


FIG. 1

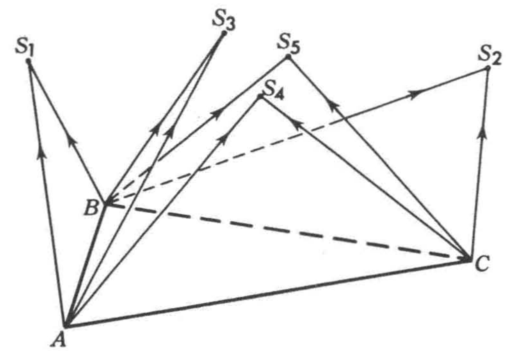


FIG. 2

oriented station triangle. Any one of these planes contains two stations and a specific satellite position. It is obvious that the just described principle can be applied to any number of triangles forming a three-dimensional triangulation scheme. Of interest is the fact that three of the five planes can be formed with only one satellite position if the location of the satellite is so chosen that its sub-point is about in the center of the triangle to be determined, as shown in Figure 3. In such a case, only 3 satellite positions are necessary.

From the viewpoint of analytical photogrammetry, the geometric feasibility of the photogrammetric satellite triangulation can be explained by the fact that the unknown ground stations and the unknown orientations of the cameras at these stations resemble the air stations in traditional aerial photogrammetry. The unknown satellite positions correspond to the relative control points of the model, and the stars play the role of absolute control points, with the obvious limitations that they are at infinity and, therefore, do not contribute to scale determination.

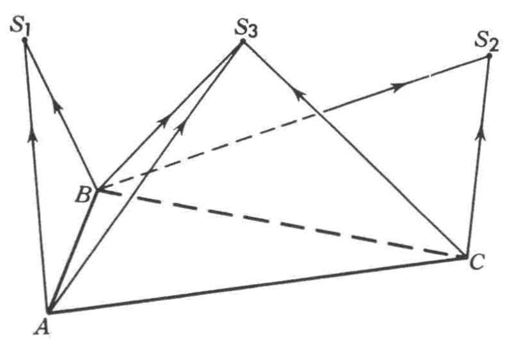


FIG. 3

Furthermore, a somewhat different geometrical interpretation of the triangulation problem is necessary in so far as the stars are used to determine the elements of interior orientation together with the three rotational components of the exterior orientation. The condition of intersection, as associated with any one satellite image, is used to determine the translatory elements of the exterior orientations (the position coordinates of the camera stations). In this way there is avoided the unfavorable correlation that exists between rotations and translations in aerial photogrammetric procedures.

The strictly geometrical considerations must now be complemented by considering a significant physical phenomenon which exercises a decisive influence on the accuracy of stellar triangulation.

The refraction problem as such enters into the method with only a second-order effect. It can be shown that the refraction of an image outside the atmosphere can be computed as astronomical refraction minus a parallactic angle, which is extremely *in*sensitive to an error in astronomical refraction. Nor does the refraction error enter significantly into the orientation of a photogrammetric record, because the orientation is established with respect to the geometrically-known star catalog values.

As a matter of fact, the basic concept of the photogrammetric star triangulation method depends not only on the geometrical interpolation method of the photogrammetric measuring procedure, but equally on the fact that, in a physically significant sense as well, the satellite images are interpolated into the astronomical refraction effect. Because of the geometrical as well as the physical interpolation, this method is essentially free of bias errors. It is this fact which distinguishes the method in principle from other spatial triangulation methods, e.g., from electronic length-measuring methods.

In order to obtain a precise and bias-free result with the stellar-triangulation method, it is, however, necessary to recognize a basic shortcoming of the method as long as the practical execution is carried out with a flashing-light satellite beacon.

The reason, very simply, is that the random scintillation (shimmer) of a short-duration light flash limits its geometrical significance to plus or minus two to three seconds of arc, somewhat depending on the geographic location of the station. It is true that with increasing aperture this effect can be reduced. But because the just-mentioned values are by no means extremes, it appears that this phenomenon can in practice only be coped with by statistical means—that is to say by producing and measuring a large number of photographic images. Present-day technology does not provide the means for producing the large number of flashes necessary to assure geometric significance of the observed target direction.

A practical answer to this problem is obtained if the trail of a sun-illuminated satellite is chopped by accurately-timed shutter actions. A precision photogrammetric system, which was developed during the past ten years for this purpose largely at the Ballistic Research Laboratories in Aberdeen, Maryland is now used by the Coast and Geodetic Survey. With such a system it is possible to produce several hundred satellite images on a single photogrammetric record.

In addition, there is much that can be said in favor of a sun-illuminated target in terms of ease of recording, reliability of a passive target vehicle, dependability of the sun as an illuminator, and overall economy of the project, as proven lately with the use of Echo I and Echo II satellites. The launching of a similar balloon-type satellite for the establishment of a world-wide geodetic reference system is now under preparation.

Such a net must be planned as a compromise between various parameters (Figure 4). First, the line of sight to the satellite must be at least 30° above the horizon in order to avoid disturbing refraction anomalies. The maximum angle between any two planes generating the spatial direction between two stations must be at least 60° in order to provide sufficient geometrical strength for the determination of three-dimensional station positions. The distance to the satellite should not be larger than the length of the sides between stations in order to avoid suffering from unfavorable scale.

Obviously, these conditions ask for as much symmetry in the net as possible with regard to the distribution of islands and land masses. Accepting as a compromise variations between 4,000–4,500 km. in the lengths of sides between selected stations, a plan for a world-wide reference net was established with 36 stations forming 68 triangles with 102 sides. The corresponding optimum satellite orbit requires a nominal 90° inclination (polar), a nominal circular shape, and a height of 3,600 km. above the earth.

The ultimate selection of a world-wide reference net obviously will be influenced by secondary considerations such as political and physical accessibility of station locations, the need for incorporating already available geodetic information established with the major world datums, the subsequent execution of astro-geodetic and gravimetric surveys in the neighborhood of the selected sites, and the incorporation of stations which are already installed in the world-wide satellite tracking network, as for example, those operated by the Smithsonian Institution Astrophysical Observatory.

It may now be of interest to discuss some of the aspects which are significant in judging the overall accuracy which can be expected from photogrammetric satellite triangulation.

The absolute reference system into which the individual satellite directions are interpolated is, as was pointed out, the right ascension and declination system. The Boss Catalog, containing more than 30,000 stars, was at its mean epoch (1900–1910) a rather accurate catalog. However, the multiplication of errors in proper motion when advancing GC-position to 1965 yields somewhat unreliable answers concerning systematic and random errors. For a photogrammetric world-wide satellite triangulation, it is especially essential that the right ascension-declination system present a homogeneous reference system covering the whole sky. Presently, only the N-30 catalog, with about 5,000 stars, and the FK-4 catalog, containing 1,535 stars, should be applied. The FK-4 catalog is, on the average, accurate to 0.″2 and as a whole more accurate than the N-30 catalog in the sense that two stars of the N-30 catalog will produce the same accuracy as one star of the FK-4 catalog. Serious attention must be paid to the fact that the precision of star positions from all catalogs decreases commencing at about −30° declination and extending southward.

The startling and unexplained manner in which the N-30 and the FK-4 systems depart from each other should be of concern for the problem of establishing a reliable geometric world reference system by photogrammetric-stellar triangulation. It is estimated that by 1975 these two systems will have deviated from each other by as much as 0.″9 in right ascension at −60° declination. Precision geodetic satellite triangulation depends on the removal or at least on the explanation of such discrepancies, emphasizing the need for an intensive series of fundamental astronomical observational programs. For the northern hemisphere the anticipated publication of the AGK 3 Catalog with 180,000 stars will be a welcome addition, but will not at present solve the problem of a homogeneous world-wide reference system.

Entirely in accordance with any other photogrammetric triangulation procedure, the collinearity condition is the basic element of measurement in stellar triangulation. In other words, any system of stellar triangulation can be conceived as a multitude of individual rays, where the direction of each of these rays is determined from a corresponding photogrammetric record.

Therefore, it is first of interest to study the accuracy with which such a direction can be obtained, in order to follow ultimately with the problem of determining the accuracy of the final triangulation as the result of propagating the errors of the individual rays into the configuration of the spatial triangulation.

The problem of how accurately an individual ray can be interpolated into the star background can be studied by considering two independent error-sources:

a. The accuracy with which the spatial orientation of the photogrammetric record can be established from the star photography, and
b. the accuracy with which the image of the satellite can be measured.

The accuracy of the determination of the orientation of the photogrammetric record is obtained from the inverse matrix of the normal equation system associated with the analytical treatment of the single photogrammetric camera. Experience has shown that, despite previously established camera calibrations, it is necessary to incorporate, in addition to the six geometrical parameters (three rotations and three translations), four to ten additional parameters, depending on the quality of the photogrammetric camera, for the purpose of describing the distortion characteristics of the specific photogrammetric record at the moment of exposure. In addition, it is obviously necessary to provide means for correcting both the plate measurements and the star catalog values in accordance with assigned weighting factors.

The diagram (Figure 5) shows the number of stars which must be measured on an 18×18 cm. photography, obtained with $f=300$ mm., as a function of the number of parameters carried in the solution, in order to obtain a certain overall orientation accuracy. It is important to note that not only the absolute number of parameters, but also the need for additional star imagery increases with the size of the area of the photogrammetric record used for the recording of the satellite information. If, for example, $\frac{2}{3}$ of the record is to be used for recording satellite images, and 9 parameters are considered necessary for describing the mathematical model of the photogrammetric camera, it is necessary to measure approximately 150 evenly distributed star images in order to obtain a minimum orientation accuracy of ±0.5 seconds of arc, if the mean error of unit weight for a single coordinate measurement is ±3 microns.

The coordinate measuring process obviously is one of the key operations during the data evaluation. The problem begins with the need for a calibration standard in the form of a calibrated grid with which the comparator calibration can be performed. In this connection the fact may be of interest that at present the national institutes for calibration—to name a few, the Bureau of Standards in the U.S.A.; the Physikalische Technische Bundesanstalt in Braunschweig, Germany; the Eidgenössische Amt für Mass und Gewicht in Bern, Switzerland—do not accept the task of calibrating a line grid of 200×200 mm. to an accuracy of $\pm 1\mu$. As a matter of fact, these institutes apparently have to apply their full capabilities in order to calibrate the positions of a sequence of lines on a 200 mm. glass scale to an accuracy of $\pm 0.5\mu$.

It was necessary, therefore, to develop a special measuring procedure in order to obtain the coordinates of a selected number of grid intersections (25 points arranged in a 5 cm. square-pattern) with the required accuracy. The method determines the coordinates of these points by trilateration, whereby all possible lengths between the selected points (sides and diagonals of squares) are measured in all possible combinations, by

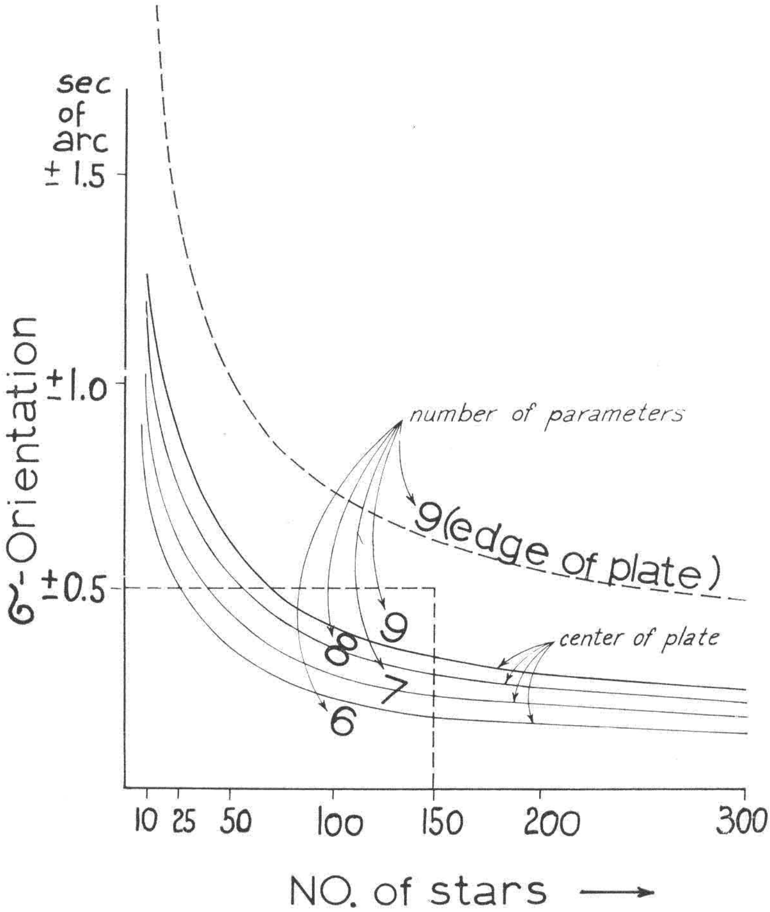


FIG. 5. The number of stars that must be measured in order to obtain an overall orientation accuracy on 18 by 18 cm. photography where $f = 0.3$ m. and $\sigma = \pm 3\mu$.

comparing each length under consideration with a calibrated glass scale. The measuring procedure is arranged as a null method in order to assure, as much as possible, the elimination of the personal equation of the observer. One hundred and sixty (160) independently-measured lengths, obtained from 2,560 pointings, are combined in a rigorous least squares adjustment. Each of these pointings is made with a precision of $\pm 0.5\mu$, leading to an accuracy of an individual length measurement of $\pm 1.0\mu$. The final grid coordinates are then obtained with a σ of $\pm 0.6\mu$ at the center and $\pm 1.0\mu$ at the corners of the grid. Repetitious calibrations with this method have given results which agree in the individual coordinates to close to 1.0 micron. Using the arithmetic mean values of these individual calibrations, it is believed that the coordinates of the 25 points on each of the calibrated grids are now determined with a sigma somewhat smaller than 1 micron.

Based on extensive tests, the accuracy of the coordinate measuring process for a star image, using the arithmetic mean of 2 sets of 4 measurements each, is $\pm 1.0\mu$ degrading in practice to not more than $\pm 1.5\mu$ (one sigma level), including identification (setting) and comparator errors. If 150 star positions are measured, the error contributed by the orientation of a single ray within the bundle ranges at present from a minimum of $\pm 0.''3$ at the center of the plate, to maximum $0.''5$ at a point 6 cm. from the center of the plate.

The standard deviation for a double setting of an individual satellite image is ± 2 microns. It is reasonable to assume that the orbit of an artificial satellite during the short period of observation (at most 2 minutes), because of its dynamic characteristics, is a curve which is smoother in nature than can be reconstructed from the corresponding plate measurements. Consequently, a high-order polynomial (mostly 4th or 5th order) is applied to smooth the random errors of the satellite imagery which originate from the measuring process and the shimmer effect.

Due to extremely high timing accuracy (± 40 micro seconds), the recording of individual satellite images at equal intervals can be considered flawless. It is possible, therefore, to smooth the x and the y coordinate measurements as independent operations, using a power series of time. The necessary least-squares fit is accomplished after the raw data (comparator measurements) are corrected for lens distortion, refraction and phase of illumination. This phase depends on the size and shape of the sun-illuminated object and the relative geometry between sun, satellite and photogrammetric tracking station.

In the case of a spherical-shaped balloon, the significance of this correction is to refer the ultimately computed satellite directions to the center of the satellite, in order to provide, from all tracking stations, geometrically consistent data for the spatial triangulation. There is a slight difference in the mathematical formulation for this correction, depending on whether the surface of the satellite produces a moon or planet-like reflection, or whether the surface acts more like a mirror providing a high-light point as a source of reflection.

Numerous results from such curve fits have been obtained which show a typical standard deviation of ± 2.5 to 3μ for a single satellite image (Figure 6 and Figure 7). Making allowance for a standard deviation of $\pm 2.0\mu$ for the measurements of the image coordinates, a contribution of $\pm 2.5\mu$ or, with $f = 300$ mm., about ± 2 seconds of arc, results from the shimmer effect. This result is in agreement with independent quantitative studies of this phenomenon.

Finally, a set of fictitious plate coordinates is computed using the coefficients of the least squares-fitted polynominal. This approach can be considered as a data compression process whereby the coordinate measurements of 500 to 600 satellite images are combined into a single set of fictitious x, y coordinates. The instant of time used to compute these coordinates is chosen so that the fictitious satellite image is as close to the center of the photograph as practical. Perhaps the greatest significance is that this procedure allows us to synchronize the various tracking stations by applying small time correction to compensate for asynchronization of the oscillators between different tracking cameras. In addition, this step makes it possible to correct for the difference in travel time of light signals from the satellite to different tracking stations.

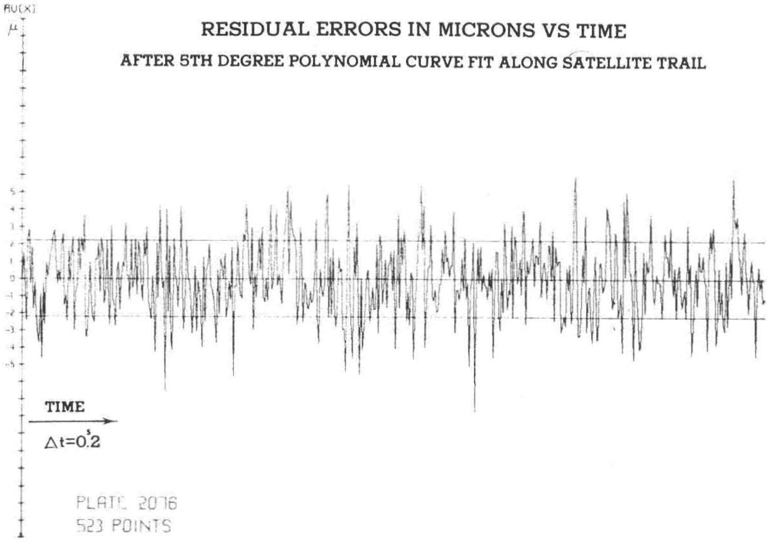


FIG. 6

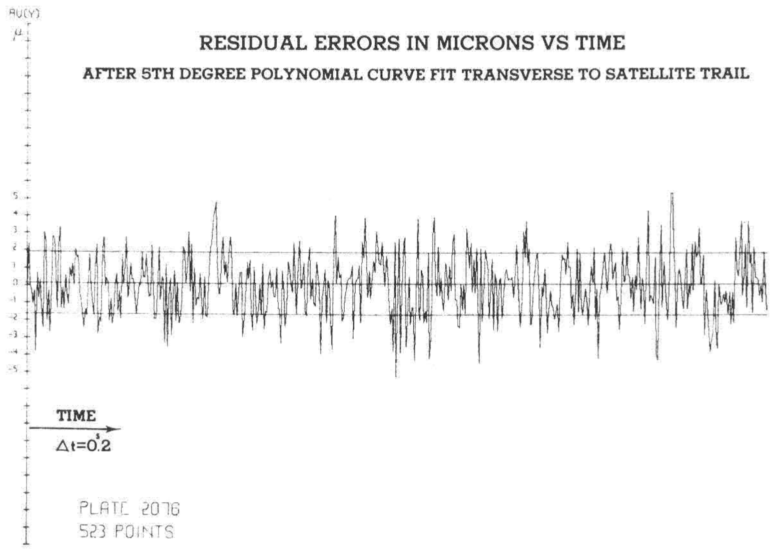


FIG. 7

The standard deviation of a computed fictitious satellite position is less than $\pm 0.2\mu$ at the center of the plate (see Figure 8) which corresponds, with a focal-length of 300 mm. to ± 0.1 seconds of arc. If combined with the aforementioned standard deviation of the orientation at the center of the plate, which is about $\pm 0.''3$, the accuracy of the final direction to the satellite should be better than $\pm 0.''5$.

Practical tests, based on multi-camera observations, indicate at present an accuracy of $\pm 0.''7$ or better. The slight degradation in the accuracy is caused by some insufficiently resolved bias errors. The most critical bias error is the one which affects the coordinate measurements between star and satellite images. Automation of the image measuring process by electronic image sensing is expected to be helpful.

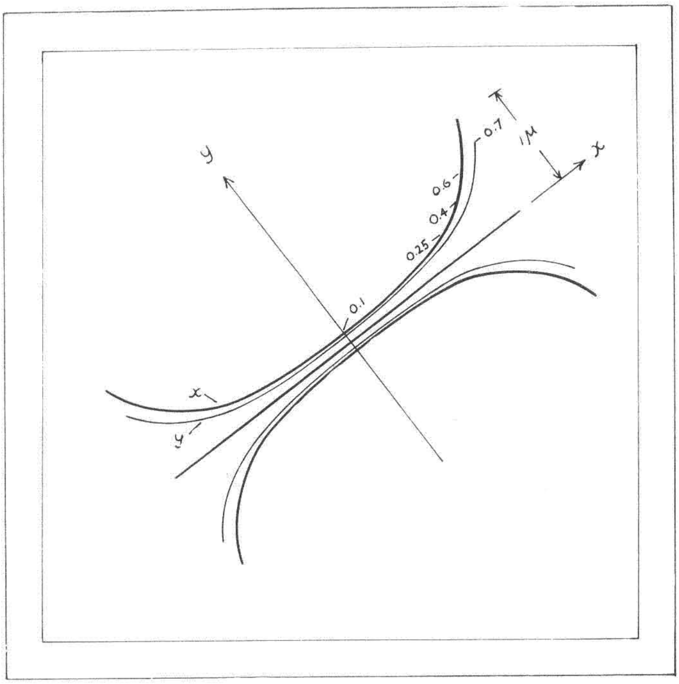
These remarks will now be complemented by a few typical results from a large number of tests which have been made in the last few months. At Aberdeen, three cameras were set up at the vertices of a 3 m. and a 25 m. triangle. The camera-systems were operated completely independently and a specific satellite (Echo I) trail was observed. The difference in right ascension-declination for about 500 satellite images for two of the stations are shown in Figure 9 and Figure 10. Figure 9 shows the differences in right ascension. The noise is again indicative for the combined errors introduced by measuring and shimmer errors. The obvious bias is caused from

1. the geometry, namely the 3 m. distance between the cameras, corresponding to $0.''3$ arc
2. an asynchronization of 0.880 micro sec between the station oscillators and
3. by the influence of the combined orientation errors of the two cameras under consideration.

After a curve fit was executed as described before, and correction for the asynchronization was made, the final result is indicated by the black dot at about $0.''10$.

The corresponding plot of the differences in declination is shown in Figure 10. Here the final result shows a difference in direction of $0.''5$. The measured spatial parallactic angle is, therefore, $0.''6$ of a second, as compared to the $0.''3$ of a second which is the true value based on the 3 meter separation of the cameras.

Figure 11 is shown to convey an idea of the super-large geometry which has become a practical possibility for geodesy with the advent of satellites. With the thin lines, the typical size of classic first-order triangulation chains is shown. The medium heavy lines delineate the first 800 mile triangles observed by the Coast and Geodetic Survey during the initial phase of its satellite triangulation pro-



ERROR BOUNDS FOR X AND Y
(ALONG AND TRANSVERSE TO SATELLITE TRAIL)
FOR 5th ORDER LEAST SQUARES—FITTED POLYNOMINALS

FIG. 8

gram with Echo I, and the heavy line is one side of the contemplated world net.

Figure 12 shows some results for missions executed over the triangle Chandler, Greenville, Aberdeen.

The column labeled "mean error after point matching" gives evidence of the accuracy of the plate measuring procedure. The mean errors are obtained by comparing, with a least squares fit, two sets of measure-

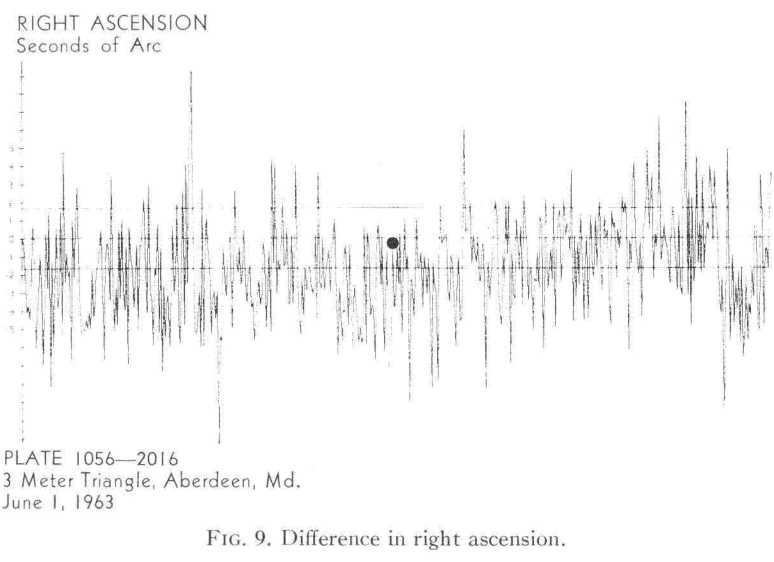


FIG. 9. Difference in right ascension.

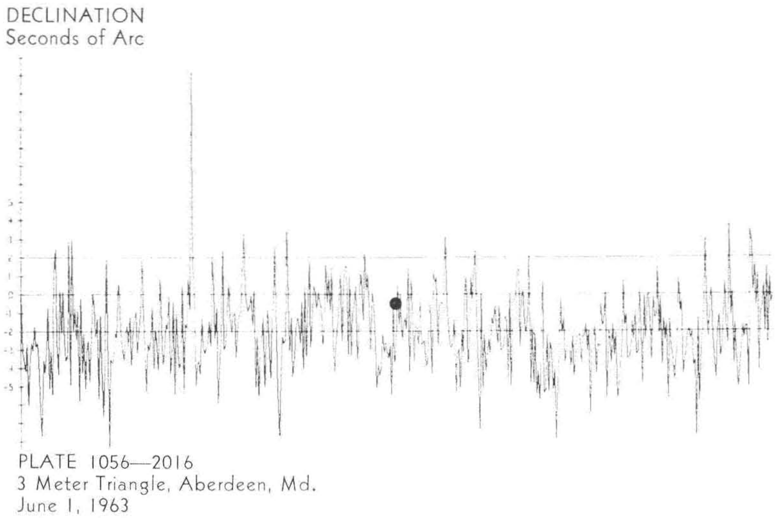


FIG. 10. Differences in declination.

ments of the same plate which are performed independently in such a way that in the second set the plate is rotated 180° in relation to the first set up.

The increase of the mean error of the comparator measuring procedure, which is typically between $\pm 1.0\mu$ and $\pm 1.5\mu$, to a mean error after camera orientation of $\pm 2.5\mu$ to 3.0μ as shown in column 6, indicates some unresolved bias errors because of a somewhat overeconomized model. The introduction of a model for tangential distortion, and the use of a point of symmetry for the radial distortion pattern in addition to the principal point, is expected to remedy the situation in the future.

Columns 8 and 9 show the random noise of the measured satellite trail. The last two columns give the mean errors a fictitious satellite position computed from 5th order polynomial least squares fits.

An upper limit of $\pm 3.0\mu$ is presently associated with the plate orientation procedure, and a typical mean error for a fictitious satellite position is $\pm 0.15\mu$, resulting in an accuracy of about $\pm 0.''7$ for an individual direction to the satellite.

Figure 13 gives the final results of the triangulation of the satellite for a triple and five double intersections. In all cases, the geodetic information as given from the United States first-order survey has been rigorously en-

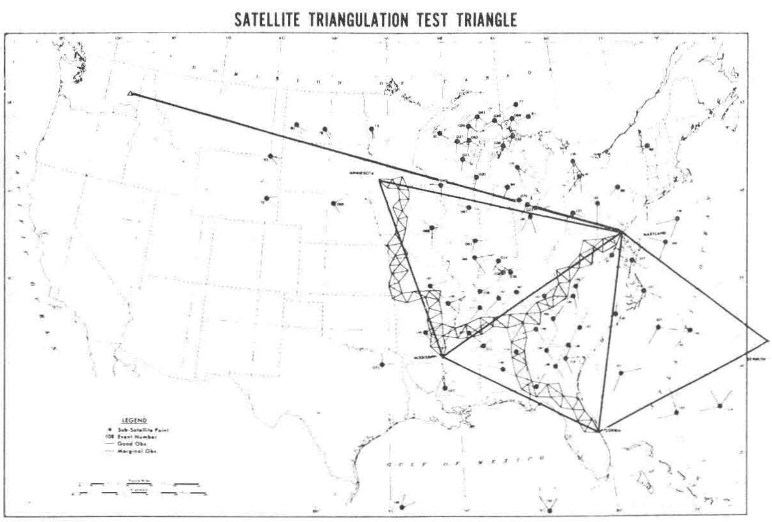


FIG. 11

EVENT NO.	PLATE NO.	LOCATION	MEAN ERROR AFTER POINT MATCHING $\pm\mu$	NO. OF STARS	MEAN ERROR AFTER CAMERA ORIENTATION $\pm\mu$	NO. OF SATELLITE IMAGES	MEAN ERROR OF 5th ORDER CURVE FIT x $\pm\mu$	y $\pm\mu$	MEAN ERROR OF FICTITIOUS SATELLITE POSITION x $\pm\mu$	y $\pm\mu$
106	1135	CHANDLER, MINN.	1.55	130	2.69	215	2.91	2.32	0.20	0.15
	2117	GREENVILLE, MISS.	1.46	144	2.66	194	2.94	2.02	0.21	0.15
	3104	ABERDEEN, MD.	1.31	163	2.10	217	2.09	2.26	0.14	0.15
068	1100	CHANDLER	1.15	149	2.77	517	2.20	1.76	0.10	0.09
	2076	GREENVILLE	1.29	168	2.97	515	2.24	1.93	0.10	0.08
	3077	ABERDEEN	1.46	168	2.93	215	3.15	2.48	0.25	0.16
041	1092	CHANDLER	1.23	144	2.49	551	3.06	2.26	0.13	0.10
	3070	ABERDEEN	1.59	141	2.69	549	3.29	2.60	0.14	0.11
011	2050	GREENVILLE	1.55	150	2.31	366	2.49	1.88	0.13	0.10
	3056	ABERDEEN	1.56	151	2.81	273	2.38	2.01	0.14	0.12

FIG. 12. Results for missions executed over the triangle Chandler, Greenville, Aberdeen.

forced as indicated with the ∞-weight for the station coordinates. In columns 5 and 6 the residual plate errors v_x and v_y are tabulated. In the last column the mean error of measurement of unit weight is shown. The maximum values of $\pm 1.5\mu$ correspond to ± 1 second of arc. On account that the geodetic information cannot be flawless, this preliminary result is consistent, if not better, with the expected accuracy of $\pm 0.''7$. A computer program for the execution of a three-dimensional spatial triangulation based on a rigorous least squares solution is in preparation, which will allow simultaneous adjustment of all observed data, with their proper constraints.

From the results so far obtained, it appears, therefore, justified to conclude that the photogrammetric satellite triangulation method will eventually give the direction to a satellite to within ± 0.4 to ± 0.5 seconds of arc or 1 part in 400,000.

The fact that such a direction has an *absolute* meaning with respect to the right ascension-declination system allows the con-

	WEIGHT MATRICES					
EVENT NO.	1 CHANDLER, MINN.	2 GREENVILLE, MISS.	3 ABERDEEN, MD.	v_x 1 2 3 μ	v_y 1 2 3 μ	σ μ
068	∞ ∞ ∞	∞ ∞ ∞	∞ ∞ ∞	+0.23 +0.06 −1.51	−1.25 +0.87 −1.48	±1.51
106	∞ ∞ ∞	∞ ∞ ∞	—	−0.86 −0.12 —	−0.02 +0.81 —	±1.19
106	∞ ∞ ∞	—	∞ ∞ ∞	+0.46 — −1.03	+1.05 — −0.39	±1.59
106	—	∞ ∞ ∞	∞ ∞ ∞	— −0.53 −0.20	— −0.17 −0.53	±0.79
041	∞ ∞ ∞	—	∞ ∞ ∞	−0.14 — −0.05	+0.008 — −0.16	±0.22
011	—	∞ ∞ ∞	∞ ∞ ∞	— −0.003 −0.01	— −0.007 +0.005	±0.01

FIG. 13. Final results of triangulation of the satellite for a triple and five double intersections.

clusion that such a result is promising if compared with the accuracy of ±0.″2 to ±0.″3 which is obtained today in first order triangulation for a *relative* direction.

In closing this consideration of photogrammetric satellite triangulation, it can be predicted that this method will not only prove useful for a world-wide triangulation scheme, but will eventually provide the necessary accuracy for increasing the geometric fidelity within individual geodetic datums.

Extensive numerical analysis on various possible schemes for the application of photogrammetric satellite triangulation is currently in progress for the purpose of studying the problem of error propagation and for establishing an optimized field operational procedure.

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tively fast test sequence, and permits one to achieve an accuracy of about ±0.5 mm. for deformations in x or z. This accuracy is sufficient for the determination of ultimate strength design parameters, which cannot be obtained reliably from strain gauges. Structural model testing plotters [3] are considerably more expensive and are very limited in size and versatility as compared to stereometric camera techniques.

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