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*Geodetic Position Determination by Stellar Camera**

FLAR and BCSR computer programs are applied for data reduction at the Atlantic Missile Range

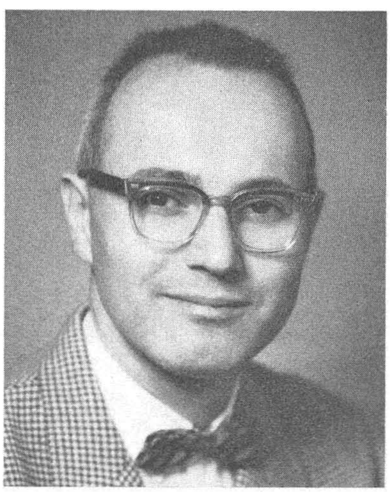
INTRODUCTION

THIS paper is the first part of a two-part paper. Part II, Test Results, is to be presented by Radm. J. P. Lushene, at the 1964 Regional Convention of the American Congress on Surveying and Mapping, to be held in Kansas City, Missouri, 24–26 September 1964. It is contemplated that Part II will be published in "Surveying and Mapping." The data reduction aspects for these techniques have been presented by F. A. Collen at the SPIE Seminar on Photo-Optical Data Reduction, St. Louis, Missouri, 2–3 March 1964, and will be published in the proceedings of that Seminar.

Geodetic position determination by means of Analytical Photogrammetry is essentially a Space Resection problem. The Manual of Photogrammetry defines Space Resection as, "The analytical determination of the three rectangular coordinates of an exposure station with reference to the ground survey coordinate system." This definition holds for both aerial and terrestrial applications of Photogrammetry. There are two variations to the space resection problem: single point in space resection, and multiple point in space resection.

At the AMR, we are engaged in the application of analytical photogrammetry to missile and space problems. One of the areas in which we are interested is the geodetic applications of photogrammetry. The field instrumentation, operations, and data processing features of this application are a normal outgrowth from the determination of missile trajectory position information by the use of ballistic cameras.

Determination of the geodetic position of a range station is a transformation calculation after the space resection of the station has been determined. We have two data reduction computer programs at the AMR for the determination of space resection and the associated geodetic transformation. These programs are used in the processing of ballistic camera data for determination of the geodetic position of range tracking stations to a high degree of geometric quality. One of these computer programs is called FLAR, for Photogrammetric Flare Triangulation, the other is called BCSR, for Ballistic Camera Space Resection. The FLAR program performs a multiple point in space resection, and the BCSR program performs a single point in space resection.



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ABSTRACT: *Geodetic position determination by means of Analytical Photogrammetry is essentially a Space Resection problem. The Manual of Photogrammetry defines Space Resection as, "The analytical determination of the three rectangular coordinates of an exposure station with reference to the ground survey coordinate system." This definition holds for both aerial and terrestrial applications of Photogrammetry. Two variations to the space resection problem are: single-point in space resection, and multiple-point in space resection.*

Determination of the geodetic position of a range station is a transformation calculation after the space resection of the station has been determined. AMR has two data reduction computer programs at the AMR for the determination of space resection and the associated geodetic transformation. One of these computer programs is called FLAR, for Photogrammetric Flare Triangulation, and performs a multiple-point in space resection. The other program is called BCSR (for Ballistic Camera Space Resection) which performs a single point in space resection.

FLAR COMPUTER PROGRAM

The FLAR computer program is based upon the mathematics presented by D. C. Brown in RCA Data Reduction Technical Report No. 46, "Photogrammetric Flare Triangulation," dated 2 December 1958. The program uses the more sophisticated approach outlined in Chapter 4 of this reference. Due to the present limitations of the IBM 7094 computer, the current program will accept a maximum of 10 cameras and of 20 relative control points. Each relative control point must be matched by a set of plate coordinates for each camera. In order to overcome loss of significance within the computer, double precision arithmetic is necessary. The program is also limited by present analysis to only one camera orientation at each station. The adjustment is performed in a common range coordinate system, about the perspective point of each camera.

Given the geodetic coordinates for the origin of the range system (Table 1), together with approximate geodetic position of each station, the local camera orientations are transformed to a common range coordinate system using double precision geodetic subroutines. The origin for the range coordinate system is commonly chosen as Cape Kennedy. In addition, the local Cartesian coordinates of the gimbal point and of the perspective point for each camera are transformed to the range coordinate system. *A priori* estimates of error for the geodetic position coordinates of all stations in the net must also be given, and have been obtained from the USC&GS. These estimates of error in ϕ λ h are converted by geodetic transformation into estimates of error in the range Cartesian coordinate system (Table 2).

The relative control points, whose images appear on each camera plate, are used for the space resection adjustment. The positions of these relative control points are considered to be known without error, and are computed in the range coordinate system by the regular Ballistic Camera Position Program using only those cameras of the net whose geodetic positions are considered to be known with high geometric quality. The observations from all camera stations are processed through this operational position program, to allow preparation of the corrected plate coordinates for even the unknwon stations. The position program allows the observations to be weighted for the geometric quality of the camera orientation, for the focal length of the camera, and for the random error of the data observations. Data from the cameras at the unknown stations may therefore be weighted out of the position adjustment.

TABLE 1

FLAR COMPUTER PROGRAM

Given Data:
1. Geodetic coordinates for origin of the range system.
2. Approximate geodetic position for each station.
3. Range coordinates of relative control points.
4. Local camera orientations.
5. Local coordinates of perspective point or gimbal point.
6. *A priori* estimates of error.

TABLE 2

FLAR CONDITION EQUATIONS

Basic condition equations:

$$x = x_p + c\frac{A(X - X^c) + B(Y - Y^c) + C(Z - Z^c)}{D(X - X^c) + E(Y - Y^c) + F(Z - Z^c)},$$

$$y = y_p + c\frac{A'(X - X^c) + B'(Y - Y^c) + C'(Z - Z^c)}{D(X - X^c) + E(Y - Y^c) + F(Z - Z^c)}.$$

Supplemental condition equations:

$$X_i^c = (X_i^c)^0 + \delta X_i^c + v_{xi}^c,$$
$$Y_i^c = (Y_i^c)^0 + \delta Y_i^c + v_{yi}^c,$$
$$Z_i^c = (Z_i^c)^0 + \delta Z_i^c + v_{zi}^c.$$

The normal equations for the space resection adjustment are established using the corrected plate coordinates, the given relative control point positions, and the approximations to the geodetic positions of the camera stations (Table 3). Relative weights for all the stations used in the adjustment are determined from the *a priori* estimates of error, and the adjustment is solved in accordance with these relative weights. The adjustment computes a correction to the XYZ range coordinates of each camera station. The correction vector is transformed to values of $\phi\ \lambda\ h$ which are then applied to the original approximations. New range orientations and perspective point positions are then computed for each station and the solution is iterated. This procedure is continued until the solution converges to either a predetermined criterion or a maximum of five iterations, whichever occurs first. The criterion for convergence is established about the standard error of unit weight estimated from the adjustment; the value of 0.5 microns is normally used to stop the iteration.

TABLE 3

FLAR NORMAL EQUATIONS

Relative weights from *a priori* estimates of error:

$$\dot{W} = \begin{bmatrix} \dot{W}_1 & 0 & \cdots & 0 \\ 0 & \dot{W}_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \dot{W}_m \end{bmatrix},$$

$$\dot{W}_1 = \begin{bmatrix} s_{X^c}^2 & s_{X^cY^c} & s_{X^cZ^c} \\ & s_{Y^c}^2 & s_{Y^cZ^c} \\ & & s_{Z^c}^2 \end{bmatrix}.$$

Solution to normal equations:

$$\delta_{X^cY^cZ^c} = (S + \dot{W})^{-1}(C + \dot{W}\epsilon)$$
(range coordinate system)

Convert

$$\delta_{X^cY^cZ^c} \text{ to } \delta_{\phi\lambda h}.$$

The standard error of unit weight estimated from the adjustment is computed from the weighted quadratic form of the residuals divided by the degrees of freedom of the adjustment (Table 4). The degrees of freedom are equal to the number of observations in excess of the minimum required for solution. The minimum number of observations required to solve the existing condition can be determined from consideration of the condition equations of the adjustment. For the FLAR reduction, the degrees of freedom is $2mn$ (m=cameras, n=data points). The estimates of error for the adjusted parameters (the space resections) are determined from the standard error covariance matrix of the adjustment (the inverse of the normal equation coefficient matrix multiplied by the square of the standard error of unit weight estimated from the adjustment). The covariance matrix from the adjustment represents the estimates of standard error in

TABLE 4

FLAR ERROR ANALYSIS

Standard error of unit weight:

$$s_0^2 = \frac{V^T\overline{W}V}{2mn}.$$

Estimates of error:

$$\Sigma_{X^cY^cZ^c} = s_0^2(s + \dot{W})^{-1}, \text{ (range coordinate system)}$$
$$\Sigma_{\phi\lambda h} = (U^{-1})\Sigma(U^{-1})^T.$$

Coefficient of correlation:

$$\rho = \frac{s_{gk}^2}{[s_{gg}^2 s_{kk}^2]^{1/2}}.$$

Convert

$$s_{\phi\lambda} \text{ to } s_{NE}{:}f(\phi).$$

Relative control point error:

$$\sigma_s = 0.3333(s_X + s_Y + s_Z).$$

Pooled estimates of error:

$$\bar{s}_{NEh}^2 = [s_{NEh}^2 + \sigma_s^2].$$

Convert

$$s_{NE} \text{ to } s_{\phi\lambda}{:}f(\phi).$$

Covariance terms:

$$\bar{s}_{gk}^2 = \rho[\bar{s}_{gg}^2 \bar{s}_{kk}^2]^{1/2}.$$

the range Cartesian coordinate system. This matrix is then transformed to obtain the covariance matrix for the goedetic coordinates.

In the space resection adjustment, the relative control-point positions are considered to be known without error. Since this is a fallacy, the error of the relative control-points must be considered in estimating the error of the resected positions.

The Ballistic Camera Position Program furnishes standard error estimates for the relative control point coordinates. An average value for the errors of these coordinates and a value for the spherical standard error is determined. It is then necessary to apply this error to the estimated standard errors of the resected positions. The FLAR program prints out the standard errors of ϕ λ h and the covariance terms in units of arc seconds. These must be converted to linear units in Northing and Easting, and pooled with the spherical standard error for the relative control points. The pooled values may then be reconverted to units of arc seconds. The coefficients of correlation are computed from the covariance terms in ϕ λ h. A new set of covariance terms for the pooled error estimates is computed using these coefficients of correlation together with the pooled standard error estimates.

When using the FLAR program to obtain geodetic positions, it is necessary that the *a priori* estimates of error for the range stations be realistic. A base datum of high geometric quality containing at least three camera stations is necessary to control the net adjustment, and to obtain relative control points of adequate geometric quality for a satisfactory resection.

As with all data reduction programs, this one is not perfect. It is recognized that the program has limitations, and future modifications are planned. Although a multiple station solution is accomplished, known relations between stations in a common datum is not enforced; thus the internal consistency of a given datum may be disrupted by the adjustment. Although the solution is simultaneous for the range stations, it is not simultaneous with respect to the positions of the space points. Consequently, the propagation from the errors of the observations to the estimated errors of the resected positions is not rigorous.

Future improvements to the program may include the ability to enforce the internal consistency between stations on a given datum. Allowing for several orientations and sets of data for a given station will give the program more capability. Programming is already planned to weight the observations for the error in the given camera orientation, and for the focal length of the cameras at the various stations. Finally, a completely new program is being planned to perform a general photogrammetric adjustment from a theoretical and rigourous standpoint.

BCSR Computer Program

The BCSR computer program is based upon the mathematics suggested by D. C. Brown in RCA Data Reduction Technical Report No. 39, "A Treatment of Analytical Photogrammetry," dated August 1957. The description below indicates differences from the FLAR program. The program will accept a maximum of 10 different camera orientations and data sets, each data set consisting of a maximum of 20 relative control points. It is possible that each orientation and data set be obtained from a different camera, however, all cameras must be operated from the same range station. The adjustment is performed in the local coordinate system at the single camera station, with origin at either the survey monument or at the gimbal point of the camera. Each set of relative control point positions may be in a different range coordinate system.

Given the geodetic coordinates for the origin of the range system for each data set (Table 5), together with the approximate geodetic position for the unknown station, the range coordinates of the relative control points are transformed to the local coordinate system at the camera station. *A priori* estimates of error for the geodetic position coordinates of the unknown station must be known. These estimates of error in ϕ λ h are converted into estimates of error in the local Cartesian coordinate system.

Table 5

BCSR Computer Program

Given Data:

1. Geodetic coordinates of range origin for each data set.
2. Range camera orientation for each data set.
3. Range coordinates of relative control points for each data set.
4. Approximate geodetic position for unknown station.
5. *A priori* estimates of error.

TABLE 6
BCSR CONDITION EQUATIONS

Basic condition equations:

$$x = x_p + c\frac{A(X - X^c) + B(Y - Y^c) + C(Z - Z^c)}{D(X - X^c) + E(Y - Y^c) + F(Z - Z^c)}$$

$$y = y_p + c\frac{A'(X - X^c) + B'(Y - Y^c) + C'(Z - Z^c)}{D(X - X^c) + E(Y - Y^c) + F(Z - Z^c)}$$

Supplemental condition equations:

$$X^c = (X^c)^0 + \delta X^c + v_{X^c},$$
$$Y^c = (Y^c)^0 + \delta Y^c + v_{Y^c},$$
$$Z^c = (Z^c)^0 + \delta Z^c + v_{Z^c}.$$

The relative control points are input into the program as Cartesian coordinates (Table 6), and each set of data may be referenced to a different range origin. The positions of these relative control points are considered to be known without error and may be obtained from any of several sources. Highest quality relative control points are obtained by simultaneous observations with ballistic cameras positioned in a high quality datum, but also may be obtained from either an orbital solution, or the short arc method. The local elements of orientation and the plate coordinates have been obtained from the regular Ballistic Camera Orientation Program, which corrects the plate coordinates for lens distortion. The atmospheric refraction correction is applied within the BCSR program.

The normal equations for the space resection adjustment are established using the corrected plate coordinates (Table 7), the transformed relative control point positions, the different camera orientations, and the approximations to the geodetic position of the local camera station. Relative weights for the geodetic positions are utilized in the adjustment. The observations are weighted in the adjustment for the geometric quality of each orientation, focal length for each orientation, and random error of the data observations. The adjustment computes a correction to the XYZ local coordinates of the camera station. If the root sum square of the correction components is greater than 1 foot, the correction vector is transformed to values of $\phi\ \lambda\ h$ which are then applied to the original approximations. New local coordinates for the relative control point positions, as well as new local camera orientations, are then computed for the new geodetic position of the local camera station, and the solution is iterated. This procedure is continued for a maximum of five iterations or until the solution converges to a change in the standard error of unit weight estimated from the adjustment, of 0.3 microns. For the BCSR reduction, the degree of freedom is $2n$ (n = data points). The estimates of error for the adjusted parameters (the space resection), in both Cartesian and geodetic coordinates, are determined from the respective standard error covariance matrices.

Standard error estimates for the relative control point coordinates must be furnished for adequate analysis (Table 8). The standard error estimates of $\phi\ \lambda\ h$ in units of arc seconds, output from the BCSR Program, must be converted to linear units in Northing and

TABLE 7
BCSR NORMAL EQUATIONS

Weights for individual observations:

$$W = \tfrac{1}{2}\left[\begin{bmatrix} \dfrac{s^2/c_L^2}{s_{00}^2/c^2} & 0 \\ 0 & \dfrac{s^2/c_L^2}{s_{00}^2/c^2} \end{bmatrix} + \begin{bmatrix} s^2/\sigma_x^2 & 0 \\ 0 & s^2/\sigma_y^2 \end{bmatrix}\right].$$

Relative weights from *a priori* estimates of error:

$$\dot{W} = \begin{bmatrix} s^2/s_{X^c}^2 & 0 & 0 \\ 0 & s^2/s_{Y^c}^2 & 0 \\ 0 & 0 & s^2/s_{Z^c}^2 \end{bmatrix}.$$

Solution to normal equations:

$$\delta_{X^cY^cZ^c} = (S + \dot{W})^{-1}(C + \dot{W}\dot{\epsilon}),\ \text{(local coordinate system)}$$
$$\delta_{X^cY^cZ^c} = X^cY^cZ^c.$$

Convert $X^cY^cZ^c$ to $\phi\lambda h$.

TABLE 8
BCSR ERROR ANALYSIS

Standard error of unit weight:

$$s_0^2 = \frac{V^T\overline{W}V}{2n}.$$

Estimates of error:

$\Sigma x^c y^c z^c = s_0^2(S + \dot{W})^{-1}$ (local coordinate system)

Easting, and pooled with the spherical standard error for the relative control points. The pooled values may then be reconverted to units of arc seconds. Coefficients of correlation and a new set of covariance terms for the pooled standard error estimates are then computed.

When using the BCSR Program to obtain geodetic positions, it is necessary that the *a priori* estimates of error for the unknown station be realistic. Relative control points of high geometric quality in the coordinate system of the base datum are necessary to control the net adjustment.

It is recognized that the BCSR program also has its limitations, and future modifications are planned. Although multiple orientations at a single station are utilized, the geodetic position of only one station can be determined. This results in a minimum number of degrees of freedom compared to the FLAR technique. The adjustment is also performed about the gimbal point of the camera; if multiple cameras have been used, the gimbal points may not all represent a common point. Furthermore, the resected position represents an interpolation between the gimbal point and the perspective points of the individual orientations. It is therefore necessary that this displacement be negligible compared to the distance to the relative control points. In addition, the errors of the relative control points are not rigorously propagated into the estimated errors of the resected positions.

Future improvements to the program may include the ability to transform from the gimbal point to the perspective points for each of the orientations. Allowing for multiple station resection, and simultaneous adjustment of the relative control points, will allow this program to be replaced by the improved FLAR Program.