Three-Dimensional Transformations of Higher Degree

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A mistake in a recently-published condition for the conformal transformation of strip coordinates is corrected.

IN A recent paper (E. M. Mikhail, PHOTOGRAMMETRIC ENGINEERING, July 1964) priority was claimed for the introduction of simultaneous three-dimensional transformations of higher degree for the adjustment of strips and blocks to external control. The claim is not valid for strips, since transformations of this type were introduced in 1958 by this writer in a paper read at the Munich Photogrammetric Week of that year and published in the *Photogrammetric Record* of October 1959.

However, since my results were somewhat concealed in a rather lengthy paper dealing mainly with other matters and since they appear to be unfamiliar to other writers on the same subject, it may be helpful to reproduce them here, together with some remarks on the results obtained by Mikhail.

The errors of a strip are the results of extremely complex stochastic processes and therefore cannot be accurately described by analytical expressions of a relatively simple nature. It is to be understood then that the expressions given below represent only the smoothed trends of the errors. This limitation holds for all formulas of this kind.

Let x, y, z be the strip coordinates and X, Y, Z the local model coordinates at the same scale. Let the smoothed strip coordinate errors of the air stations be

$$\left.\begin{aligned} \Delta x_0 &= \theta(x) \\ \Delta y_0 &= \phi(x) \\ \Delta z_0 &= \psi(x) \end{aligned}\right\} \qquad (1)$$

and let the smoothed absolute lateral tilt error be

$$\Delta\xi = \omega(x). \qquad (2)$$

Then the local scale error is

$$\Delta\mu = \theta'(x). \qquad (3)$$

The local longitudinal tilt error is

$$\Delta\zeta = \phi'(x). \qquad (4)$$

and the local azimuth error is

$$\Delta\eta = \psi'(x). \qquad (5)$$

Moving now from the line of air-stations without changing x, through the displacements Y and Z, and taking account of (2), (3), (4) and (5), we have the strip coordinate errors:

$$\begin{aligned} \Delta x &= \Delta x_0 - Y\Delta\zeta - Z\Delta\eta, \\ \Delta y &= \Delta y_0 + Y\Delta\mu + Z\Delta\xi \\ \Delta z &= \Delta z_0 - Y\Delta\xi + Z\Delta\mu. \end{aligned}$$

These are valid for a right-handed system with z and Z downwards. Substituting, we finally obtain for the strip errors:

$$\left.\begin{aligned} \Delta x &= \theta(x) - Y\phi'(x) - Z\psi'(x) \\ \Delta y &= \phi(x) + Y\theta'(x) + Z\omega(x) \\ \Delta z &= \psi(x) - Y\omega(x) + Z\theta'(x) \end{aligned}\right\}. \qquad (6)$$

If the strip x-axis is adjusted so that it coincides closely with the line of air-stations, Y and Z in (6) may be replaced with the strip values y and z. We then have

$$\left.\begin{aligned} \Delta x &= \theta(x) - y\phi'(x) - z\psi'(x) \\ \Delta y &= \phi(x) + y\theta'(x) + z\omega(x) \\ \Delta z &= \psi(x) - y\omega(x) + z\theta'(x) \end{aligned}\right\}. \qquad (7)$$

It is simple enough to convert (7) into a direct relation between the uncorrected strip coordinates and the external ground control system, but there is no need to detail such a development here. The above result is considerably more general for the strip than is Mikhail's. The relations are not conformal and are not intended to be. Rather they represent the correct conditioning of the errors. In practice the functions θ, ϕ, ψ, ω would be represented by polynomials in x of any required degree.

I am not entirely convinced of the merits of conformalism for the adjustment of the discrepancy-free block to external control. Certainly, for large blocks, rather complex expressions would be necessary and the conformalism might introduce some rather sharp local "kinks" in scale and orientation which might not be foreseeable. In any case Mikhail has mistated the condition for conformalism. It is *not* the orthogonalism of the matrix

$$\begin{bmatrix} \frac{\partial X}{\partial x}, & \frac{\partial X}{\partial y}, & \frac{\partial X}{\partial z} \\ \frac{\partial Y}{\partial x}, & \frac{\partial Y}{\partial y}, & \frac{\partial Y}{\partial z} \\ \frac{\partial Z}{\partial x}, & \frac{\partial Z}{\partial y}, & \frac{\partial Z}{\partial z} \end{bmatrix}$$

which is required, but rather the orthogonalism of the matrix

$$\begin{bmatrix} \frac{1}{f}\frac{\partial X}{\partial x}, & \frac{1}{f}\frac{\partial X}{\partial y}, & \frac{1}{f}\frac{\partial X}{\partial z} \\ \frac{1}{f}\frac{\partial Y}{\partial x}, & \frac{1}{f}\frac{\partial Y}{\partial y}, & \frac{1}{f}\frac{\partial Y}{\partial z} \\ \frac{1}{f}\frac{\partial Z}{\partial x}, & \frac{1}{f}\frac{\partial Z}{\partial y}, & \frac{1}{f}\frac{\partial Z}{\partial z} \end{bmatrix}$$

where f is an arbitrary function of x, y, and z. This condition is considerably less restrictive, and Mikhail's fears about the non-existence of second and third degree conformal transformation in three dimensions may be quite without foundation. That Mikhail's condition is too restrictive is easily seen when we consider that it rules out even the general transformation of rectangular coordinates, which is certainly conformal.

In any case, it is certain that there exist non-linear conformal transformations in three dimensions. As an example I give the well-known inversion with respect to a sphere, which in its simplest form may be written

$$X = k^2x/(x^2 + y^2 + z^2),$$
$$Y = k^2y/(x^2 + y^2 + z^2),$$
$$Z = k^2z/(x^2 + y^2 + z^2),$$

in which k is radius of inversion.

Kodak Bimat Process (*continued from page 128*)

under consideration in the manufacturing division, and it and a suitable imbibant are expected to be available for testing later this year. The photographic results obtained from the Type 2401 Film in the laboratory with this reflection-type material are comparable with those illustrated in Figures 1 and 2 and Table 1 for the SO-111 Bimat Film and the MX-572 Imbibant except for the positive sensitometric curve.

Additional investigations on laboratory equipment indicate that, for certain applications, access times can be considerably reduced below the 15 minutes needed with the other processes described. With the reflection-type Bimat Film and a special negative film suitable for cathode-ray-tube recording, complete processing of both films has been achieved in 2 minutes. In another instance, the SO-111 Bimat Film has been used to obtain access to the positive image in about 10 seconds, with the same type of negative film. The negative film requires further treatment if it is to be retained.

Thus the convenience of Bimat processing can be considered for many applications where high-quality results are required and free solutions are undesirable or cannot be tolerated.

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