Image Motion and Its Compensation for the Oblique Frame Camera

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Image blur due to aircraft velocity, camera vibration, roll, pitch and yaw can be minimized by film translation during exposure and by a judicious shutter selection.

(*Abstract on page 157*)

IN STUDIES concerned with aerial photography, it is necessary to consider the effect of aircraft movement upon photographic resolution, for any motions during exposure will reduce the sharpness of an image. Resolution is, of course, a vital factor in reconnaissance missions and sources of image blur must be eliminated if possible.

As the aircraft travels in flight the ground below moves with respect to the aircraft, and images of the ground move in the photographic plane. To compensate this movement the photograph is translated forward with respect to the lens during exposure at the same speed as the image, which is image-motion compensation. Thus there is not net movement of the image on the film if the IMC mechanism is perfect and the camera is vertical. Alternatively the image-motion compensation can be achieved by moving the lens rather than the film, or by rotating film and lens about an axis transverse to the direction of flight.

There is still image-motion with an IMC mechanism. No IMC device is so perfect that it can travel at a desired velocity with a hundred per cent accuracy. In addition the velocity of this mechanism is based upon information about the plane's flight, in particular the ratio of velocity to height, which is measured with a device known as the V/H sensor. This device has limited accuracy, the error ranging at present from 0.1% achieved in the laboratory with high-quality equipment to as high as 10% in flight under low ground contrast conditions.

A third source of image-motion arises from the fact that the images in the photoplane are not moving at the same speed. The film on the other hand must move at a single velocity, therefore, it cannot compensate the velocities of all the images and residual image motion results. There are two reasons why all the images may not move at the same speed—the ground points are at varying heights and the camera is not vertical. This paper analyzes the image velocity for an intentionally nonvertical camera, and explores methods of achieving the optimum image-motion compensation.

Image Motion Compensation for a Tilted Frame Camera

For a vertical camera the image velocity is the same over the format because the scale is constant, assuming that the ground is level. If the photograph is tilted the scale becomes variable within the field of view and the image-velocity must vary also. If this variable image-velocity is compensated by a movement of the film or lens with a between-the-lens type shutter, the film will have only one velocity at the moment of exposure, thus there must be residual image-motion. In general this image motion is sufficient to decimate the resolution at some sections of the film for any significant amount of tilt. For this reason it is desirable to use a focal-plane shutter with a graded or variable image-motion compensation velocity whenever the camera is intentionally tilted.

A focal-plane shutter consists of a curtain with a slit opening placed in front of the film. The curtain is moved over the film with a constant speed, thereby exposing it. Since different parts of the film are exposed at different times, the velocity of the film may be varied for different positions of the slit, and the image-motion can be considerably reduced. If the slit is oriented at any one time such that the image velocities of points within it are approximately the same, and the film or lens is moved at this velocity, then the residual image-motion is minimized. However, it can never be completely eliminated, even with an accurate IMC mechanism, because the image-velocity varies both in magnitude and direction within the slit opening for any slit orientation or position. An exception is the side-oblique camera for which the image-velocity is in only one direction, but there is still a small residual motion due to the varying magnitude of velocity within the finite slit width.

The problem of determining the optimum mode of image-motion compensation for the tilted frame camera therefore becomes a problem of finding (1) the best direction of image-motion compensation, (2) the best orientation of the exposure slit and (3) the magnitude of the IMC velocity for any position of the slit. This problem can be solved by examination of the image-velocity for any camera orientation. The slit orientation and the IMC velocity which results in the least image blur over the film is the optimum choice.

For two special cases the proper image-compensation is easy to decide. If the camera is tilted in the forward/oblique position the direction of compensation should be in the direction of flight which is along the principal line, i.e., the line from the center of the photograph to the nadir, and the slit should be oriented perpendicular to this line. Since the image-velocity component in the IMC direction is constant for any position of the slit, the IMC velocity is simply this component.

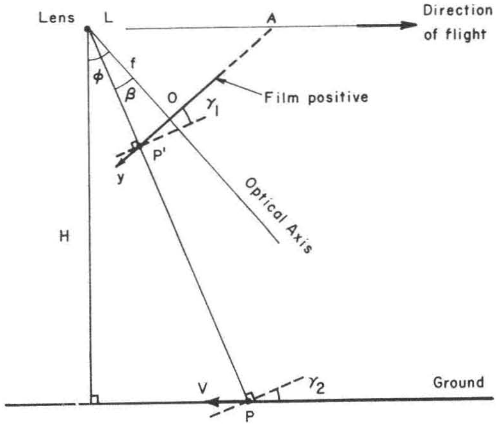

FIG. 1. Side view of the geometry for the forward oblique frame camera.

The image-motion compensation does not mean that the image-velocity is completely compensated for the forward oblique case; on the contrary there is still considerable image-motion perpendicular to the IMC direction which cannot be compensated. The magnitude of this motion varies linearly with distance from the principal line. To acquire some idea of the variation of image-velocity over the format examine a photograph of a rectangular grid as sketched in Figure 2. The pattern of the lines reveals the direction and magnitude of the image-velocity. Since the lines converging to A are grid lines in the direction of flight, the image-velocity at any point on the photograph is along these lines, and the magnitude of the velocity is represented by the spacing between the grid lines in the direction perpendicular to flight. This is true because the spacing between adjacent lines on the ground is a constant, and the distance the aircraft has moved during the exposure interval is also a constant for all points on the photograph.

Lines T_1T_2 and $T_1'T_2'$ are the image-motion at those respective points if the aircraft has moved through the distance represented by T_1T_2 during exposure and there is no IMC. It is apparent from the diagram that the velocity component in the y direction is constant for any value of y, while the x velocity is zero at the center and increases toward the sides.

This grid technique can be used for a camera in any orientation.

If the camera is tilted to the side the image-velocity over the whole photograph is in the same direction, perpendicular to the principal line, hence the IMC should be in this direction. Since the magnitude is constant along any line perpendicular to the principal line, the slit should also be in this direction, and the IMC velocity set equal to the single image-velocity within the slit. The equation for the image-velocity is derived later.

With a very narrow slit and an accurate IMC mechanism there is no residual image-motion. However, for many cameras the picture must be taken quickly, necessitating a fast curtain speed and a correspondingly wide slit to maintain the desired exposure time. If the slit has a finite width there is a variation of the image-velocity within the slit in the direction of image-motion compensation resulting in a slight residual velocity. This residual velocity is also present when the camera is pitched forward, but it is greatly overshadowed by the image-motion due to variation of the direction of the image-velocity; and so it is not necessary to consider it.

For all other directions of camera tilt (double oblique case) the optimum IMC velocity and slit orientation are not as apparent and must be determined from an analysis of the image-velocity equations.

DERIVATION OF THE IMAGE-VELOCITY EQUATIONS

The derivation of the image-velocity equations for the camera tilted in any random direction consists of a derivation of the equations for the forward and side-oblique cases and a combination of these equations. If the reader desires to pass over the derivation he should consult the equations numbered (7) through (10). The symbols will be defined first. The film-positive mentioned in the definition is used as a reference rather than the actual negative behind the lens, and it is positioned between the lens and the object space.

PARAMETERS:

f = focal-length
V = aircraft velocity
H = altitude
y = photocoordinate in a fixed direction on the film, in the forward and side oblique cases, the axis is arbitrarily defined to be in the plane of tilt, positive toward the nadir.
x = photocoordinate in direction perpendicular to y axis, forming a right-handed system on the film positive.
y' = photocoordinate axis which always points toward the nadir or vertical (same as principal-line).
x' = photocoordinate perpendicular to y' axis
v_x', v_y', v_x, v_y = image velocity in x', y', x, and y directions
ϕ = camera pitch-angle, positive when camera is tilted forward
ω = camera roll-angle, positive, when viewing to the right.
t = camera tilt-angle, between optical axis and vertical.
α = azimuth-angle, clockwise from North to direction of tilt of film positive. (Not used in derivations).
α' = modified azimuth-angle, clockwise from direction of flight to direction of tilt. Differs from α by the angle known as the heading of the vehicle.
s = swing-angle in focal-plane clockwise from y to y' axis, defining the orientation of the film about the optical axis.

If the camera is tilted forward the image-velocity in the y direction is derived with the help of Figure 1, a simplified side view of a pitched frame camera. L is the position of the lens, LO the optical axis and Oy the film. A ground point P images at P' on the film. The angles γ_1 and γ_2 are formed by the perpendicular to LP and the photoplane and groundplane respectively. As the aircraft travels to the right the point P moves to the left with respect to the aircraft, and ray LP has an angular velocity of

$$\dot{\beta} = \frac{v_y \cos\gamma_1}{f \sec\beta} = \frac{V\cos\gamma_2}{H\sec(\phi-\beta)} \tag{1}$$

Or

$$v_y = \frac{Vf}{H}\,\frac{\cos\gamma_2}{\cos\gamma_1}\,\frac{\cos(\phi-\beta)}{\cos\beta}$$

Since

$$\gamma_2 = \phi-\beta, \quad \gamma_1 = \beta, \quad \&\ \tan\beta = \frac{y}{f}$$

$$v_y = \frac{Vf}{H}\,\frac{\cos^2(\phi-\beta)}{\cos^2\beta} = \frac{Vf}{H}(\cos\phi + \sin\phi\tan\beta)$$

$$v_y\ (\text{pitch}) = \frac{Vf}{H}\left(\cos\phi + \frac{y}{f}\sin\phi\right)^2 \tag{2}$$

The velocity in the x direction is related to the image-velocity in the y direction by the factor $\tan \gamma_3$ as shown in Figure 2. As mentioned earlier, this represents a forward-oblique photograph of a rectangular grid pattern with images moving in the directions of the lines diverging from A, the vanishing point on the horizon. It is apparent that

$$v_x = v_y \tan \gamma$$

and

$$\tan \gamma_3 = \frac{x}{y + AO}$$

and from Figure 1, $AO = f \text{ ctn } \phi$. Therefore,

$$v_x = v_y\left(\frac{x}{y+f \text{ ctn } \phi}\right) = \frac{v_y}{f}\left(\frac{x \sin \phi}{\frac{y}{f} \sin \phi + \cos \phi}\right)$$

$$v_x(\text{pitch}) = \frac{V}{H} \cdot x \sin \phi \left(\cos \phi + \frac{y}{f} \sin \phi\right) \qquad (3)$$

Equations (2) and (3) give the velocity components if the camera is tilted forward. If the camera is tilted to the side (rolled) then the image-velocity is in one direction, perpendicular to the principal-line, for all parts of the photograph. The image-velocity is the product of vehicle-velocity and scale and from Figure 3, a simplified diagram of the side-oblique geometry, the scale in the y direction (perpendicular to the plane of the diagram) is the ratio of LP' to LP.

$$v_x = -V \cdot \frac{LP'}{LP} = -\frac{Vf \sec \beta'}{H \sec (\omega - \beta')}$$

$$= -\frac{Vf}{H} (\cos \omega + \sin \omega \tan \beta')$$

$$v_y = -v_x' = -\frac{Vf}{H}\left(\cos \omega + \frac{y}{f} \sin \omega\right) \qquad (5)$$

And

$$v_y = 0 \qquad (6)$$

Roll Only

For the general case of the camera pitched and rolled, the equations for the image-velocity are derived by replacing the ground-velocity vector by two components, one in the direction of tilt, V'_y, and the other perpendicular to it, V'_x, and treating each separately. The resolution of the velocity vectors is illustrated in Figure 4, a top view of the vector and its components.

Abstract: *There are three main causes of motion blur in aerial photography: the aircraft velocity, rotations of the aircraft, and vibration. Each of these produces a movement of the image which results in a significant resolution degradation, if not compensated. The image motion from the first two causes is examined in this three part series. The first part consists of an analysis of the image motion arising from the aircraft velocity for the oblique frame camera, with or without image motion compensation. The optimum type of shutter and IMC velocity for the tilted camera are analyzed, and the photographic distortions arising from the use of a focal-plane shutter are examined. Conversion equations from pitch, roll and yaw to tilt, swing and azimuth are included. In the second part, equations for the image-motion from aircraft rotations are derived for the tilted frame camera, from which the resolution loss can be computed. The image-motion equations for the panoramic camera from the aircraft velocity and rotations are derived in part 3.*

Consider the velocity component V'_y. If point P has this velocity, then the aircraft is moving in the opposite direction, the direction of camera-tilt. Therefore the situation is equivalent to the forward oblique case with the y' direction (along the principal line) replacing the y, the tilt angle t replacing ϕ, and V_y' replacing V. Since the velocity equations for this case have already been derived, the equations for the image-velocity due to V_y' are easily obtained.

If the point P has the velocity V_x' only, the situation is analogous to the side-oblique case, and the equations for the image-velocity due to V_x' are also readily available. Since the two velocities can be treated separately, the resulting image-velocity due to the aircraft velocity is the sum of the velocities from V_x' $(= V \sin \alpha')$ and V_y' $(= V \cos \alpha')$:

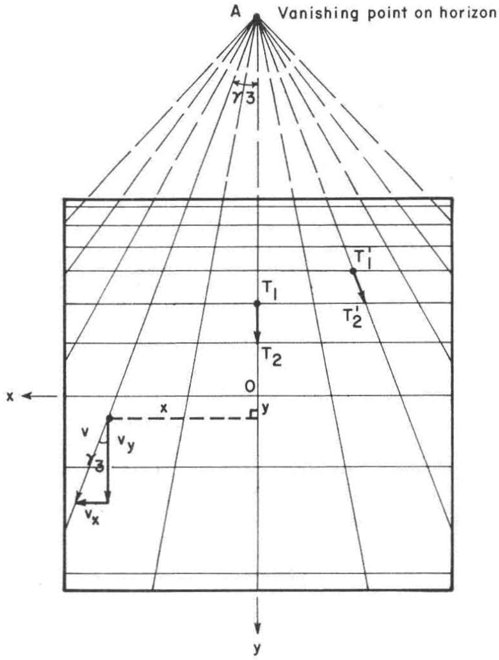

FIG. 2. Forward oblique photograph of a rectangular grid, no IMC.

$$v_x' = \frac{Vf}{H}\left(\cos t + \frac{y'}{f}\sin t\right)\left(\frac{x'}{f}\cos\alpha'\sin t - \sin\alpha'\right) \tag{7}$$

$$v_y' = \frac{Vf}{H}\cos\alpha'\left(\cos t + \frac{y'}{f}\sin t\right)^2 \tag{8}$$

The coordinates x' and y' are related to the film coordinates x and y by a matrix rotation through the swing angle s:

$$\begin{aligned} x' &= x\cos s - y\sin s \\ y' &= x\sin s + y\cos s \end{aligned} \tag{9}$$

The image-velocity components in the x and y directions are:

$$\begin{aligned} v_x &= v_x'\cos s + v_y'\sin s \\ v_y &= -\,v_x'\sin s + v_y'\cos s \end{aligned} \tag{10}$$

In the derivation of the equations, the ground velocity was resolved into two components, one in the direction of camera tilt and the other perpendicular to it. By treating the velocity in this manner, it is possible to include any component of velocity perpendicular to the intended direction of flight, such as might be due to the "crabbing" of an aircraft from a crosswind or the rotation of the Earth below a satellite. For these cases other velocities are substituted for $V\cos\alpha'$ and $V\sin\alpha'$ in the image velocity equations:

$$V'\cos\alpha' - V''\sin\alpha' \quad \text{for } V\cos\alpha'$$
$$V'\sin\alpha' + V''\cos\alpha' \text{ for } V\sin\alpha\,.$$

where V'' is the perpendicular velocity component, positive to the right.

CONVERSION FROM PITCH, ROLL AND YAW TO TILT, SWING AND AZIMUTH

The equations for the image-velocity due to the forward movement of the vehicle have been derived for the general case of the camera tilt in any direction. However, these equations are in terms of the tilt-swing-azimuth description of camera orientation. In most instances the mechanics of the camera system allow the camera orientation to be more easily specified in terms of pitch, roll and yaw. Unfortunately, these angles are not defined as concisely as tilt, swing, and azimuth; in fact, given a pitch and a roll angle the optical axis of the lens can wind up in three possible positions. For example, a camera after undergoing a pitch-angle, can either be rolled around the new roll axis or rolled around the original axis, the direction of flight. Obviously, it will wind up at different orientations for the two cases, and will have different tilt and azimuth-angles. Or it can be rolled first from its vertical position, then pitched about either the pitch-axis in the rotated position or the axis in its original position. Actually the latter two pairs of rotations are the same as the first two.

A third set of pitch and roll-angles consist of the component angles of a four sided pyra-

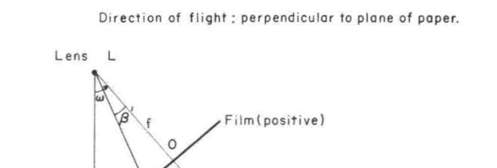

FIG. 3. Rear view of the geometry for the side oblique frame camera.

mid formed by the optical axis, the vertical and the direction of flight as illustrated in Figure 5. This set of angles does not correspond to any actual rotations of pitch and roll but offers an approximate description of camera orientation if it is not known which of the first two sets is applicable to the actual camera system. The resulting positions of the optical-axis for the three cases is shown in Figure 5.

Conversion Equations:

a) Pitch first, then roll around the new pitched axis.

$$\cos t = \cos\phi\cos\omega$$
$$\tan\alpha' = \frac{\tan\omega}{\sin\phi} \qquad (11)$$
$$\tan s = \frac{\sin\omega}{\tan\phi}$$

b) Pitch first, then roll around "original" or vehicle-axis.

$$\cos t = \cos\phi\cos\omega$$
$$\tan\alpha' = \frac{\sin\omega}{\tan\phi} \qquad (12)$$
$$\tan s = \frac{\tan\omega}{\sin\phi}$$

c) Pitch and roll-angles form side-angles of a pyramid (see Figure 2). (These angles are not the same as photogrammetric x-tilt and y-tilt since the latter are referenced to the photo x and y axes rather than the ground X and Y axes.)

$$\tan t = (\tan^2\phi + \tan^2\omega)^{1/2}$$
$$\tan\alpha' = \frac{\tan\omega}{\tan\phi} \qquad (13)$$
$$s = \alpha'$$

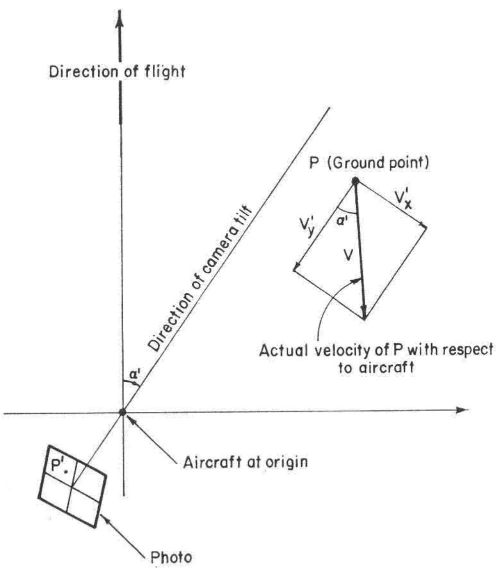

FIG. 4. Resolution of velocity vector into two components for double oblique case (top view).

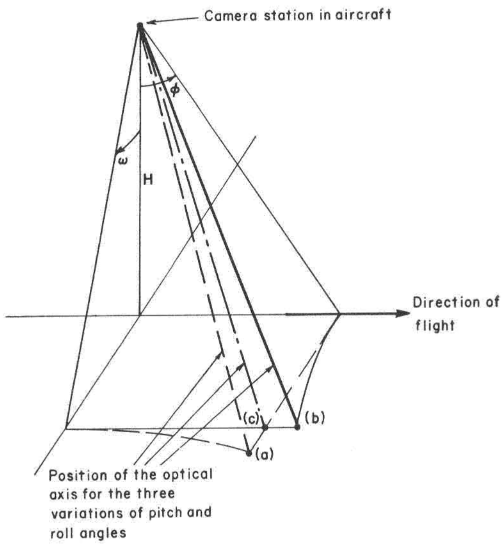

FIG. 5. Camera orientations for three possible cases of pitch and roll angles.

For the definitions and sign conventions of the angles see the section on symbols. The proper quadrant for angles α' and s is determined from the signs of the trigonometric functions in the numerator and denominator. The swing-angle is in the same quadrant as α'.

In obtaining the swing-angle equations for cases (a) and (b), it is assumed the y axis of the film positive is opposite to the direction of flight before the camera is pitched or rolled and that the camera does not twist about the optical-axis as it undergoes these excursions. For case (c) it is assumed that the camera is rotated directly from the vertical to the tilted position.

There are two reasons why the type of pitch and roll angles of the camera system should be known. The first reason is obvious, that the computed swing and azimuth-angles are not exact if the wrong case is chosen, and the IMC velocity, based on these angles, will be slightly in error. The second reason concerns the direction of image-motion compensation. If the camera system is to be built to achieve the optimum IMC, then the type of pitch and roll must be known so that the system can be built to move the film and orient the slit in the directions which minimize the overall blur.

COMPUTATION OF THE RESIDUAL IMAGE MOTION

Equations (7) through (10) give the image-velocity from the forward movement of the aircraft for any point on an oblique photograph. With these equations the residual image-velocity for any type of image-motion compensation can be computed and the resulting loss of resolution determined. Thus it is possible to establish the requirements of the IMC mechanism and to determine the minimum resolution loss which exists despite the accuracy of components.

The residual image-velocity from aircraft movement at any point is the vector difference between the image-velocity without IMC, computed from Equations (7) through (10), and the velocity of the film at the moment the point is exposed, which depends on the image-motion compensation system. Since this residual-velocity is usually constant during exposure, the distance of blur is simply the product of the velocity and the exposure time. At some parts of the film, notably where the residual image-motion is small, the velocity is not only variable but actually reverses direction. This reversal of direction arises from the finite slit width. However, the actual blur in such cases is so small that its effect on resolution is negligible.

The loss in resolution may be analytically determined from the total distance of blur and lens-film resolution by use of the modulation transfer function[1,2,3]. The blur ordinarily is uniform. The approximate magnitude of the degradation can be obtained from the reciprocal square principle

$$1/R^2 = D^2 + 1/R_0^2$$

where R and R_0 are the dynamic and static resolutions, respectively, and D is the total blur distance.

IMAGE MOTION COMPENSATION FOR THE FORWARD AND SIDE OBLIQUE SPECIAL CASES

It was stated earlier that the curtain slit should be perpendicular to the principal-line for the forward oblique case and the direction of compensation should be parallel to the principal-line. This becomes apparent from an examination of the image-velocity Equations (2) and (3) and also from the photograph of a rectangular grid pattern shown in Figure 2. For any value of y in the equations the image velocity component in the y direction is the same for all values of x, therefore there is no residual velocity in the y direction if the slit is oriented parallel to the x axis and the film given the proper velocity. But let us examine the equation for this velocity:

$$v_{imc} = \frac{Vf}{H}\left(\cos\phi + \frac{y''}{f}\sin\phi\right)^2$$

which is a second-degree equation in the coordinate y'', where y'' is in the IMC direction but does not move as the film moves. In terms of the time t the velocity is:

$$v_{imc} = \frac{Vf}{H}\left(\cos\phi + \frac{u\sin\phi}{f}t\right)^2$$

u is the curtain speed, assumed to be a constant, and is zero when the slit is at the principal point.

The mechanization to use to duplicate this velocity depends upon accuracy requirements, cost and complexity of the equipment, and the camera itself. High accuracy is achieved if a digital computer is available in the vehicle and its programmed output guides a servo system. If the focal length is comparable to the film width the variation in velocity may cause stability to be a problem. An electrical analog system of lower accuracy may also be used. Perhaps the simplest system is a mechanical cam which drives the platen when rotated at a constant velocity, but it can only be used when the pitch angle of the camera is fixed, unless several cams can be interchanged. The equation for the cam is of the form

$$r = a + b\theta + c\theta^2 + d\theta^3$$

where r and θ are the polar coordinates and a, b, c and d are constants.

If none of these systems is tenable then the IMC velocity might be varied linearly with y'' at the cost of accuracy:

$$v = \frac{Vf}{H}\left(\cos^2\phi + \frac{2\sin\phi\cos\phi}{f}y'' + \frac{\sin^2\phi}{f^2}y''^2\right)$$

$$\cong \frac{Vf\cos^2\phi}{H} + \frac{2V\sin\phi\cos\phi}{H}y''.$$

A velocity which varies linearly with y'' is easier to mechanize and the error is small if the film width is small in comparison to the focal-length. For example, if the focal-length were 12″, the film-width $4\frac{1}{2}$″ and the pitch-angle 30°, the maximum error is one per cent.

The image-motion in the y direction therefore depends on the accuracy of the equipment. In the x direction, perpendicular to the direction of IMC, there is image-movement and it cannot be compensated in any way because it is in opposite directions for the two sides of the photograph. It exists whenever the camera is tilted forward, and it is reduced only by shortening the exposure time. The equation for this velocity is Equation (3).

For the side-oblique camera the slit should be perpendicular to the principal-line, and the IMC velocity should also be in this direction. From Equation (5) it is apparent that this velocity varies linearly with the coordinate and should not present difficulties in mechanizing it. If the IMC is accurate there should be no residual image motion except that which arises from the finite slit width. This image motion is small.

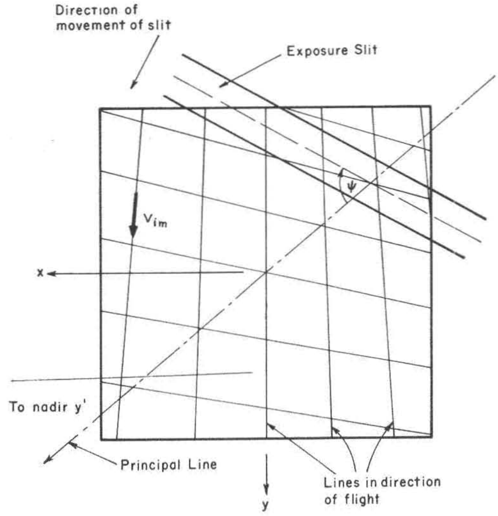

FIG. 6. Tilted photograph of a square grid (No IMC). Optimum orientation of slit is also shown. . . . Camera is viewing forward and to the right.

IMC FOR THE DOUBLE OBLIQUE FRAME CAMERA

This is the general case of camera-tilt, in which the camera is both pitched and rolled. Determining the proper IMC involves finding the solutions to the three problems posed earlier in this article: (1) what is the best direction of film movement, (2) in what direction should the exposure slit be oriented, and (3) what velocity should the film have for any position of the slit?

One might think that the direction of slit movement should be specified in addition to its orientation, but it does not make any difference which direction the slit moves in as long as its orientation is fixed. To demonstrate this consider an infinitely long slit parallel to the x axis of an xy coordinate system. Now, regardless of the actual direction of movement the slit appears to move in the y direction. Therefore, the only effect of variation of the direction of movement is a variation of the slit velocity in the y direction. In general, however, it is best to move the slit in the direction perpendicular to its orientation, otherwise the required velocity in some cases becomes very large. The only situation in which the direction of movement is not arbitrary is if the slit is curved.

The optimum IMC for the tilted frame camera is as follows:

1) *The best direction of film movement is the direction of image-movement at the principal-point.*

The best direction is that which results in the minimum blur in the direction perpendicular to movement. Since the image velocity in the latter direction varies approximately linearly with the coordinate in that direction, the velocity is minimized if the velocity at the center is zero. Perhaps this is more easily demonstrated with a tilted photograph of a rectangular grid, shown in Figure 6. As explained earlier the images move along the grid lines in the direction of flight, and their velocities are proportional to the spacing of the grid lines perpendicular to the flight direction. It is apparent that if the film is moved in the direction of the image-velocity at the center of the photograph the velocity perpendicular to this direction is minimized. This direction also has the advantage of eliminating the image-motion at the center of the photograph, which is desirable.

Now that the direction has been chosen as the direction of the velocity vector at the center, the angle of this direction to reference lines on the film will be specified. First, it will be shown that velocity at the center is always in the direction of the y axis on the film if the camera is pitched and rolled and the pitch rotation is about the rotated axis (case b of the pitch and roll angles described earlier). To prove this pass a plane through the longitu-

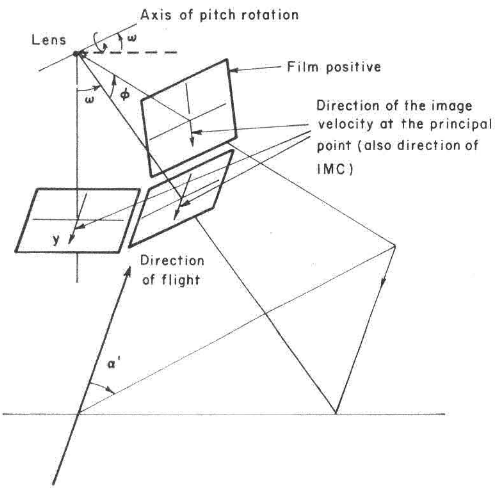

FIG. 7. Illustrations showing that the velocity vector at the principal point remains in the direction of the photo y axis as the camera is pitched about the rotated axis (Case b). (*See text on page 161.*)

dinal axis of the aircraft and the optical axis of the camera as illustrated in Figure 7. It will now be demonstrated that the intersection of this plane with the photograph is both the photo y axis and the direction of image-velocity. The y axis is always in the plane as the camera is rolled and remains in it as the camera is pitched. The image-velocity is also along the intersection because the intersection of this plane with the ground is the direction of the ground velocity vector, and the projection of this velocity on the film must also be in this plane.

Thus, if the camera is pitched about the rotated axis and rolled about the longitudinal axis of the aircraft (case b), the IMC always winds up in the right direction. This direction is at an angle s' to the principal-line, where s' corresponds to s in Equation (12),

$$\tan s' = \sec t \tan \alpha'. \tag{14}$$

If the mechanics of the system cause the roll rotation to occur about the rotated axis (case a), the IMC mechanism must always be turned to a new direction as the camera is pitched or rolled. The correction is given by this formula:

$$\tan \Delta s = -\sin \omega \tan \phi$$

which is derived by subtracting the swing-angle in Equation (11) from s' in Equation (14) with the substitutions for t and α' from (11) into (14).

2. *The exposure-slit should be oriented at an angle ψ to the principal-line,* where ψ is given by

$$\tan \psi = 2 \cos t \operatorname{ctn} \alpha' + \sec t \tan \alpha'. \tag{15}$$

The angle ψ is shown in Figure 6, in which the slit is superimposed on the tilted photograph. Analysis indicates that the slit should be straight and its orientation kept fixed as it is moved at the constant velocity over the film. The tilt and azimuth angles, t and α', can be obtained in terms of pitch and roll angles by Equations (11), (12) or (13) depending on the mechanical geometry of the system.

Once the direction of image-motion compensation is decided, there is no control over the image-velocity perpendicular to this direction. This velocity was minimized by prudently choosing the direction of IMC, but it cannot be eliminated. Hence the orientation of the slit has no effect on the image-motion in this direction. However, the slit-orientation does exert a large influence over the residual image-velocity in the direction of IMC, since this velocity is variable over the photograph. The slit-orientation should be chosen so that for any position of the slit the velocity component in the direction of IMC for all points within the slit is a constant, and the film is moved at this velocity when these points are exposed, thereby nullifying the image velocity. In other words, the loci of points on the photograph having constant components of velocity in the direction of IMC must be determined and the slit must be shaped such that each of these loci is exposed instantaneously.

The velocity in the direction IMC is v_y in Equation (10), with v_x' and v_y' from Equations (7) and (8) substituted into (10). The resultant equation is a long second degree equation for v_y in terms of x', y' and the camera parameters, and from this the lines of constant velocity are obtained. In terms of y_0' the resultant equation for the lines is

$$y' = -A + Bx' + [A^2 + B^2x'^2 + (C - 2AB)x' + y_0'^2 + 2Ay_0']^{1/2}$$

where

$$A = f\,(\operatorname{ctn} t + \tfrac{1}{2} \tan \alpha' \tan s \csc t)$$
$$B = \tfrac{1}{2} \tan s$$
$$C = f \operatorname{ctn} t \tan s$$

y' and x' form a coordinate system in and perpendicular to the direction of the principal line, respectively, and y_0' is the coordinate at which the locus crosses the y' axis.

If the focal length is larger than the film size the above formula may be approximated by series expansion to

$$y' = y_0' + \frac{C}{2A} x'$$

which is the equation of a straight line. This line forms an angle ψ with the y' axis,

$$\tan \psi = \frac{dx'}{dy'} = \frac{2A}{C} = 2 \operatorname{ctn} s + \tan \alpha' \sec t$$

where s is given by Equation (14). By substitution,

$$\tan \psi = 2 \cos t \operatorname{ctn} \alpha' + \sec t \tan \alpha'$$

which is Equation (10).

The equation for the angle from the principal-line to the slit has been derived. The angle from the direction of IMC to the slit may be more useful. This angle is simply $s+\psi$, with s given by Equation (14). Hence,

$$\tan \psi' = \tan (s + \psi) = 2 \cos t \operatorname{ctn} \alpha'. \quad (16)$$

The angle ψ changes each time the camera orientation is changed which means the orientation of the slit must always be adjusted. If the mechanization of this is complicated and the resolution requirement is low, an approximate constant direction may suffice.

3. *The IMC velocity varies with the second power of the coordinate* according to the equation

$$v_{\text{imc}} = K[a + by'' + c(y'')^2] \quad (17)$$

where y'' is the coordinate of the slit in the direction perpendicular to the slit and K, a, b and c are constants depending on the camera orientation and its parameters.

The problem of mechanizing the velocity is discussed in the paragraph on image-motion compensation for the forward oblique camera. The mechanization to use depends on the required accuracy, its cost and complexity, and on the system parameters. If accuracy can be sacrificed and the tilt-angle is not too large, the same approximation of varying the velocity linearly with the coordinate can be made.

The constants in Equation (17) depend upon which image-velocity within the slit the IMC is to be matched to. In selecting the optimum slit orientation the guideline used was the loci of points which have the same velocity component in the direction of IMC. Then the slit is oriented to these loci so that for any position of the slit, the velocity is a constant. If this can be done there should be no problem of selecting the film velocity and evaluating the constants. However, approximations were made in the formulas for the loci, and actually there are small variations in image-velocity for the prescribed slit orientation. To minimize the residual image-motion the film should be moved at the velocity of points at the middle of the slit or on the principal line since this is near the middle. The choice of the line enables the IMC velocity to be derived.

Equation (17) is derived by substituting Equations (7) and (8) into (10). The IMC velocity is v_y in Equation (10) with the swing-angle specified in terms of α' and t by Equation (14) and with $x'=0$ since the velocity along the principal-line (y' axis) is to be matched. The coordinate perpendicular to the slit (in the direction of movement of the curtain), y'', is introduced by substituting y'' $\csc \psi$ for y'. With the substitutions Equation (17) is obtained, and the constants K, a, b and c are:

$$\left.\begin{aligned} K &= \frac{Vf}{H} \cos t \cos \alpha' (\cos^2 t + \tan^2 \alpha')^{-1/2} \\ a &= \cos^2 t + \tan^2 \alpha' \\ b &= \frac{1}{f \sin \psi} (\tan t \tan^2 \alpha' + 2 \sin t \cos t) \\ c &= \frac{\sin^2 t}{f^2 \sin^2 \psi} \end{aligned}\right\} \quad (18)$$

The angle ψ is given by Equation (15).

If the curtain is moved in the direction of the principal-line $y' \sin \psi$ is substituted for y'' in the equation for v_{imc}, regardless of the slit orientation.

The three statements in italics specify the behavior of an IMC mechanism: the magnitude and direction of velocity and the orientation of the focal-plane slit for any tilt of the camera. The purpose of image-motion compensation is to eliminate motion blur, but in the process the photograph is distorted because the focal-plane shutter does not expose the whole film simultaneously.

Photographic Distortions

If a photograph is taken with a between-the-lens type shutter which exposes the whole film at once, the only distortion (other than film shrinkage, lens aberration and related imperfections) is due to the tilt of the camera. With the focal-plane shutter the whole photograph is not exposed simultaneously, and two additional distortions arise. One exists because the aircraft has moved in the time it takes to expose the whole film. Hence, an instantaneous picture of the ground is not obtained, rather the images are displaced by amounts depending on the moment of occurrence of exposure. This displacement is commonly called the sweep positional displacement.

The other distortion is due to the movement of the film or lens (IMC) and it exists whenever a focal-plane shutter is used and the camera has image-motion compensation. It results from the changing position of the lens relative to the film as the slit exposes different parts of the film. The amount of this displacement depends on the distance the film or lens is displaced from some reference position, and therefore it also is dependent on the time of exposure. This displacement is known as the IMC displacement.

The IMC displacement does not cancel the sweep positional displacement; this occurs only if the camera is vertical. The sole purpose of IMC is to counteract the image-movement due to aircraft movement and in doing so it cannot cancel the sweep positional displacement, although the two are in opposite directions. Therefore, there is a net displacement, the sum of the two displacements.

The sweep positional displacement is the product of the image-velocity and the time of exposure of the image since the velocity is approximately constant. A still photograph taken at the moment the slit is at the center, is treated as an imaginary reference so that the time of exposure is the time difference between the moments when the image is exposed and when the center of the photograph is exposed. This time is the quotient of the y'' coordinate of the image divided by the curtain speed in this direction, and the displacement is:

$$d_{x\,\mathrm{swp}} = v_x\left(\pm\frac{y''}{u}\right)$$

$$d_{y\,\mathrm{swp}} = v_y\left(\pm\frac{y''}{u}\right)$$

where u is the curtain speed, the sign depending on the direction of movement of the curtain (+ for movement toward the vertical), v_x and v_y are given by Equations (10), (7), (8) and (9), and y'' is a function of the image-point location:

$$y'' = x\cos(s+\psi) - y\sin(s+\psi)$$

with ψ specified by Equation (15) and s by Equation (11), (12) or (13). If the camera is pitched forward only, Equations (2) and (3) may be used for the velocity, and if it is tilted to the side, Equations (5) and (6) can be used.

The IMC displacement represents the distance the film is from its position when the exposure slit is at the center of the film. Since the IMC velocity varies the displacement is computed as the integral of the lens velocity over the time interval from the moment the image is exposed to the moment the slit is at the center of the photograph,

$$d_{x\,\mathrm{imc}} = d_{\mathrm{imc}}\sin(s'-s)$$
$$d_{y\,\mathrm{imc}} = d_{\mathrm{imc}}\cos(s'-s)$$

where

$$d_{\mathrm{imc}} = \int_0^t v_{\mathrm{imc}}\,dt = \int_0^{y''} v_{\mathrm{imc}}\left(\pm\frac{dy''}{u}\right)$$

where v_{imc} is the IMC velocity given by Equation (2) for the forward oblique case, by Equation (5) for the side oblique case, or by Equations (17) and (18) if the camera is both pitched and rolled. The angle s' is given by Equation (14). The resultant correction of image-position on the unrectified photograph is $(d_{\mathrm{swp}}+d_{\mathrm{imc}})$, which is subtracted from the coordinates of the image-point.

Summary

In this part the compensation of the image-motion due to the forward movement of the aircraft was investigated for the tilted frame camera. The analysis consists of finding what the image-motion is at any point in the focal-plane, and determining how this image-motion can best be compensated. In determining the optimum mode of image-motion compensation, three questions had to be answered: (1) What is the best direction of movement of the film? (2) How should the exposure-slit of the focal-plane shutter be oriented? and (3) What should the velocity of the film be?

For a vertical camera there is no residual image-motion if the IMC is accurate. For a tilted camera on the other hand the image-motion cannot be completely compensated because the magnitude and direction of the image-velocity varies. Therefore there will be degradation of the resolution from the residual image-motion over the film. (An exception is the side-oblique tilt position for which the only residual image-motion with accurate IMC is due to the width of the exposure slit.)

If the accuracy of the IMC mechanism and V/H sensor and the orientation of the camera are known the distance of image-blur from the movement of the aircraft may be computed for any point on the photograph, and from this the resolution degradation can be determined. In computing the blur distance all sources must be taken into account. If the aircraft is flying at low altitude varying heights of the ground terrain and aircraft rotation will contribute to the image blur.

Whenever the film is not instantaneously exposed, two distortions will arise. For a vertical frame camera the distortions cancel, but for any tilt orientation there is a net image displacement which is important if the camera is used for mapping. Equations for the image-velocity, IMC velocity, and the image-displacements due to the forward movement of the vehicles are given in this series for both the frame and panoramic cameras.

Any vehicle in flight is subjected to rotations which results in some blurring of the image. Stabilization systems can attenuate this motion, but cannot entirely eliminate it. In the second part of this series the image-motion from aircraft rotation for the oblique frame camera is investigated.

The panoramic camera offers important advantages over the frame as a type of sensor. Its continuous and unlimited field of view contrast with the more restricted coverage of the frame. An analysis of the image motion from aircraft velocity and rotations, and the proper image motion compensation for the vertical and forward oblique panoramic cameras, is contained in the last part of the series.

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FORUM

T. W. Norcross—Editor Emeritus

Theodore W. Norcross, who has served as the Editor and Advertising Manager of Photogrammetric Engineering since 1948, has been awarded the new title of Editor Emeritus. In his new assignment he serves as consultant and advisor to the President of the American Society of Photogrammetry, and to the editorial staff.

Not only did "Ted" serve as editor for 17 years, but also until 1953 the position included the functions as Secretary and Treasurer.

The Society is greatly indebted to "Ted's" efforts throughout the years. Although he accepted the position as a half-time job, he devoted more than full-time to the work, including week ends and evenings. Moreover, the Society's character and modus operandi will continue to reflect for many years to come the influence of "Ted's" devotion to its needs.

The Society congratulates "Ted" on a most successful term of service and thanks him for performing all those extra odd jobs that have been so necessary in the operation of the organization.

New Editor and Advertising Manager

Beginning with this issue, the new editor is Mr. G. C. Tewinkel, 11612 Michale Court, Silver Spring, Maryland 20904; telephone 301-MA 2-0409. The new Advertising Manager is Mrs. Barbara Muir, 2618 Spencer Road, Chevy Chase, Maryland 20015; telephone 301-JU 7-6107.

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