Slope Corrections in Aerotriangulation Adjustments

G. C. TEWINKEL
U. S. Coast & Geodetic Survey

A modified set of polynomial corrections not only yields improved accuracy but also facilitates an interrelated simultaneous horizontal and vertical solution by electronic computer.

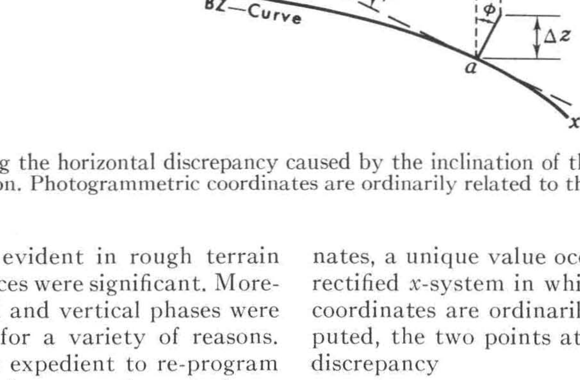
INTRODUCTION

THE COAST AND GEODETIC SURVEY has been applying a computational method for adjusting strips of aerotriangulation as explained by Harris [1,2,3] as early as 1958, and then the same program continued to be used in analytic aerotriangulation. Although the general technique gave gratifying results, discrepancies became evident in rough terrain where relief differences were significant. Moreover, the horizontal and vertical phases were treated separately for a variety of reasons. Recently it became expedient to re-program the entire routine, whereupon the two phases were combined and corrections were incorporated to compensate for the inclination of the strip.

In the Coast and Geodetic Survey application using the stereoplanigraph, a sufficient amount of vertical ground control is almost never supplied in order to level the first stereoscopic model. Operators ordinarily use their best judgment, such as assuring themselves that the stream beds decrease in elevation. But in the analytic aerotriangulation system being used, this judicious precaution is not applicable. Instead, the first model is arbitrarily assigned a zero tilt. This in itself may not be of any consequence if the strip is properly rotated and transformed as a solid body in space. Neither is it of any consequence if the terrain is fairly flat or, at least, not mountainous. But if the strip is inclined considerably, such as an angle ϕ in the figure, inasmuch as a rigorous treatment is not practical because of the large number of control points required, the top and bottom of an elevated object have different horizontal coordinates. In the strip x' -system of coordinates, a unique value occurs at a, but in the rectified x-system in which photogrammetric coordinates are ordinarily measured or computed, the two points at b and c introduce a discrepancy

$$\Delta x = \Delta z \tan \phi$$

in the horizontal dimension. This introductory explanation is an over-simplified version of the subject of the paper.



Sketch illustrating the horizontal discrepancy caused by the inclination of the strip combined with a difference in elevation. Photogrammetric coordinates are ordinarily related to the horizontal x-axis.

A SOLUTION

The Coast and Geodetic Survey has been applying the following polynomial transformation to the photogrammetric coordinates, first in x and y simultaneously and later in z almost independently [1,2,3]:

$$x' = x + ax^3 + bx^2 + cx - 2dxy - ey +$$
$$y' = y + 3ax^2y + 2bxy + cy + dx^2ex + g$$
$$z' = z + hx^3 + ix^2 + jx + kx^2y + lxy + my + n. \quad (1)$$

One wishes that these three equations could all be solved simultaneously with ease.

ABSTRACT: *The effect of the longitudinal and transverse inclinations of a strip of spatial aerotriangulation on the resultant horizontal coordinates of elevated objects is corrected through the application of factors derived from a preliminary solution of the vertical phase prior to the horizontal and incorporating both phases into a single computer program. The application has resulted in a significant reduction of horizontal discrepancies in areas of mountainous terrain. The corrections apply particularly to any system of aerotriangulation where the initial model is not leveled to fit ground data.*

But the only way, evidently, is to furnish a sufficient number of control points, which one tends to avoid if possible because of their high cost. The first two equations are interrelated: they are quasi-conformal. It would be helpful if all three are conformal, but Schut [4] and Mikhail [5] indicate that only *linear* transformations in three dimensions can be conformal. Clearly, experience has shown, and the literature universally supports the idea, that the transformations are essentially quadratic, if not cubic. Consequently, an iterative or piece-meal solution is applied.

The third equation z is of particular interest. One notes that the polynomial correction is composed of two primary parts:

$$hx^3 + ix^2 + jx \quad \text{and} \quad kx^2y + lxy + my. \qquad (2)$$

Without attempting any formal proof, it is readily apparent that the first part comprises the equation of the longitudinal *BZ*-curve of the center line of the strip, and the second part relates to the transverse inclination perpendicular to the center line. Then from calculus, the slope of the *BZ*-curve at any abscissa x is given by the first derivative of the equation of the curve:

$$\tan\phi = 3hx^2 + 2ix + j \qquad (3)$$

If the y term is factored out of the second expression,

$$(kx^2 + lx + m)y, \qquad (4)$$

it is evident that the polynomial factor in x is the slope of the strip across flight:

$$\tan\omega = kx^2 + lx + m \qquad (5)$$

Then the resultant strip inclination can be derived from the approximate expression:

$$\sec\tau = (1 + \tan^2\phi + \tan^2\omega)^{\frac{1}{2}} \qquad (6)$$

where the license for the approximation is based on the relatively small sizes of ϕ and ω and small elevational differences Δz.

With these terms known, corrections to the strip coordinates can be applied before they are adjusted to fit the ground control:

$$\begin{aligned} \Delta x &= \Delta z \tan\phi \\ \Delta y &= \Delta z \tan\omega \\ z'' &= z' \sec\tau. \end{aligned} \qquad (7)$$

Equations (1) are therefore modified:

$$\begin{aligned} x' &= x - \Delta z(3hx^2 + 2ix + j) + ax^3 + bx^2 + cx \\ &\quad - 2dxy - ey + f \\ y' &= y - \Delta z(kx^2 + lx + m) + 3ax^2y + 2bxy + cy \\ &\quad + dx^2 + ex + g \\ z' &= z\sec\tau + hx^3 + ix^2 + jx + kx^2y + lxy \\ &\quad + my + n. \end{aligned} \qquad (8)$$

The scheme for handling the computation in practice includes a single preliminary iteration: preliminary values of $h \ldots n$ are computed using the third formula of Equation 1, then the horizontal adjustment in x and y is executed applying the first two formulas of Equation 8, and finally the third formula of Equation 8 is resolved inasmuch as (1) a preliminary value for sec τ and (2) the scale information from the horizontal solution are available. A question may arise in the event that insufficient vertical ground data are known for solving the third part of Equation 1, which occurs if the aerotriangulation is purely planimetric and the vertical phase is not required. In such instances approximate vertical ground data are supplied through the use of existing maps, or simply "invented" from a knowledge of the area being mapped. In the Coast and Geodetic Survey application the approximate data consist of seven or eight elevations broadly distributed throughout the strip on both sides of the center line.

CONCLUSIONS

The slope correction has been incorporated in the Coast and Geodetic Survey program and tested. Several former adjustments have been recomputed using the new idea where the relief differences were sufficiently large that some improvement might be possible. Obviously, a fixed rate of improvement cannot be specified because the occurrence of ϕ and ω is accidental and any improvement also depends on the elevation difference of the specific point.

Two examples involving analytic aerotriangulation are quoted. The scale of photography was 1:40,000 in both instances. In one instance where maximum elevational differences of 2,000 feet occurred, the larger discrepancies in the horizontal position of tie points in the common area of two controlled strips was reduced from the neighborhood of 20–30 to 10–15 feet. Smaller discrepancies were essentially unaffected as might be expected on the basis that their values were subject mainly to accidental errors rather than this systematic cause. In the second instance, a horizontal control station 700 feet about the average elevation of the others indicated a horizontal discrepancy of 6 feet, which was the maximum value of seven points used. This maximum was reduced to 4 feet by the slope corrections with a consequential reduction of the root-mean-square error for the strip.

It is pointed out that even though the values of ϕ and ω vary continuously and independently throughout the strip, the values that are applied are those that are effective at the specific location in the strip where the particular image occurs; also the correction depends on the elevation of the particular object.

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