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*Radar Antenna Calibration using Half-Base Convergent Photography**

Introduction

Different photogrammetric methods have been suggested for the calibration of radar antennae. These previous studies have shown that photogrammetry provides a practical, reliable, and accurate means of calibrating antennae. It not only reduces somewhat the human errors but also introduces a method which is independent of environmental conditions. Also, it involves fewer man-hours work as compared to the direct methods of measurement. It can be used anywhere, under almost any given climatic conditions and, necessitates a minimum amount of fieldwork, which is highly desirable in certain geographic regions such as the Arctic.

Abstract: *Photogrammetric methods provide a practical, reliable and accurate means for calibrating radar antennae. Not only are human errors reduced to a minimum, but also the system is independent from environmental conditions and requires fewer man-hours of effort than the classical, direct measuring techniques. Inasmuch as the photogrammetric method is nearly independent from climatic conditions, the procedure can be applied in Arctic regions, requiring a minimum of exposure during field work. A novel technique known as "half-base convergent photography" can yield reliable results for calibrating the finished surface of an antenna reflector.*

In this paper a new photogrammetric method is proposed for the calibration of the finished surface of an antenna reflector using half-base convergent photography. Also, it will be shown that, with the proposed method, satisfactory and reliable results can be obtained.

Presentation of the Problem

Three principal requirements must be satisfied by any calibration method. These are:

1. The positioning of the antenna must be determined within specified limits of accuracy.
2. The acquisition and processing of data must be done within a relatively short period of time.
3. The determination of deformations due to settlement, wind, and temperature changes which can take place after construction, should be possible.

Advantages of Photogrammetric Methods

The first of the above three requirements is the most important. The accuracy within which we intend to check the positioning of an antenna will be given by the allowable tolerances of construction, i.e., $\pm\frac{1}{8}$ to $\pm\frac{1}{4}$ of an inch. This tolerance is small but can be achieved successfully using the proposed photogrammetric method. However, since the accuracy of the measurements depends on other factors, these will be mentioned briefly and reference to the original studies given.

That photogrammetric methods are less time consuming than the traditional field methods is well known. Other advantages are: the measurements can be performed in the office; fewer hazards, as far as the safety of the personnel is concerned, are encountered; and, photogrammetric methods reproduce a complete instantaneous record of all data which may be used at any time in the future.

A structure is never rigid. It deforms according to the stresses produced by external forces, the most important being settlements wind, and temperature changes. Because such movements might appreciably affect the performance of a radar antenna these

* This paper received the Bausch & Lomb Photogrammetric Award for the Best Paper on Photogrammetry by a College (Graduate) Student at the Annual Meeting of the Society in Washington, D. C., March 17–20, 1964.

deformations must be determined. In order to achieve this the author suggests the construction of permanent camera stations so that referenced periodic photographs can be taken. These photographs would provide a means of checking the radar antenna under various climatic conditions and at different time intervals.

Consequently, the use of photogrammetric methods for antennae calibration provides many advantages. The reduction in time consumed and hazards involved results in lower costs, while the flexibility of the photogrammetric method allows efficient data acquisition at any desired time.

Proposed Photogrammetric Method

Half-base convergent photography is proposed. Three basic reasons support the selection of this type of photography: first, a base-height ratio less than unity is needed (Mr. Warren Marks[1] in his proposed method used B/H = 0.60); secondly, high quality, high contrast photography is required. That half-base convergent photography satisfies this second condition is indicated by test results obtained by Mr. Edmund Swasey,[2] which can be summarized as follows:

1. "Readily apparent is the increase in illumination provided in the farthest corner of the half-base set-up and the improvement of the uniformity of intensity throughout the model. These factors not only provide a more pleasing stereo model with which to work, but also mean that fewer adjustments in light balance are required during the stereo plotting operation."
2. "The consensus of those who compared the stereo models of both types was that the resolution of the half-base model was much superior to that of the vertical models."
3. "The advantageous use of the more favorable part of the lens cone by the half-base technique is obvious."

Thirdly, an accurate determination of position is needed. That this can be better achieved using convergent photography was shown by Mr. Ake Jonsson.[3] His results can be summarized as follows:

1. "The accuracy in determination of model elevation increases all the time with the convergence angle, if we assume a constant standard error of unit weight in parallax measurements in the model."
2. "The planimetric accuracy will be somewhat greater in the vertical case than in the convergent one. The mean planimetric accuracy differs very little between the vertical and convergent case."

For purposes of demonstration, the case of a 60-foot diameter parabolic antenna which has its feed-horn tower at 60 feet from the reflector surface will be treated. The geometry of the general case will be analyzed and then a particular case will be studied for computing the theoretical accuracy of the measurements.

Equipment and Method

CAMERA

The Bausch and Lomb Altimar camera is selected. The following characteristics apply to this camera

aperture	f/5.0
field	75°
focal length	8.25 inches
distortion	0.03 mm.
format	9″×9″

The author realizes that since this is an aerial camera some difficulties will arise for focusing objects at less than one hundred feet. Modifications in the camera would be needed for adequate focusing.

CAMERAS SUPPORT

It is suggested that the two camera stations be positioned on permanent supports whose coordinates must be determined. This can be done in several ways but it is proposed that the camera stations should be connected by steel supports to the feed-horn tower whenever practicable and feasible.

ORIENTATION OF THE CAMERAS

It is proposed that 20 degree twin oblique convergent photography should be used.

COMPILATION

A first order universal stereoplotter such as the Wild A-7 Autograph is suggested.

GROUND CONTROL

Readily identifiable points on the reflector surface should be selected. The corners of the reflector surface are convenient and a minimum of five control points should be surveyed using conventional methods.

Geometry of the Proposed Method

For half-base convergent photography:

$$B/H = 0.62$$

where:

B = base distance

H = perpendicular distance from datum plane to the camera station.

Since in our example $H=60$ feet,

$$B = 0.62 \times 60 = 37.2 \text{ feet.}$$

GENERAL CASE. (See Figure 1)

In triangle $L'PQ$

$$(H - h)/\cos \alpha = L'P \qquad (1)$$

where $\alpha = 20° + \text{arctg}\,(O'P'/f)$.

In triangle LPL^1

$$L'P/\sin \beta = B/\sin \gamma \qquad (2)$$

But

$$a = 180° - (90° + \alpha) = 90° - \alpha$$
$$\gamma = 90° - b - a = 90° - b - (90° - \alpha)$$
$$\gamma = \alpha - b$$

Also

$$b = \text{arctg}\,(OP/f) - 20°.$$

Therefore

$$\gamma = [20° + \text{arctg}\,(O'P'/f)] - [\text{arctg}\,(OP/f) - 20°]$$
$$= 40° + \text{arctg}\,(O'P'/f) - \text{arctg}\,OP/f$$

and

$$\beta = 90° + b = 90° + \text{arctg}\,(OP/f) - 20°$$
$$= 70° + \text{arctg}\,(OP/f).$$

From Equation 1 and 2:

$$(H - h)/\cos \alpha = B \sin \beta/\sin \gamma$$

or:

$$\mathbf{h = H - B \sin \beta \cos \alpha/\sin \gamma.} \qquad (3)$$

PARTICULAR CASE

The case for which $\beta = 90°$ is now considered. For this value of β it is easy to see that $\gamma = \alpha$. Consequently Equation 3 becomes:

$$h = H - B \cos \alpha/\sin a$$

or

$$\mathbf{h = H - B\ \text{cotg}\ \alpha.} \qquad 4)$$

ERROR ANALYSIS

PARTICULAR CASE

Considering Equation 4 we see that the accuracy in the value of h depends only on the accuracy with which $O'P'$ can be measured. An accuracy of ± 0.01 mm. can be obtained with the Wild Autograph and this value will be used for the error analysis that follows.

Let $O'P' = r$. The expression for α becomes:

$$\alpha = 20° + \text{arctg}\,(r/f).$$

Applying a series analysis,

$$\alpha = 20° + \frac{r}{f} + \frac{1}{3}\left(\frac{r}{f}\right)^3 + \frac{1}{5}\left(\frac{r}{f}\right)^5 \cdots.$$

Differentiating this expression with respect to r:

$$d\alpha = \frac{1}{f}\,dr - \frac{1}{f}\left(\frac{r}{f}\right)^2 dr + \frac{1}{f}\left(\frac{r}{f}\right)^4 dr.$$

The error in α will then be given by:

$$\Delta\alpha = \frac{1}{f}\,\Delta r - \frac{1}{f}\left(\frac{r}{f}\right)^2 \Delta r + \frac{1}{f}\left(\frac{r}{f}\right)^4 \Delta r. \qquad (5)$$

Considering the case of $\omega = 32°$ (approximately measured α to point R on Figure 1),

$$r = f\,\text{tg}\,12° = 8.25\,\text{tg}\,12°$$
$$r \cong 1.46 \text{ inches.}$$

Since r/f is small in this case we can safely

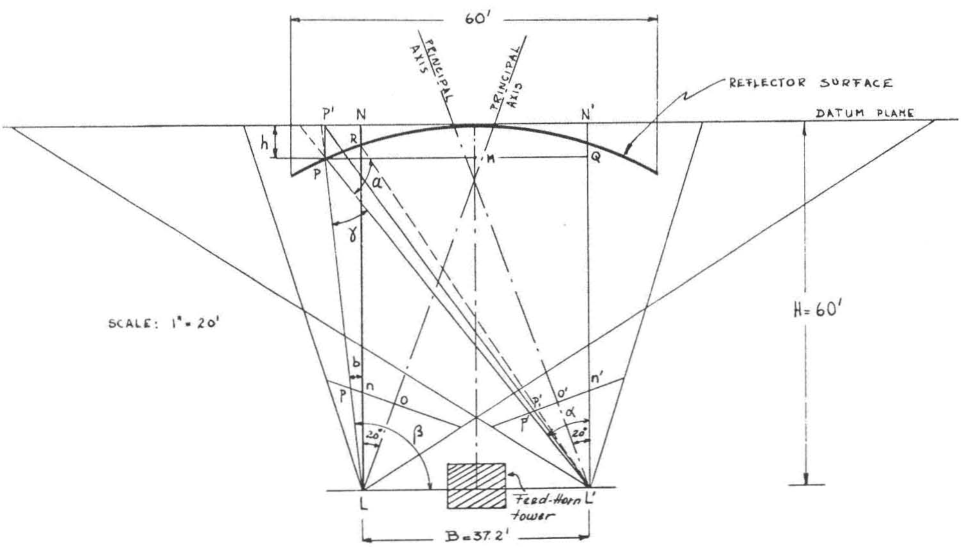

FIG. 1

neglect all the terms in Equation 5 except the first. Therefore:

$$\Delta\alpha \approx \Delta r/f$$

Using $\Delta r = 0.01$ mm.,

$$f = 8.25'' = 209.55 \text{ mm.}$$

$$\Delta\alpha = (1/209.55) \times 0.01 = 0.48\ 10^{-4} \text{ radians.}$$

Taking the derivative of Equation 4 we get:

$$dh = B(1/\sin^2\alpha)d\alpha.$$

The error in h can be expressed as:

$$\Delta h = (B/\sin^2\alpha)\Delta\alpha. \quad (6)$$

Using $\alpha = 32°$

$$\Delta h = \pm(37.2 \times 12/\sin^2 32°) \times 0.48\ 10^{-4}$$
$$\cong \pm 0.0763 \text{ inches.}$$

As one can see, this value of Δh, the error in h, is less than the specified maximum of ± 0.125 to ± 0.25 inches. Hence, it has been shown that theoretically one can expect very good results.

GENERAL CASE

The method for deriving the expression for Δh is similar to the one used in the particular case. However, the expression is more complicated. Differentiating Equation 3 we get:

$$dh = -B\frac{[(\cos\beta\, d\beta)\cos\alpha - \sin\beta(\sin\alpha\, d\alpha)]\cdot\sin\gamma - \sin\beta\cos\alpha(\cos\gamma\, d\gamma)}{\sin^2\gamma}.$$

Since $\Delta\alpha = \Delta\beta$,

$$\Delta h = B\frac{\cos\beta\cos\alpha - \sin\beta\sin\alpha)\sin\gamma\Delta\alpha - (\sin\beta\cos\alpha\cos\gamma)\Delta\gamma}{\sin^2\gamma}$$

or,

$$\Delta h = B\frac{\cos(\beta-\alpha)\sin\gamma\Delta\alpha - (\sin\beta\cos\alpha\cos\gamma)\Delta\gamma}{\sin^2\gamma}. \quad (7)$$

The evaluation of Equation 7 is quite complicated but could be accomplished on an electronic computer for all desired points on the photographed surface.

OTHER ERRORS

The influence of relative orientation[3]

From investigations performed by Mr. Ake Jonsson, the influence of the relative orientation on the elevation of the model is expressed by the formula:

$$dH = \frac{y}{B}[H\cos\phi_2 + (x-B)\sin\phi_2]d\chi_2 - \left(1-\frac{x}{B}\right)dbz_2 - \frac{H^2+(x-B)^2}{B}\cdot d\phi_2 + \frac{y}{B}[(x-B)\cos\phi_2 - H\sin\phi_2]d\omega_2.$$

The influence of compensation at the absolute orientation[3]

"In an arbitrary point $P(x, y)$ the elevation error will be in terms of errors in the elements of relative orientation,"

$$dh_r = y\sin\phi_2\left(\frac{2x-B}{2B}\right)d\chi_2 + \left(-\frac{x^2}{B} - x - \frac{B}{20}\right)d\phi_2 + y\cos\phi_2\left(\frac{2x-B}{2B}\right)d\omega_2.$$

The influence of elevation measurements[3]

"The mean square value of elevation errors is computed according to:"

$$M_{hm} = S\sqrt{\frac{H^2}{25d^2} + \frac{41H^2}{20B^2}}$$

where

S_0 = standard error in elevation measuring
d = perpendicular distance between the x photo axis and the orientation points outside the x axis.

VALUE OF THE CONSTANTS[3]

"The mean square value of the standard elevation error will vary with the inclination according to the scheme" shown in Table 1 in which ϕ is the angle of convergence. Although the above table doesn't apply to half-base convergent photography, a similar variation in M_{hm}/S_0 with respect to ϕ can be expected.