PROF. GOTTFRIED KONECNY
University of New Brunswick
Fredericton, N. B., Canada

*Structural Engineering Application of the Stereometric Camera**

A camera system developed for traffic accident analysis finds an application in the materials testing laboratory.

(*Abstract on page 100*)

INTRODUCTION

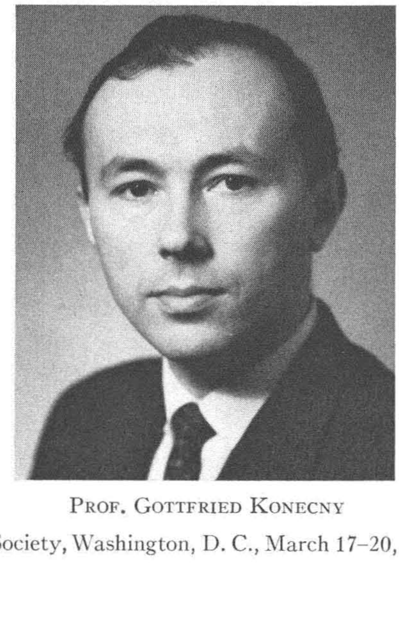
NON-TOPOGRAPHICAL use of photogrammetry has led to a variety of methods and special instruments; [6], pp. 97–102. The Wild C12 stereometric camera was designed specifically for the survey of traffic accidents (Figure 1). It consists of a fixed base of 1.20 m. length onto which two identical cameras are mounted. Their principal distance is 90 mm. and their glass plate format is 90×65 mm. The lens shutters are synchronized for exposures varying from 1 to 1/300 sec. The *f*-stop can be varied from 12 to 36. The camera unit can be rotated for azimuth and for ω-inclination at fixed settings of +15, 0, −15 and −25 grads. The unit can be mounted on a tripod; it can be leveled and vertically raised.

The stereophotographs taken of traffic accidents can be evaluated in the so-called "police-autograph" Wild A4 which was specifically designed for the plotting of stereometric camera photos; or they can be plotted on an ordinary first order stereoplotter, in which a multiple of the camera principal distance has to be introduced, resulting in a z-affinity of the stereomodel. For the Wild A-5 stereoautograph this ratio is usually chosen as 2:1.

It suggests itself that the stereometric camera can be applied in a similar manner to other non-topographical uses of photogrammetry: [4]; Figure 90 in [6].

Structural engineering makes use of various model tests to determine material properties and design criteria. Many of these tests restrict themselves to the measurement of strains. As long as the loads are kept small, the tested material (concrete, reinforced concrete, or steel) can be considered to have elastic properties. Elastic range tests lead to so-called "elastic design criteria" [1]. Elastic design, however, has its shortcomings, inasmuch as it is not valid for greater loads under which plastic properties of the tested material become noticeable. This is always the case near failure. Strain gauges fail to register in the plastic range so that deflection measurements of the structural model can lead to a much more accurate indirect determination of design parameters [2, 5].

The measurement of deflections should be fast, so that changes due to settling of the structure during the measurement are ineffective. Photogrammetric techniques are ideally suited for this purpose. They also give the advantage of not having to disturb the



PROF. GOTTFRIED KONECNY

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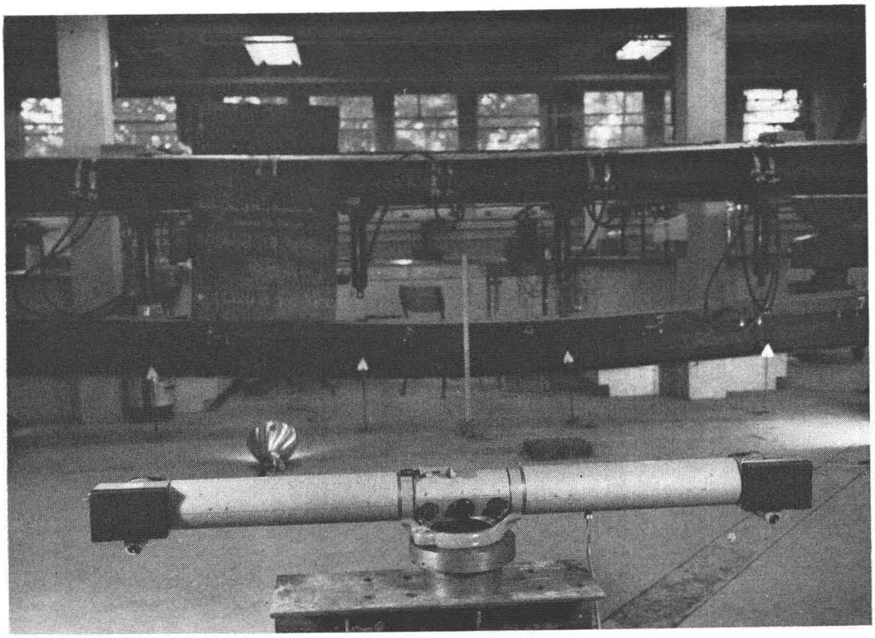

FIG. 1. The Wild C-12 Stereometric Camera used in testing a composite concrete-steel beam.

object by a physical measurement, and of having no accessibility problems for the measurement itself.

The Wild C12 stereometric camera was applied to two separate structural tests.

Composite Concrete-Steel Beam Test

In the first test a composite concrete-steel beam (Figure 2) was subjected to loads varying from 0, 450 psi., 600 psi., 800 psi. to 1,200 psi., at which intervals photographic stereo-exposures were made. The load was applied by five hydraulic pressure pumps subtended on a steel structure on top of the beam, simulating a uniform load. The top part of the composite beam consisted of a concrete slab, which was connected to the bottom part composed of steel by shear connectors. Failure of the composite beam occurred at 700 psi. when one of the shear connectors broke, freeing the concrete part from the steel.

The objective of the test was to measure the deflections of the beam at 21 points which were marked by chalk. Secondly, an attempt was made to measure horizontal differences between points 1 and 2 etc. situated alternately on concrete and on steel (Figure 3.)

A dial indicator was used as a check measurement of the deflection at midspan each time photographs were taken.

The camera was set in horizontal position at a distance of 7 m. from the beam,

Reinforced Concrete Slab Test

The second test was directed toward determining the deflections of a reinforced concrete slab under varying loads (Figure 4). The slab, 4.5 by 4.5 feet square, rested along the edges on a 3-inch diameter steel frame, leaving a net span between supports of 4 by 4 feet. The bottom of the slab contained 64 regularly distributed hooks connected to the reinforcement (Figure 5). Sixty-four cylindrical metal containers were suspended on the hooks. These were filled with sand and hung freely in a water tank 36 inches deep. By draining the water tank at intervals of 6 inches, varying "uniform" load stages could be achieved, increasing the load up to failure of the slab.

The 4 by 4-foot net span was marked by lines of white chalk on the slab which, for this purpose, had been previously painted black (Figure 6). A 6-inch grid further subdivided the net span into a total of 81 points. However only every second point was intended for measurement of the deflection, thus leaving 25 points to be measured (Figure 7). For the determination of absolute orientation, four control points, *A* to *D*, were determined using a level.

Photography of this arrangement was made for varying load stages. The slab (4 by 4 feet) was tested using *oblique* photography. The stereometric camera was set up on the tripod, 5 m. to one side of the test structure and at a

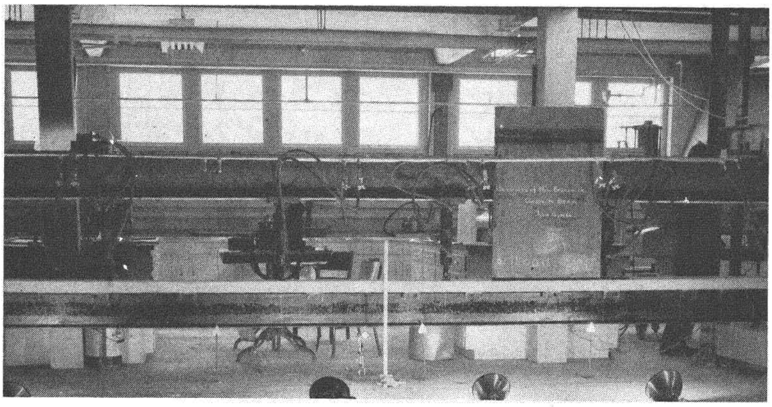

FIG. 2. A composite concrete-steel beam being tested.

height of 1 m. above the slab. An inclination of -15^g was used for the exposures (+ is considered as upward from the horizontal).

Another slab (4 by 6 feet) was tested using *vertical* photography. A metal frame was erected outdoors directly above the test structure. The stereometric camera was directly supported by the frame, 3.5 m. above the slab. Again four level points, *A* to *D*, were used to determine absolute orientation of the photographs.

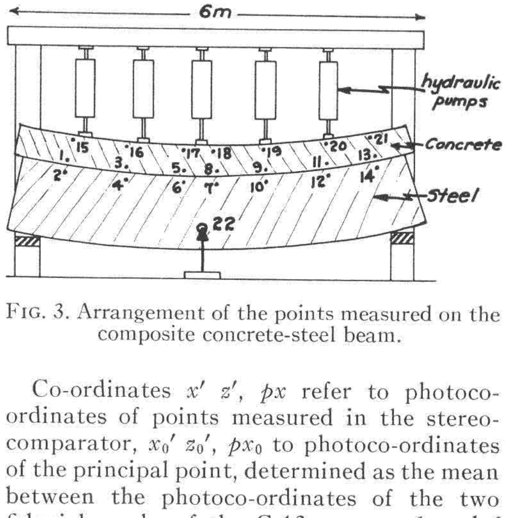

FIG. 3. Arrangement of the points measured on the composite concrete-steel beam.

ANALYTICAL EVALUATION OF THE PHOTOGRAPHS

Since all measurements needed referred to discreet points rather than areas, consideration was given to analytical procedures. Measurements of photo co-ordinates of the left photograph and of the *x*-parallax between right and left photograph were made on the Jena 1818 stereocomparator for each point. From double measurement of all plates the following standard errors were obtained for the mean of both determinations:

$$m_x' = \pm 3.0\mu$$
$$m_z' = \pm 4.9\mu$$
$$m_{px} = \pm 3.7\mu$$

m_z' for oblique photography is equivalent to m_y' for vertical photography.

Ground co-ordinates were computed on the LGP-30 electronic computer according to the formulas:

$$y = b \times f/(px - px_0)$$
$$x = (y/f)(x' - x_0')$$
$$z = (y/f)(z' - z_0')$$
$$Z = z \cos\omega + y \sin\omega$$
$$Y = y \cos\omega - z \sin\omega$$
$$X = x$$

Co-ordinates x' z', px refer to photoco-ordinates of points measured in the stereocomparator, x_0' z_0', px_0 to photoco-ordinates of the principal point, determined as the mean between the photoco-ordinates of the two fiducial marks of the C-12 camera. b and f represent base and principal distance of the camera. x, y, z are inclined ground co-ordinates, while X, Y, Z are horizontal ground co-ordinates. ω can be determined from two con-

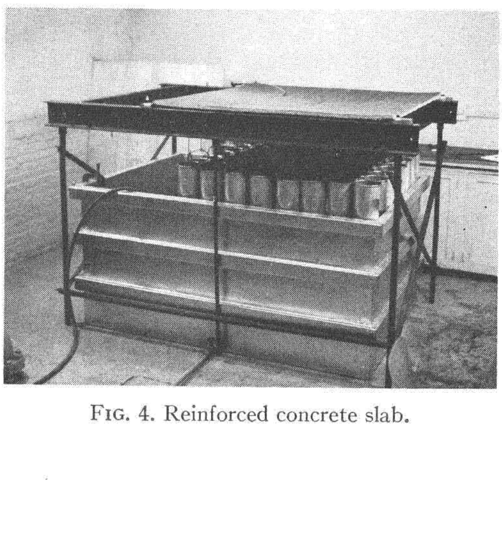

FIG. 4. Reinforced concrete slab.

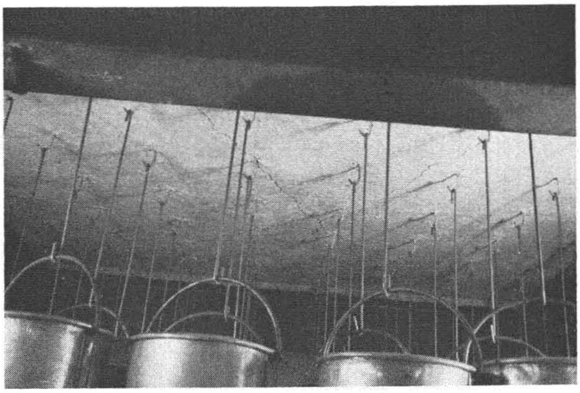

FIG. 5. Suspension of weights on reinforced concrete slab.

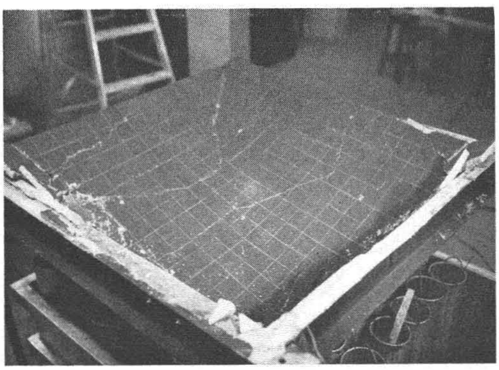

FIG. 6. Concrete slab at failure, showing grid lines.

trol points, at which level readings were taken (Figure 8); the formulae valid for oblique photographs become:

$$\operatorname{tg} \xi = (Z_B - Z_A)/(Y_B - Y_A)$$

$$\sin \eta = (Z_A - Z_B)/(Y_B - Y_A) \cos \xi$$

$$\xi + \eta = -\omega$$

In the case of four control points A, B, C, D (Figure 7), ω can be obtained as the mean from two determinations via A, B and C, D.

The co-ordinates X, Y, Z for each of the points should then be compared with the corresponding co-ordinates X, Y, Z at various loading intervals. The differences in Z co-ordinates will represent the deflections. These are the most significant deformations.

According to measurement accuracy, it should be possible for the values m_x', m_z' and m_{px} (as quoted previously) (in the absence of other influencing factors such as distortion or interior orientation errors) to reach the following co-ordinate accuracies for the conditions of the experiment:

$$m_x = \pm\ 0.23 \text{ mm.}$$

$$m_y = \pm\ 1.7 \text{ mm.}$$

$$m_z = \pm\ 0.38 \text{ mm.}$$

The deflection ΔZ should be determinable with an accuracy of $m_z\sqrt{2}$ or ± 0.54 mm. This presumes, however, that the orientation of the photography remains constant, or that an eventual change in exterior or interior orientation can be taken into account. This is al-

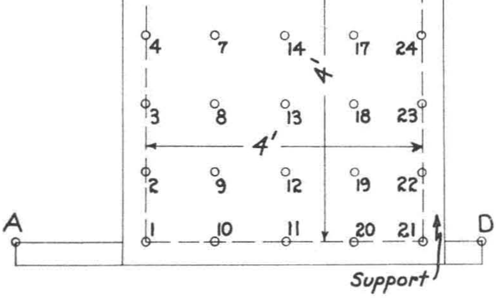

FIG. 7. Arrangement of points measured on the reinforced concrete slab.

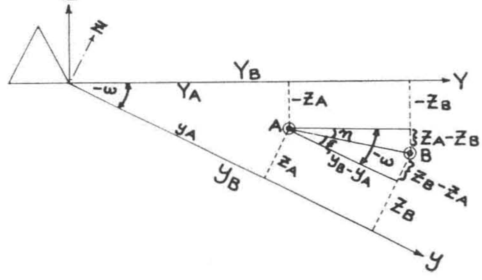

FIG. 8. Determination of ω in oblique photographs from control points.

ways possible if four, or at least three, control points are used.

Interior orientation changes are without effect in this case. For parallel photographs, directed normal to the base, such changes only affect a change in apparent absolute orientation and these effects can be eliminated with the aid of control points.

Also distortion, which is constant for subsequent photographs of the same exposure station, does not influence the accuracy by which the deflection can be determined.

The control points thus allow a check on the stability of the stereometric camera orientation within the accuracy of measurement (m_x, m_y, m_z).

Table 1 gives a listing of co-ordinate differences for point *A* of the reinforced concrete slab test; for this test the camera rested on the tripod. Table 2 gives a similar listing for point 22 of the composite beam test, for which the stereometric camera rested by its own weight on a wooden support.

The differences ΔX, ΔY, and ΔZ are caused both by changes in interior orientation (nuclear definition of fiducial marks on the plates during measurement) and in exterior orientation (small rotations of the camera during the change of plates). The change in *Y*-coordinates applies to the former, the change in *X* and *Z* coordinates to the latter, or for both. The following formulas demonstrate this.

The differential relations containing interior and relative orientation elements become for terrestrial photogrammetry:

$$dx = -y\left(1 + \frac{x^2}{y^2}\right)d\phi + \frac{xz}{y}d\omega - zd\kappa - \frac{x}{y}dby - dbx + \frac{y}{f}dx' - \frac{x}{f}df$$

ABSTRACT: *The paper describes the application of the Wild C12 stereometric camera for the study of conditions of failure for a concrete slab and a composite concrete-steel beam. Experiments were designed so that successive stereoexposures could be made at varying loads up to the moment of failure. Both the slab and the composite beam carried a painted grid system, which allowed point measurements on a Jena 1818 stereocomparator. The concrete slab was photographed both as a vertical as well as a high oblique picture to allow a comparison of accuracies for both methods. The usual numerical solutions had to be modified in such a way that slight motions of the camera had no effect on the computed ground co-ordinates from one exposure to the next. The results obtained analytically were also compared by plotting and contouring the deflections on the Wild A-5 Autograph in an affine model. Observation accuracy for photoco-ordinates was ±5 μ. The deflections could be determined with an accuracy of ±0.5 mm., which would correspond to an error of ±10 μ in the negative. The methods employed proved to be more reliable and more accurate to assess ultimate strength design criteria than methods previously used. Strain gauges fail to register outside of the elastic range, especially near failure.*

TABLE 1

COORDINATE DIFFERENCES FOR POINT *A* OF CONCRETE SLAB TEST, INDICATING THE DEGREE OF STABILITY OF THE C-12 CAMERA

	Dead Load	*Load Stage 4*	*Load Stage 12*
Y in mm.	4,147.624	4,163.614	4,165.461
X in mm.	−583.894	−588.504	−587.539
Z in mm.	− 61.386	− 61.160	− 60.168
ω in degrees	− 13.91°	− 13.78°	− 13.86°
Δ*Y* in mm.	—	− 15.990	− 17.837
Δ*X* in mm.	—	+ 4.610	+ 3.645
Δ*Z* in mm.	—	− 0.219	− 1.183
Δω in degrees	—	− 0.13°	− 0.05°

$$dz = -\frac{xz}{y}d\phi + y\left(1+\frac{z^2}{y^2}\right)d\omega + xd\kappa$$
$$- zdby - dbz + \frac{y}{f}dz' - \frac{z}{f}df$$
$$dy = -\frac{y^2}{b}\left(1+\frac{x^2}{y^2}\right)d\phi + \frac{xyz}{b}d\omega - \frac{yz}{b}d\kappa$$
$$- \frac{x}{b}dbz - \frac{y}{b}dbx + \frac{y^2}{bf}dx' - \frac{xy}{bf}df.$$

However no change of relative orientation elements occurs for the C-12 camera. Including only approximate terms for absolute orientation changes, the formulas become for the camera assembly:

$$dx = -yd\phi + \frac{y}{f}(dx_L' - dx_R') - \frac{x}{f}(df_L - df_R)$$
$$dz = +xd\kappa + yd\Omega + \frac{y}{f}(dz_L') - \frac{z}{f}(df_L - df_R)$$
$$dy = +\frac{y^2}{bf}(dx_L' - dx_R') - \frac{xy}{bf}(df_L - df_R).$$

It is to be noted that for dz only, dz_L' becomes influenced, but not dz_R', since it is not measured in the stereocomparator. Expressions $zd\omega$, $zd\kappa$ and $xd\phi$ were omitted since they are negligible. Inserting approximate numerical values would give:

$$dx = -7000\cdot d\Phi + 77(dx_L' - dx_R') - 11(df_L - df_R)$$
$$dz = +100\cdot d\kappa + 7000 d\Omega + 77(dz_L') - 3(df_L - df_R)$$
$$dy = 450(dx_L' - dx_R') - 0.6(df_L - df_R).$$

This indicates that principal point errors of 30 to 100μ (caused by the unclear definition of the fiducial marks) can well be responsible for the discrepancies of co-ordinates, rather than rotations of the instrument of ± 1 to 2 minutes during changes of the plates. This is partly confirmed by the use of two operators who obtained constant differences of up to 25μ for the co-ordinate readings $(x'-x_0')$ in the same model, mainly due to a different interpretation of the fiducial mark, resulting in a different x_0' reading.

It is obvious that due to these discrepancies the determination of deflections without use of control points will have large inherent errors. It therefore becomes necessary to reduce all readings or discrepancies to the plane determined by the control points. This reduction, equivalent to leveling the model, amounts to a space coordinate transformation:

$$\begin{pmatrix}\bar{x}\\ \bar{y}\\ \bar{z}\end{pmatrix} = \lambda\cdot A\cdot\begin{pmatrix}x\\ y\\ z\end{pmatrix}.$$

in which $\bar{x}\,\bar{y}\,\bar{z}$ are the transformed coordinates, λ equals 1 since the model is to be leveled only. $\bar{x}$ and $\bar{y}$ are not as important as the deflection coordinate $\bar{z}$, which can be expressed as:

$$\bar{z} = a_{31}x + a_{32}y + a_{33}z.$$

Three control points will allow the determination of the coefficients a_{31}, a_{32}, a_{33} from three such equations. For four control points a least squares adjustment will lead to a unique solution. By the aid of the determined coefficients z-coordinates of a subsequent load stage can be referred back to the initial dead load stage.

If all points to be transformed lie approximately in one plane, a simple correction graph procedure can also be applied.

Results of Composite Beam Test

Since all points used for measurement of composite beam deflections were situated along a straight line in x-direction, and since the stability of the camera in κ-direction was assured only one control point, point 22, was needed for the evaluation. Figure 9 shows the obtained deflection curves for various loads. It can be seen that the maximum deflection for load stage 800 psi. and 1,200 psi. is slightly left of half span, indicating the failure of a shear connector which was actually noticed at 700 psi. The results indicate that photo-

Table 2

Coordinate Differences for Point 22 of Composite Beam Test, Indicating the Degree of Stability of the C-12 Camera

(mm.)	Zero	450 psi	520 psi	600 psi	800 psi	1,200 psi
Y	7,093.595	7,127.771	7,121.896	7,123.070	7,126.361	7,140.495
X	141.634	138.712	139.271	139.849	138.091	138.287
Z	−277.912	−282.261	−283.847	−280.491	−279.750	−283.279
ΔY	—	− 34.176	− 28.301	− 29.475	− 32.766	− 46.900
ΔX	—	+ 2.922	+ 2.363	+ 1.785	+ 3.543	+ 3.347
ΔZ	—	− 4.349	− 5.935	− 2.579	− 1.838	− 5.367

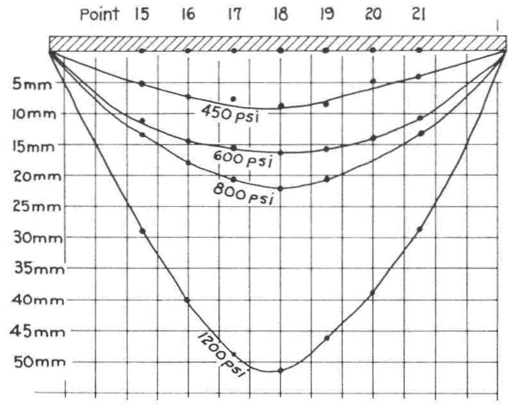

FIG. 9. Deflections of the composite beam for varying loads.

grammetric determination of the deflection is possible to ± 0.5 mm. with the stereometric camera. (Table 2.)

Table 3 shows x-coordinate differences between points located on concrete and points located on steel. The assymetry of the results shows that the break of the shear connector forced the concrete slab to the left.

Table 4 compares differences of X-coordinates between points 1 and 5, or 13 and 9, located on concrete. This indicates the concrete slab is actually lengthening when it is compressed.

RESULTS OF REINFORCED CONCRETE SLAB TEST

OBLIQUE PHOTOGRAPHY—ANALYTICAL TREATMENT

Satisfactory results from the oblique reinforced concrete slab test were obtained especially for the higher load stages, for which the deflections were sufficiently large. The relatively low, but still sufficient Z-accuracy of ± 0.5 mm. is mainly due to difficulties in stereoscopic vision for the oblique photographs. For an object distance of $y=5$ m. at a base-distance ratio b/y of 1:4 the undisturbed stereoscopic range is limited to $dy=25$ cm. due to the 70-second stereoscopic perception condition:

$$dy = n \cdot b \text{ tg } \tfrac{1}{2}(\delta + 70')$$

in which

$$\text{tg } \delta/2 = y/(b \cdot n)$$

n being the magnification of the stereoviewing device. The stereovision is thus severely limited to one to two rows of the 6″ grid.

This very fact makes it inadvisable to plot contours on the Wild A-5 stereoautograph from this type of photography, which, on account of the range, would still be possible in the way terrestrial photographs are usually plotted.

Figure 10 is an example for the evaluated deflections of load stage 12. The maximum deflection, in the center, is +13.8 mm. The corners lift up by −1 to −2 mm. due to an arching effect.

TABLE 3

X-COORDINATE DIFFERENCES BETWEEN POINTS ON CONCRETE AND POINTS ON STEEL

(mm.)	*Dead Load*	*1,200 psi*	*Difference* $d\Delta X$
ΔX:1, 2	11.293	4.922	6.371
ΔX:3, 4	12.574	11.537	+1.037
ΔX:5, 6	13.050	12.611	+0.339
ΔX:9, 10	12.843	13.905	−1.062
ΔX:11, 12	12.480	12.905	−0.425
ΔX:13, 14	12.747	12.951	−0.204

TABLE 4

X-COORDINATE DIFFERENCES BETWEEN POINTS ON CONCRETE

(mm.)	*Dead Load*	*800 psi*	*Difference* $d\Delta X$
X:1, 5	1,212.985	1,218.284	−5.299
X:13, 9	1,218.202	1,223.127	−4.925

VERTICAL PHOTOGRAPHY—ANALYTICAL TREATMENT

Due to much better stereoscopic vision, the evaluation of vertical photography results in greater ease and speed of measure-

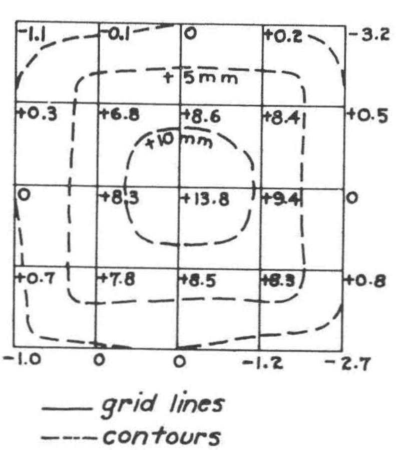

FIG. 10. Deflections of the reinforced concrete slab during load stage 12.

ment as well as in greater accuracy for the measurement of px and of $mpx = \pm 3\mu$. At a distance of 3.5 m this leads to

$$m_z = \pm (z^2/b \cdot f) m_{px} = \pm 0.34 \text{ mm.}$$

which on account of the larger base distance ratio is comparable to the accuracy with which deflections can be determined from oblique photographs. (For an equal object distance, vertical photography is ordinarily less accurate.)

The analytical evaluation of vertical photography is identical to that described for oblique photography, except that in the formula-systems z and y become interchanged.

VERTICAL PHOTOGRAPHY—STEREOPLOTTING

Vertical photography lends itself much easier for plotting in the Wild A-5 stereoautograph. Due to difficulties to insert the small 90×65 mm. negative plates with proper interior orientation into the picture carriers, relative orientation in by_R, ϕ_R, κ_L and κ_R will become necessary, as well as absolute orientation in Φ and Ω to control points.

For evaluation of the test a principal distance of $2f = 180.00$ mm. was chosen. The plotting scale was 1:5, with a model scale of 1:10 in the xy-direction and 1:5 in z-direction. Point measurements resulted in a height accuracy of ± 0.3 mm., comparable to analytical methods.

Figure 11 shows 5 mm. contours of the deflections for load stage 5 of the rectangular concrete slab, plotted on the A-5 autograph. (Actually plotting of 2 mm.-contours would have been feasible.)

Since the slab was rectangular rather than square, larger deflections of up to +27 mm. were encountered. An arching effect of −0.7 mm. was observed on the corners.

CONCLUSIONS

As has been shown, photogrammetric methods are capable of measuring structural model deformations. The use of the stereometric camera allows one to conduct a rela-

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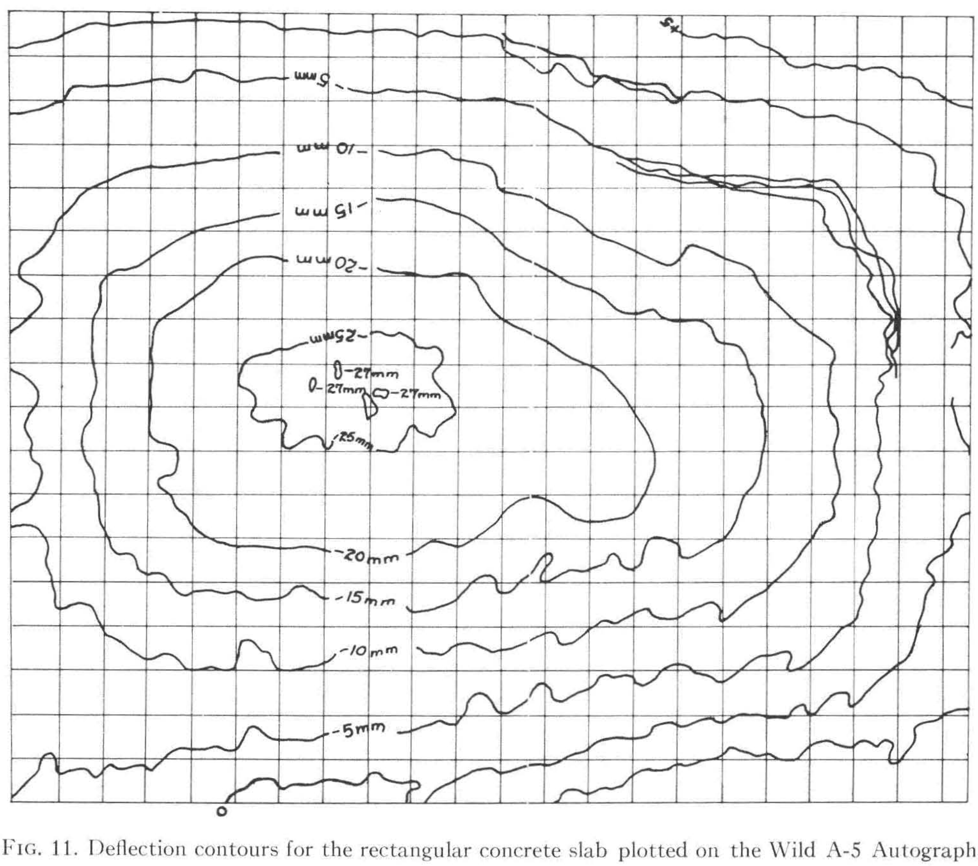

FIG. 11. Deflection contours for the rectangular concrete slab plotted on the Wild A-5 Autograph.

clusion that such a result is promising if compared with the accuracy of ±0.″2 to ±0.″3 which is obtained today in first order triangulation for a *relative* direction.

In closing this consideration of photogrammetric satellite triangulation, it can be predicted that this method will not only prove useful for a world-wide triangulation scheme, but will eventually provide the necessary accuracy for increasing the geometric fidelity within individual geodetic datums.

Extensive numerical analysis on various possible schemes for the application of photogrammetric satellite triangulation is currently in progress for the purpose of studying the problem of error propagation and for establishing an optimized field operational procedure.

References

1. Brown, Duane C., *An Advanced Reduction and Calibration for Photogrammetric Cameras*, AFCRL-64-40, Jan. 10, 1964.
2. Hynek, J. Allen, *On the Effects of Image Motion on the Accuracy of Measurement of a Flashing Satellite*, Cambridge, Mass., Smithsonian Institution Astrophysical Observatory, Research in Space Science, Special Report No. 33, Feb. 1, 1960.
3. Schmid, H. H., *The Influence of Atmospheric Refraction on Directions Measured to and from a Satellite*, U. S. Army Geodesy, Intelligence & Mapping Research and Development Agency, GIMRADA Research Note #10, Feb. 7, 1963.
4. Schmid, H. H., *Photogrammetry Applied to Three-Dimensional Geodesy*, invited paper to the Congress of the International Society of Photogrammetry, Lisbon, 1964, published in: "le Bulletin de la Société Francaise de Photogrammétrie."
5. Schmid, H. H., *Reformatory and Revoluticnary Aspects in Geodesy*, presented at the American Geophysical Union, Washington, D. C., April, 1964.
6. Väisälä, Y., *An Astronomical Method of Triangulation*, Sitzungsberichte der Finnischen Akademie der Wissenschaften 1946, Helsinki, 1947.
7. Veis, George, *Geodetic Uses of Artificial Satellites*, Smithsonian Contributions to Astrophysics, Vol. 3, No. 9, Smithsonian Institution Astrophysical Observatory.
8. Yeager, J. Austin, *A Report on Satellite Triangulation Field Operations in the Coast and Geodetic Survey*, American Geophysical Union, April, 1964.

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tively fast test sequence, and permits one to achieve an accuracy of about ±0.5 mm. for deformations in x or z. This accuracy is sufficient for the determination of ultimate strength design parameters, which cannot be obtained reliably from strain gauges. Structural model testing plotters [3] are considerably more expensive and are very limited in size and versatility as compared to stereometric camera techniques.

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References

[1] P. M. Ferguson, "Reinforced Concrete Fundamentals." Wiley, New York 1958.
[2] E. J. Grant, "The Ultimate Strength of Two-Way Reinforced Concrete Slabs." Association of Professor Engineers of New Brunswick, January 1964.
[3] N. W. Hanson and J. E. Carpenter, "Structural Model Testing—A Profile Plotter." *Journal of the PCA Research and Development Laboratories*, September 1963.
[4] U. V. Helava, "Canadian National Report to 9th International Congress of Photogrammetry Commission V." *Canadian Surveyor*, May 1960.
[5] K. W. Johansen, "Yield Line Theory." Cement and Concrete Association, London 1962.
[6] M. Zeller, "Lehrbuch der Photogrammetrie," pp. 97 to 102, Zurich, 1947, Orell-Füssli.

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