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X- and Y-Parallax Observations

ABSTRACT: *An experimental study determines the comparative precision of X- and Y-parallax observations in stereo-photogrammetric models. Supporting theories are presented. It was determined empirically that the standard error of X-parallax observations is about 0.4 times the standard error of Y-parallax observations. The effect of using dove prisms to change Y-parallax to X-parallax is also studied.*

INTRODUCTION

THE FORMATION of a three dimensional stereo-model involves the intersection of the optical rays, originating from the two corresponding picture points, at each point in the surface of the model. The intersection is performed when at the model-point (in the model space) both the rays have the same X-coordinate (model coordinate) and also the same Y-coordinate. The mathematical expression for this is that at a point P in the model-space, $X_{P_{\text{I}}}=X_{P_{\text{II}}}$, where $X_{P_{\text{I}}}$ is the X-coordinate with respect to the ray from the left-camera of the stereo-plotting instrument and $X_{P_{\text{II}}}$ is the X-coordinate with respect to the ray from the right-camera. Similarly, for the Y-coordinates, $Y_{P_{\text{I}}}=Y_{P_{\text{II}}}$.

The failure of obtaining an intersection means the existence of parallaxes (X or Y), which are defined by:

X-parallax (error): $\quad Px = X_{P_{\text{I}}} - X_{P_{\text{II}}}$ and,

Y-parallax (error): $\quad Py = Y_{P_{\text{I}}} - Y_{P_{\text{II}}}$.

Such parallaxes are due to the errors in the orientation elements. The parallaxes existing at a model-point expressed as functions of the errors in the orientation elements have been derived and established by various photogrammetrists earlier (e.g., Brandenberger). Any text book may be referred to for such derivations (e.g., Hallert, Schweidefsky, or Zeller). Considering a right-handed coordinate system with clockwise rotations, with the origin of the system at the left side projection center and with regard to the use of negatives (and not diapositives) the functions are:

$$
\begin{aligned}
-Px = {} & dbx_{\text{I}} - \frac{X}{Z}\,dbz_{\text{I}} - Y\cdot d\kappa_{\text{I}} - \frac{XY}{Z}\,d\omega_{\text{I}} \\
& + Z\left(1+\frac{X^2}{Z^2}\right)d\phi_{\text{I}} - dbx_{\text{II}} + \frac{(X-b)}{Z}\,dbz_{\text{II}} \\
& + Y\cdot d\kappa_{\text{II}} + \frac{(X-b)Y}{Z}\,d\omega_{\text{II}} \\
& - Z\left\{1+\frac{(X-b)^2}{Z^2}\right\}d\phi_{\text{II}}
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
-Py = {} & dby_{\text{I}} - \frac{Y}{Z}\,dbz_{\text{I}} + X\cdot d\kappa_{\text{I}} - Z\left(1+\frac{Y^2}{Z^2}\right)d\omega_{\text{I}} \\
& + \frac{XY}{Z}\,d\phi_{\text{I}} - dby_{\text{II}} + \frac{Y}{Z}\,dbz_{\text{II}} - (X-b)d\kappa_{\text{II}} \\
& + Z\left(1+\frac{Y^2}{Z^2}\right)d\omega_{\text{II}} - \frac{(X-b)Y}{Z}\,d\phi_{\text{II}}
\end{aligned}
\tag{2}
$$

It is apparent that the use of either or both of the parallaxes would result in solving the errors in the elements of relative orientation. The author was curious about it and went deeper into these aspects in a research sponsored by the U. S. Army Engineer (GIMRADA, Fort Belvoir, Virginia).

The X-parallax at a model point is related to the corresponding elevation error according to an expression $-Px=\delta Z(bx/Z)$; where δZ is the elevation error. For all practical purposes, one can assume that the model base (b) is identical with the X-component (bx) of the base. In practice, this is almost always true. Then

$$-Px = \delta Z\,\frac{b}{Z} \tag{3}$$

With a view to studying the feasibility of using X-parallaxes for the relative orientation of a stereo-model and to analyze the accuracy obtained thereby as compared to the situation with the use of Y-parallaxes, a series of experimental observations were made in the Photogrammetric laboratory of the Department of Geodetic Science, The Ohio State University. Below is given an account of the studies.

Description of the Experiment

For these investigations a Wild Autograph A7 was used. The photographs were taken with a Wild RC5 Aerial Camera; $f=152.36$ mm.; format: 23 cm.×23 cm.; standard photography. Picture scale was 1:8000 approximately. Model Scale was 1:4000.

Observations were made at or very near each of *three* different points of the model with undulating terrain. One of the points was on a flat side-walk that was running East-West (parallel to the X-axis), the second point was on another side-walk that was running North-South (parallel to the Y-axis) and the third point was on a grassy tennis lawn with lines crossing each other at right angles. Three hundred repeat-observations were made at each point, thus making the total number of observations 900. The 300 observations at each point were broken down as follows:

Case A1: 50 observations of Y-parallax elimination with the element by'' and corresponding readings of the by'' counter by moving the right side measuring mark from front to rear;

Case A2: 50 observations of Y-parallax elimination with the element by'' and corresponding readings of the by'' counter by moving the right side measuring mark from rear to front. Next, after changing the Y-parallax into X-parallax with Dove-prisms,

Case B1: 50 observations of Y-parallax elimination with the element by'' and corresponding readings of the by'' counter by approaching the model surface with the measuring mark from above;

Case B2: 50 observations of Y-parallax elimination with the element by'' and corresponding readings of the by'' counter by approaching the model surface with the measuring mark from below. Next, after coming back to the normal situation, by setting the Dove-prisms back to their original situations,

Case C1: 50 observations of X-parallax elimination (i.e., reading of spot elevations) with the foot-disk and the corresponding Z-counter readings by approaching the model surface with the measuring mark from above;

Case C2: 50 observations of X-parallax elimination (i.e., reading of spot elevations) with the foot-disk and the corresponding Z-counter readings by approaching the model surface with the measuring mark from below.

The relevant elements were set at the instrument to the following values: $f=152.36$ mm.; $Z \cong 304.7$ mm.; $bx=157.20$ mm.; $by'=by''=bz'=bz''=100.00$ (i.e., zero value) mm.

Assuming the arithmetic mean to be the correct value for each case (separately for A1, A2, B1, B2, C1 and C2), the standard error for each case was computed. These are, as averages obtained from all the three points:

Case A1: 15.7 microns, and Case A2: 13.5 microns; their average gives the standard error for *Case A: 14.6 microns.*

Case B1: 5.1 microns and Case B2: 5.7 microns; their average gives the standard error for *Case B: 5.4 microns.*

Case C1: 10.9 microns and Case C2: 18.8 microns; their average gives the standard error for *Case C: 14.8 microns.*

However this Case C gives the standard error of elevation determination in the model, which is related to the X-parallax according to the expression (3). This gives the standard error of X-parallax determination from case C to be, with proper substitutions in the expression (3):

$$14.8 \frac{157.2}{304.7} = 7.5 \textit{ microns.}$$

All the above are expressed as in the model. In these studies the model scale was approximately twice the picture scale. From this consideration, for the sake of standardization, it may finally be expressed that at the picture scale (in this particular case):

The standard error of Y-parallax determination is 7.3 microns.

The standard error of Y-parallax determination, by changing them to X-parallax with Dove-prisms, is 2.7 microns.

The standard error of X-parallax determination (from elevation observations) is 3.8 microns.

The above shows that the standard error of X-parallax determination is between 0.37 and 0.52 (i.e., between 2.7/7.3 and 3.8/7.3) $\cong$0.44, depending on the element used for the purpose) times the standard error of Y-parallax determination. This means, generally speaking, the co-factors are related to each other as follows:

$$Q_{Px,Px} = (0.44)^2 Q_{Py,Py} = 0.2 Q_{Py,Py}.$$

In case, however, Dove-prisms are used for changing the Y-parallax to X-parallax, generally speaking, $Q_{Px,Px} \cong Q_{Py,Py}$.

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ESSA, New Federal Agency, Includes the Coast & Geodetic Survey

President L. B. Johnson sent to Congress for approval on May 13, 1965 a plan for the reorganization of two major agencies in the Department of Commerce: the Weather Bureau and the Coast & Geodetic Survey.

Under the plan the Weather Bureau, the Coast & Geodetic Survey, and the Central Radio Propagation Laboratory of the National Bureau of Standards are to be combined in an agency known as the Environmental Science Services Administration (ESSA).

"The new agency will enable us to give the public, business and industry better and faster service, and combining certain management functions will provide the improved services at lower cost," Secretary of Commerce John T. Conner said. "But even more important, it will improve the scientific capabilities of the Commerce Department."

"Our studies range over such fields as meteorology, hydrology, climatology, seismology, geodesy, geomagnetism, oceanography, hydrography, aeronomy, tropospheric electromagnetic propagation and telecommunications," he said. "If we are to understand our environment and use it to the fullest advantage, a thorough knowledge of these fields and how they interact with one another is essential. This will be expedited if the scientists are under one organizational roof."

The Secretary said the proposal for the new agency grew out of suggestions made by Weather Bureau Chief Dr. Robert M. White, NBS Director Dr. Allen V. Astin, and C&GS Director Admiral H. Arnold Karo, plus studies conducted under the direction of Dr. J. Herbert Hollomon, Assistant Secretary of Commerce for Science and Technology.

The Commerce Department produces nautical and aeronautical charts, and forecasts of weather and ionospheric conditions (important for radio communications). The Department also issues warnings of natural hazards—hurricanes, tornadoes, floods, earthquakes, seismic sea waves, and solar disturbances. In addition, it provides the scientific community with a wide variety of information on the environment.