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V/H Image Motion in Aerial Cameras

The forward motion of a flying camera platform results in a movement of the image in the focal plane causing image blur.

(*Abstract on page 311*)

Introduction

Shortly before World War II, aerial camera and aircraft performance had reached a level where the image blur caused by forward motion of the aircraft became a limiting factor, and the technique of IMC (Image Motion Compensation) was born. Techniques so far devised make corrections for image motion at the center of the format. This is correct for certain cases, and sufficiently good for some other cases.

Modern trends in aerial camera design and application, where other than vertical or simple oblique pointings are involved and resolution levels are substantially higher, have focused attention on image motion effects which were previously considered of second order and of insignificant magnitude.

A number of articles have been published on image motion and image motion compensation in special circumstances. A general treatment of the phenomenon is considered worthwhile.

The forward motion of a flying camera platform results in a movement of the image in the focal plane, which is the reflected velocity of the ground point being imaged.

The apparent ground velocity vector is equal in magnitude to the platform velocity vector and opposite in direction. It is projected onto the camera focal plane to give a resultant image motion whose magnitude and direction depend upon the camera focal length, the location of the point on the image plane, the camera orientation, and the velocity-height ratio (V/H), where V is the platform velocity and H the height above terrain. V/H is the apparent angular velocity of a point at nadir when V is the velocity component parallel to the ground.

The resultant image motion is completely predictable from a knowledge of camera geometry, camera orientation, and V/H. Under certain conditions it can be compensated by appropriate mechanical motions of camera, lens or film. Under other conditions, i.e., when its magnitude or direction varies over the format, it cannot be completely compensated, but in some cases an approximate compensation can be made.

This report derives expressions for the components of this image motion, identifies those cases where complete compensation is possible, and discusses the degree of approximation which can be achieved in other cases, giving expressions for the residual uncompensated motion which may be used to determine whether these residuals are acceptable for a particular case.

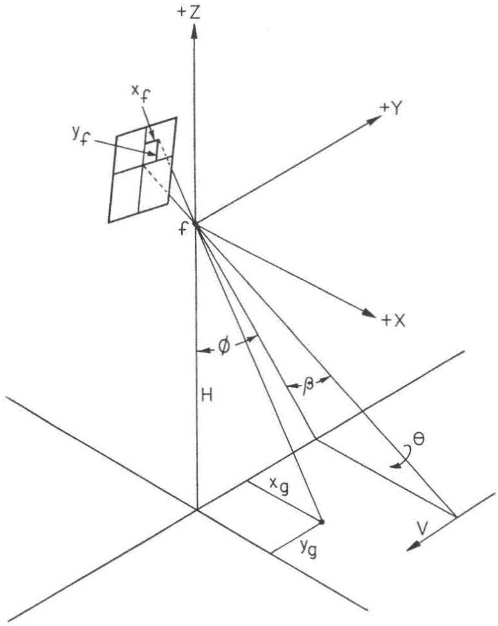

FIG. 1

GENERAL EQUATIONS FOR IMAGE MOTION

Consider the camera with its entrance pupil at the origin of a pair of rectangular coordinate systems (Figure 1). The ground coordinate system has its Z axis vertical, and its Y axis in the direction of flight. The camera coordinate system has its Z axis along the optical axis.

We take the two coordinate systems as initially identical (camera pointed at nadir) and establish the equations connecting ground and image points. We then rotate the camera system with respect to the ground system (point the camera) and determine the effect of this rotation on the equations.

With the camera system vertical, the focal plane is defined by the equation

$$z_f = f \tag{1}$$

in camera system coordinates, and the ground plane by the equation

$$z_g = -H \tag{2}$$

in ground system coordinates.

With the camera vertical, the image in the focal plane is an orthographic projection of the ground plane,* so that, for any image point $(x_f y_f z_f)$ in the focal plane, we may write

$$x_f = f\,(x_g/z_g) \tag{3}$$

$$y_f = f\,(y_g/z_g) \tag{4}$$

$$z_f = f \tag{5}$$

where $(x_g y_g z_g)$ is the ground point conjugate to the image point $(x_f y_f z_f)$. In all of the ensuing discussion, we assume a distortion-free optical system.

* It is assumed that the camera lens is distortion-free. Usually this is true to the necessary degree of accuracy. In cases where distortion needs to be considered, the values of x_f, y_f should be adjusted to reflect the values which correspond to Equation (3).

If the ground points are in motion, there will be image motion in the focal plane, which is obtained by differentiating (3) and (4)

$$S_x = \frac{dx_f}{dt} = f\frac{z_g\,(dx_g/dt) - x_g\,(dz_g/dt)}{z_g^2} \tag{6}$$

$$S_y = \frac{dy_f}{dt} = f\frac{z_g\,(dy_g/dt) - y_g\,(dz_g/dt)}{z_g^2}\,. \tag{7}$$

These are the equations for image motion for a ground point whose location, in camera system coordinates, is $(x_g y_g z_g)$. Image motion along the optical axis (change in magnification) is not considered in this discussion.

In general, for a rotated camera system, the camera system coordinates of the ground point $(x_g y_g z_g)$ are related to the ground system coordinates of the ground point $(x_g' y_g' z_g')$ by the matrix equation

$$\begin{bmatrix} x_g \\ y_g \\ z_g \end{bmatrix} = \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ A_z & B_z & C_z \end{bmatrix} \begin{bmatrix} x_g' \\ y_g' \\ -H \end{bmatrix} \tag{8}$$

where we have put $z_g' = -H$ by definition of the ground plane per equation (2). $\boldsymbol{M}$ will be used to symbolize the square rotation matrix that performs the coordinate transformation corresponding to a particular pointing of the camera system with respect to the initial vertical orientation.

Differentiating gives

$$\begin{bmatrix} dx_g/dt \\ dy_g/dt \\ dz_g/dt \end{bmatrix} = \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ A_z & B_z & C_z \end{bmatrix} \begin{bmatrix} dx_g'/dt \\ dy_g'/dt \\ = dH/dt \end{bmatrix}. \tag{9}$$

The apparent vector is in the $-y$ direction when $+y$ is defined as the direction of camera (vehicle) motion, so

$$dx_g'/dt = 0, \qquad dy_g'/dt = -\,V, \quad \text{and} \quad dH/dt = 0; \tag{10}$$

and expansion of (9) yields

$$dx_g/dt = -\,B_x V, \tag{11}$$

$$dy_g/dt = -\,B_y V, \quad \text{and} \tag{12}$$

$$dz_g/dt = -\,B_z V. \tag{13}$$

Substituting (11), (12) and (13) in (6) and (7) gives

$$S_x = -\,fV\,(z_g B_x - x_g B_z)/z_g^2 \tag{14}$$

and

$$S_y = -\,fV\,(z_g B_y - y_g B_z)/z_g^2 \tag{15}$$

and using the relations in (3) and (4), we obtain

$$S_x = -\,V\,(fB_x - x_f B_z)/z_g \tag{16}$$

and

$$S_y = -\,V\,(fB_y - y_f B_z)/z_g. \tag{17}$$

It remains to eliminate z_g from (16) and (17). We solve (8) for z_g, make the substitutions from (3) and (4), and obtain

$$z_g = \frac{fH[A_z(B_yC_x - B_xC_y) + B_z(A_xC_y - A_yC_x) + C_z(A_yB_x - A_xB_y)]}{f(A_xB_y - A_yB_x) - A_z(B_yx_f - B_xy_f) + B_z(A_yx_f - A_xy_f)}. \tag{18}$$

The bracketed term in the numerator is the determinant expansion of $\boldsymbol{M}$ and is equal to unity as shown in the following section. Substitution of z_g, as given by (18), in (16) and (17) gives

$$S_x = -(V/Hf)[fB_x - x_fB_z][f(A_xB_y - A_yB_x) - A_z(B_yx_f - B_xy_f) + B_z(A_yx_f - A_xy_f)] \tag{19}$$

and

$$S_y = -(V/Hf)[fB_y - y_fB_z][f(A_xB_y - A_yB_x) - A_z(B_yx_f - B_xy_f) + B_z(A_yx_f - A_xy_f)]. \tag{20}$$

ABSTRACT: *Expressions for the image motion in the focal plane of an aerial camera resulting from the forward motion of the vehicle are derived for the general case of a camera pointed in any direction. The expressions applicable to various pointings of frame, panoramic and strip cameras are discussed in detail, and the possibilities for introducing partial or complete compensation are explored. Effects due to pointing errors and/or residual camera motions will be treated in a later paper.*

These are the general image motion equations giving the image motion rate components along the X and Y axes as functions of the focal plane coordinates x_fy_f, the parameters V, H and f, and the orientation of the camera coordinate system relative to the ground as given by $\boldsymbol{M}$.

COORDINATE ROTATIONS

The coordinate rotational matrix $\boldsymbol{M}$ in Equation 8 is the matrix product of a series of single-axis rotations described by matrices of the form

$$\boldsymbol{M}_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \tag{21}$$

which identifies rotation of a rectangular coordinate system about the X axis through the angle θ. Rotations about Y and Z axes are described by

$$\boldsymbol{M}_y = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \tag{22}$$

$$\boldsymbol{M}_z = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{23}$$

We restrict ourselves to successive rotations about the orthogonal coordinate axes and to three rotations. Any camera pointing can be so defined. The six permutations of the product matrix are designated as $\boldsymbol{M}_{XYZ}$ in which the product sequence is indicated by the relative order of X, Y and Z:

$$\boldsymbol{M}_{XYZ} = \begin{bmatrix} \cos\theta\cos\beta & (\cos\theta\sin\beta\sin\phi \quad \sin\theta\cos\phi) & (\cos\theta\sin\beta\cos\phi + \sin\theta\sin\phi) \\ \sin\theta\cos\beta & (\cos\theta\cos\phi + \sin\theta\sin\beta\sin\phi) & (\sin\theta\sin\beta\cos\phi - \cos\theta\sin\phi) \\ -\sin\beta & \cos\beta\sin\phi & \cos\beta\cos\phi \end{bmatrix}$$

$$\boldsymbol{M}_{XZY} \quad \begin{bmatrix} \cos\theta\cos\beta & (\sin\beta\sin\phi - \sin\theta\cos\beta\cos\phi) & (\sin\theta\cos\beta\sin\phi + \sin\beta\cos\phi) \\ \sin\theta & \cos\theta\cos\phi & -\cos\theta\sin\phi \\ -\cos\theta\sin\beta & (\sin\theta\sin\beta\cos\phi + \cos\beta\sin\phi) & (\cos\beta\cos\phi - \sin\theta\sin\beta\sin\phi) \end{bmatrix}$$

$$\boldsymbol{M}_{YXZ} \quad \begin{bmatrix} (\cos\beta\cos\theta - \sin\beta\sin\theta\sin\phi) & -\sin\theta\cos\phi & (\sin\beta\cos\theta + \cos\beta\sin\theta\sin\phi) \\ (\sin\beta\cos\theta\sin\phi + \cos\beta\sin\theta) & \cos\theta\cos\phi & (\sin\beta\sin\theta - \cos\beta\cos\theta\sin\phi) \\ -\sin\beta\cos\phi & \sin\phi & \cos\beta\cos\phi \end{bmatrix}$$

$$\boldsymbol{M}_{YZX} = \begin{bmatrix} \cos\theta\cos\beta & -\sin\theta & \cos\theta\sin\beta \\ (\sin\theta\cos\beta\cos\phi + \sin\beta\sin\phi) & \cos\theta\cos\phi & (\sin\theta\sin\beta\cos\phi - \cos\beta\sin\phi) \\ (\sin\theta\cos\beta\sin\phi - \sin\beta\cos\phi) & \cos\theta\sin\phi & (\cos\beta\cos\phi + \sin\theta\sin\beta\sin\phi) \end{bmatrix}$$

$$\boldsymbol{M}_{ZXY} = \begin{bmatrix} (\cos\theta\cos\beta + \sin\theta\sin\beta\sin\phi) & (\cos\theta\sin\beta\sin\phi \quad \sin\theta\cos\beta) & \sin\beta\cos\phi \\ \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ (\sin\theta\cos\beta\sin\phi \quad \cos\theta\sin\beta) & (\cos\theta\cos\beta\sin\phi + \sin\theta\sin\beta) & \cos\beta\cos\phi \end{bmatrix}$$

$$\boldsymbol{M}_{ZYX} = \begin{bmatrix} \cos\theta\cos\beta & -\sin\theta\cos\beta & \sin\beta \\ (\sin\theta\cos\phi + \cos\theta\sin\beta\sin\phi) & (\cos\theta\cos\phi - \sin\theta\sin\beta\sin\phi) & -\cos\beta\sin\phi \\ (\sin\theta\sin\phi - \cos\theta\sin\beta\cos\phi) & (\sin\theta\sin\beta\cos\phi + \cos\theta\sin\phi) & \cos\beta\cos\phi \end{bmatrix}$$

where ϕ, β and θ are the rotations about the X, Y, Z axes, respectively.

It is seen that all of the single-axis rotation matrices, $\boldsymbol{M}_x$, $\boldsymbol{M}_y$ and $\boldsymbol{M}_z$, when expanded as determinants have the value unity; hence any of the six product matrices also have unit value. This fact was used in simplifying Equation 18.

Image Motion Equations for the Several Cases

By applying the six permutations of coordinate rotation, Equations 19 and 20 give sixteen cases for investigation:

a. One case of no coordinate rotation: Case 1

b. Three cases of single-axis rotation:

X rotation	Case 2
Y rotation	Case 3
Z rotation	Case 4

c. Six cases of two-axis rotation:

XY rotation	Case 5
XZ rotation	Case 6
YX rotation	Case 7
YZ rotation	Case 8
ZX rotation	Case 9
ZY rotation	Case 10

d. Six cases of three-axis rotation:

XYZ rotation	Case 11
XZY rotation	Case 12
YXZ rotation	Case 13
YZX rotation	Case 14
ZXY rotation	Case 15
ZYX rotation	Case 16.

Evaluation of Equations 19 and 20 for these sixteen cases gives the following results.

Case 1. Zero Rotation:

$$S_x = 0 \tag{24}$$

$$S_y = (V/H)f. \tag{25}$$

Case 2. X Rotation:

$$S_x = (V/Hf)(-fx_f \sin\phi \cos\phi + x_f y_f \sin^2\phi) \tag{26}$$

$$S_y = (V/Hf)(f^2 \cos^2\phi - 2fy_f \sin\phi\cos\phi + y_f^2 \sin^2\phi). \tag{27}$$

Case 3. Y Rotation:

$$S_x = 0 \tag{28}$$

$$S_y = (V/H)(f\cos\beta + x_f \sin\beta). \tag{29}$$

Case 4. Z Rotation:

$$S_x = -(V/H)f\sin\theta \tag{30}$$

$$S_y = (V/H)f\cos\theta. \tag{31}$$

Case 5. XY Rotation:

$$S_x = (V/Hf)[f^2 \sin\beta\cos\beta\sin\phi\cos\phi + fx_f \sin\phi\cos\phi(\sin^2\beta - \cos^2\beta) - fy_f \sin\beta\sin^2\phi - x_f^2 \sin\beta\cos\beta\sin\phi\cos\phi + x_f y_f \cos\beta\sin^2\phi] \tag{32}$$

$$S_y = (V/Hf)[f^2 \cos\beta\cos^2\phi + fx_f \sin\beta\cos^2\phi - fy_f \sin\phi\cos\phi(1 + \cos^2\beta) + y_f^2 \cos\beta\sin^2\phi - x_f y_f \sin\beta\cos\beta\sin\phi\cos\phi]. \tag{33}$$

Case 6. XZ Rotation:

$$S_x = (V/Hf)[-f^2 \sin\theta\cos^2\phi - fx_f \sin\phi\cos\phi(1 + \sin^2\theta) + fy_f \sin\theta\cos\theta\sin\phi\cos\phi - x_f^2 \sin\theta\sin^2\phi + x_f y_f \cos\theta\sin^2\phi] \tag{34}$$

$$S_y = (V/Hf)[f^2 \cos\theta\cos^2\phi + fx_f \sin\theta\cos\theta\sin\phi\cos\phi - fy_f \sin\phi\cos\phi(1 + \cos^2\theta) + y_f^2 \cos\theta\sin^2\phi - x_f y_f \sin\theta\sin^2\phi]. \tag{35}$$

Case 7. YX Rotation:

$$S_x = (V/Hf)(-fx_f \cos\beta\sin\phi\cos\phi - x_f^2 \sin\beta\sin\phi + x_f y_f \cos\beta\sin^2\phi) \tag{36}$$

$$S_y = (V/Hf)(f^2 \cos\beta\cos^2\phi + fx_f \sin\beta\cos\phi - 2fy_f \cos\beta\sin\phi\cos\phi + y_f^2 \cos\beta\sin^2\phi - x_f y_f \sin\beta\sin\phi). \tag{37}$$

Case 8. YZ Rotation:

$$S_x = (V/Hf)(-f^2 \sin\theta\cos\beta - fx_f \sin\theta\cos\theta\sin\beta - fy_f \sin^2\theta\sin\beta) \tag{38}$$

$$S_y = (V/Hf)(f^2 \cos\theta\cos\beta + fx_f \cos^2\theta\sin\beta + fy_f \sin\theta\cos\theta\sin\beta). \tag{39}$$

Case 9. ZX Rotation:

$$S_x = (V/Hf)(-f^2 \sin\theta\cos\phi - fx_f \cos\theta\sin\phi\cos\phi + fy_f \sin\theta\sin\phi + x_f y_f \cos\theta\sin^2\phi) \quad (40)$$

$$S_y = (V/Hf)(f^2 \cos\theta\cos^2\phi - 2fy_f \cos\theta\sin\phi\cos\phi + y_f^2 \cos\theta\sin^2\phi). \quad (41)$$

Case 10. ZY Rotation:

$$S_x = (V/Hf)(-f^2 \sin\theta\cos^2\beta - 2fx_f \sin\theta\sin\beta\cos\beta - x_f^2 \sin\theta\sin^2\beta) \quad (42)$$

$$S_y = (V/Hf)(f^2 \cos\theta\cos\beta + fx_f \cos\theta\sin\beta - fy_f \sin\theta\sin\beta\cos\beta - x_f y_f \sin\theta\sin^2\beta) \quad (43)$$

Case 11. XYZ Rotation:

$$\begin{aligned} S_x = (V/Hf)\{&f^2 \cos\beta\cos\phi(\cos\theta\sin\beta\sin\phi - \sin\theta\cos\phi) \\ &fx_f[\sin\theta\cos\theta\sin\beta(\sin^2\phi - \cos^2\phi) + \sin\phi\cos\phi(\cos^2\theta\sin^2\beta - \sin^2\theta\cos^2\beta)] \\ &+ fy_f[\sin\theta\cos\theta\sin\phi\cos\phi(1 + \sin^2\beta) - \sin\beta(\cos^2\theta\sin^2\phi + \sin^2\theta\cos^2\phi] \\ &- x_f^2 \cos\beta\sin\phi(\cos\theta\sin\beta\cos\phi + \sin\theta\sin\phi) \\ &+ x_f y_f \cos\beta\sin\phi(\cos\theta\sin\phi - \sin\theta\sin\beta\cos\phi)\} \end{aligned} \quad (44)$$

$$\begin{aligned} S_y = (V/Hf)\{&f^2 \cos\beta\cos\phi(\cos\theta\cos\phi + \sin\theta\sin\beta\sin\phi) \\ &+ fx_f[\sin\theta\cos\theta\sin\phi\cos\phi(1 + \sin^2\beta) + \sin\beta(\cos^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi)] \\ &+ fy_f[\sin\theta\cos\theta\sin\beta(\cos^2\phi - \sin^2\phi) + \sin\phi\cos\phi(\sin^2\theta\sin^2\beta - \cos^2\theta\cos^2\beta)] \\ &+ y_f^2 \cos\beta\sin\phi(\cos\theta\sin\phi - \sin\theta\sin\beta\cos\phi) \\ &- x_f y_f \cos\beta\sin\phi(\sin\theta\sin\phi + \cos\theta\sin\beta\cos\phi)\}. \end{aligned} \quad (45)$$

Case 12. XYZ Rotation:

$$\begin{aligned} S_x = (V/Hf)\{&f^2[\cos^2\theta\sin\beta\cos\beta\sin\phi\cos\phi - \sin\theta(\sin^2\beta\sin^2\phi + \cos^2\beta\cos^2\phi)] \\ &+ fx_f[2\sin\theta\sin\beta\cos\beta(\sin^2\phi - \cos^2\phi) + \sin\phi\cos\phi(\sin^2\beta - \cos^2 B)(1 + \sin^2\theta)] \\ &+ fy_f \cos\theta\sin\phi(\sin\theta\cos\beta\cos\phi - \sin\beta\sin\phi) \\ &- x_f^2[\sin\beta\cos\beta\sin\phi\cos\phi(1 + \sin^2\theta) + \sin\theta(\sin^2\beta\cos^2\phi + \cos^2\beta\sin^2\phi)] \\ &+ x_f y_f \cos\theta\sin\phi(\sin\theta\sin\beta\cos\phi + \cos\beta\sin\phi)\} \end{aligned} \quad 46)$$

$$\begin{aligned} S_y = (V/Hf)\{&f^2 \cos\theta\cos\phi(\cos\beta\cos\phi - \sin\theta\sin\beta\sin\phi) \\ &+ fx_f \cos\theta\cos\phi(\sin\theta\cos\beta\sin\phi + \sin\beta\cos\phi) \\ &+ fy_f[\sin\theta\sin\beta\cos\beta(\sin^2\phi - \cos^2\phi) + \sin\phi\cos\phi(\sin^2\theta\sin^2\beta - \cos^2\beta\cos^2\theta)] \\ &+ y_f^2 \cos\theta\sin\phi(\cos\beta\sin\phi - \sin\theta\sin\beta\cos\phi) \\ &- x_f y_f[\sin\beta\cos\beta\sin\phi\cos\phi(1 + \sin^2\theta) + \sin\theta(\sin^2\beta\cos^2\phi + \cos^2\beta\sin^2\phi)]\}. \end{aligned} \quad (47)$$

Case 13. YXZ Rotation:

$$\begin{aligned} S_x = (V/Hf)\{&-f^2 \sin\theta\cos\beta\cos^2\phi \\ &- fx_f[\cos\beta\sin\phi\cos\phi(1 + \sin^2\theta) + \sin\theta\cos\theta\sin\beta\cos\phi] \\ &- fy_f \sin\theta\cos\phi(\sin\theta\sin\beta - \cos\theta\cos\beta\sin\phi) \\ &- x_f^2 \sin\phi(\sin\theta\cos\beta\sin\phi + \cos\theta\sin\beta) \\ &- x_f y_f \sin\phi(\sin\theta\sin\beta - \cos\theta\cos\beta\sin\phi)\} \end{aligned} \quad (48)$$

$$
\begin{aligned}
S_y = (V/Hf)\{&f^2 \cos\theta \cos\beta \cos^2\phi + fx_f \cos\theta \cos\phi(\sin\theta \cos\beta \sin\phi + \cos\theta \sin\beta) \\
&- fy_f[\cos\beta \sin\phi \cos\phi(1 + \cos^2\theta) - \sin\theta \cos\theta \sin\beta \cos\phi)] \\
&+ y_f^2 \sin\phi(\cos\theta \cos\beta \sin\phi - \sin\theta \sin\beta) \\
&- x_f y_f \sin\phi(\cos\theta \sin\beta + \sin\theta \cos\beta \sin\phi)\}.
\end{aligned} \tag{49}
$$

Case 14. *YZX* Rotation:

$$
\begin{aligned}
S_x = (V/Hf)[&-f^2 \sin\theta(\cos\beta \cos\phi + \sin\theta \sin\beta \sin\phi) \\
&- fx_f(\sin\theta \cos\theta \sin\beta + \cos\theta \cos\beta \sin\phi \cos\phi + \sin\theta \cos\theta \sin\beta \sin^2\phi) \\
&+ fy_f \sin\theta(\cos\beta \sin\phi - \sin\theta \sin\beta \cos\phi) - x_f^2 \cos^2\theta \sin\beta \sin\phi \\
&+ x_f y_f \cos\theta \sin\phi(\cos\beta \sin\phi - \sin\theta \sin\beta \cos\phi)]
\end{aligned} \tag{50}
$$

$$
\begin{aligned}
S_y = (V/Hf)\{&f^2 \cos\theta \cos\phi(\cos\beta \cos\phi + \sin\theta \sin\beta \sin\phi) + fx_f \cos^2\theta \sin\beta \cos\phi \\
&+ fy_f[\sin\theta \cos\theta \sin\beta(\cos^2\phi \quad \sin^2\phi) - 2\cos\theta \cos\beta \sin\phi \cos\phi] \\
&+ y_f^2 \cos\theta \sin\phi(\cos\beta \sin\phi - \sin\theta \sin\beta \cos\phi) - x_f y_f \cos^2\theta \sin\beta \sin\phi\}.
\end{aligned} \tag{51}
$$

Case 15. *ZXY* Rotation:

$$
\begin{aligned}
S_x \quad (V/Hf)\{&f^2 \cos\beta \cos\phi(\cos\theta \sin\beta \sin\phi - \sin\theta \cos\beta) \\
&+ fx_f[\cos\theta \sin\phi \cos\phi(\sin^2\beta - \cos^2\beta) - 2\sin\theta \sin\beta \cos\beta \cos\phi] \\
&- fy_f \sin\phi(\cos\theta \sin\beta \sin\phi - \sin\theta \cos\beta) \\
&- x_f^2 \sin\beta \cos\phi(\cos\theta \cos\beta \sin\phi + \sin\theta \sin\beta \cos\phi) \\
&+ x_f y_f \sin\phi(\cos\theta \cos\beta \sin\phi + \sin\theta \sin\beta)\}
\end{aligned} \tag{52}
$$

$$
\begin{aligned}
S_y \quad (V/Hf)\{&f^2 \cos\theta \cos\beta \cos^2\phi + fx_f \cos\theta \sin\beta \cos^2\phi \\
&- fy_f[\cos\theta \sin\phi \cos\phi(1 + \cos^2\beta) + \sin\theta \sin\beta \cos\beta \cos\phi] \\
&- y_f^2(\sin\theta \sin\beta \sin\phi - \cos\theta \cos\beta \sin^2\phi) \\
&- x_f y_f \sin\beta \cos\phi(\cos\theta \cos\beta \sin\phi + \sin\theta \sin\beta)\}.
\end{aligned} \tag{53}
$$

Case 16. *ZYX* Rotation:

$$
\begin{aligned}
S_x = (V/Hf)\{&-f^2 \sin\theta \cos^2\beta \cos\phi \\
&- fx_f \cos\beta[\cos\phi(\sin\theta \sin\beta \cos\phi + \cos\theta \sin\phi) + \sin\theta \sin\beta] \\
&+ fy_f \sin\theta \cos^2\beta \sin\phi - x_f^2 \sin\beta(\sin\theta \sin\beta \cos\phi + \cos\theta \sin\phi) \\
&+ x_f y_f \cos\beta \sin\phi(\sin\theta \sin\beta \cos\phi + \cos\theta \sin\phi)\}
\end{aligned} \tag{54}
$$

$$
\begin{aligned}
S_y = (V/Hf)\{&f^2 \cos\beta \cos\phi(\cos\theta \cos\phi - \sin\theta \sin\beta \sin\phi) \\
&+ fx_f \sin\beta(\cos\theta \cos\phi - \sin\theta \sin\beta \sin\phi) \\
&+ fy_f[\sin\theta \sin\beta \cos\beta(\sin^2\phi - \cos^2\phi) - 2\cos\theta \cos\beta \sin\phi \cos\phi] \\
&+ y_f^2 \cos\beta \sin\phi(\sin\theta \sin\beta \cos\phi + \cos\theta \sin\phi) \\
&- x_f y_f \sin\beta(\sin\theta \sin\beta \cos\phi + \cos\theta \sin\phi)\}.
\end{aligned} \tag{55}
$$

Equations 24 through 55 give the image motions in the focal plane for all possible cases of one-, two-, or three-axis pointing. The following sections discuss a number of specific camera types and the application of these equations thereto.

These effects are those due to static pointing of prescribed angles. Further sources of error are *errors* in pointing and residual angular motions of the camera platforms. These effects will be treated as a subsequent paper.

Types of Aerial Cameras

Aerial cameras may be divided into three types: frame cameras, panoramic cameras, and strip cameras.

The frame camera takes a photograph of a finite two-dimensional area; the panoramic camera has a slit parallel to the Y axis of the camera which is scanned transversely by suitable means of rotation about this axis; the strip camera has a slit parallel to the X axis of the camera and does not scan, obtaining its coverage by forward motion of the vehicle.

The application of the equations of Section IV to each of these types of cameras is discussed in the following sections.

Frame Cameras

VERTICAL FRAME CAMERA

When the optical axis is vertical, Case 1 applies. Equations 24 and 25 describe the image motion. The motion is solely in the Y direction and is constant over the entire format. Conventionally, it is compensated by orienting the magazine so the film travels in the Y direction and driving the film at the rate $(V/H)f$ during the exposure period. Alternatively, the film may be fed in a direction transverse to the forward velocity and the platen driven at the rate $(V/H)f$. Also, the lens may be moved in the opposite direction at the same rate. Rocking the camera about the X axis in its mount has been used as a means for image motion compensation. It is easy to see, however, that this is not a rigorous form of correction; the motion produced is that described by Equation 27 and is correct only at nadir. Compensation by means of counter-rotating prisms has been advocated and analyzed. This does not provide complete correction because the image motion generated by counter-rotating prisms contains second order effects at finite field angles.

THE OBLIQUE FRAME CAMERA

Frame cameras are frequently mounted in an oblique direction. Two standard arrangements are the tri-metrogon array, in which three wide angle cartographic cameras are mounted in a fan—one vertical with an oblique on either side; and the split vertical, in which two cameras are directed to left and right of the flight line. Both arrangements increase transverse coverage—the tri-metrogon array gives horizon-to-horizon coverage when the cameras have an angular field of 75 degrees or more. The split vertical coverage depends upon the angular field of the cameras involved. Transverse fans of more than three cameras have also been used to obtain horizon-to-horizon coverage with cameras having a relatively small field of view.

Another standard camera installation is the forward oblique strike reconnaissance camera in which the optical axis is directed slightly (about 5 degrees) below the forward horizon.

For side oblique cameras, Case 3 applies. The image motion is in the Y direction only and has the value $(V/H)f \cos \beta$ at the center of the format. This motion is readily corrected by film or lens motion. When this is done there is a residual uncorrected motion which produces an image blur,

$$a = \frac{V}{H} e x_f \sin \beta \tag{56}$$

(where e is the exposure time), at points distant from the center of the format. It is interesting to note that this residual error is independent of the camera focal length, but depends only on the forward velocity, the tilt angle, and the distance from the center of the format.

Illustrative Example:

To provide a common example for illustration, the following parameters have been selected:

Focal length:	12 inches
Format:	$4\frac{1}{2}\times 4\frac{1}{2}$, or 5-inch film
Exposure Time:	1/1,000 second

The figures which follow are based upon this example. Application to other camera or operating parameters can be made by simple extrapolation using the governing equations.

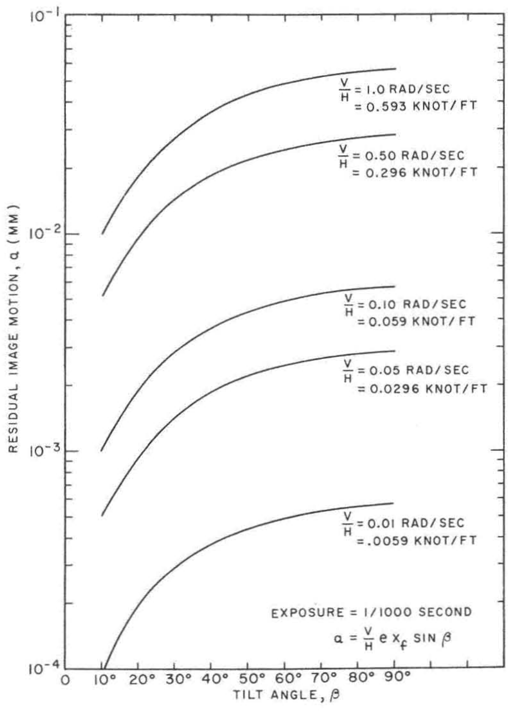

FIG. 2

Side Oblique Example:

Figure 2 shows the residual image motion in the side oblique camera as a function of the tilt angle, β, for various values of the V/H ratio. The units of this ratio in the equations are, of course, radians/second. It is customary to state V/H in knots/foot since aircraft velocity is commonly expressed in knots. The conversion ratio is

$$V/H\ (\text{knots/foot}) = 0.593V/H\ (\text{radians/second}).$$

The actual image degradation resulting from a residual image motion of a stated amount depends upon the resolution level for which the camera is designed. As a very rough rule of thumb, it may be said that if the residual image motion is the dominant factor in establishing the resolution performance, then the resolution will be limited to twice the motion. Thus, from Figure 2, for a V/H of 0.3 knot/ft and a tilt angle of 20 degrees, the resolution would not be better than 50 lines/mm at the edge of the format for an exposure time of one millisecond; for an exposure time of two milliseconds, the corresponding resolution limit would be 25 lines/mm. There are so many other factors contributing to resolution that this criterion should be used only as a "ball-park" estimate.

For the forward oblique camera, Case 2 applies, and Equations 26 and 27 describe the image motion, which has components in both X and Y directions. The motion at the center of the format is in the Y direction only, and is equal to $(V/H)f\cos^2\phi$. If this is corrected by film motion, the residual uncorrected motions are

$$S_x = -(V/Hf)(fx_f \sin\phi\cos\phi - x_f y_f \sin^2\phi) \quad (57)$$

$$S_y = -(V/Hf)(2fy_f \sin\phi\cos\phi - y_f^2 \sin^2\phi). \quad (58)$$

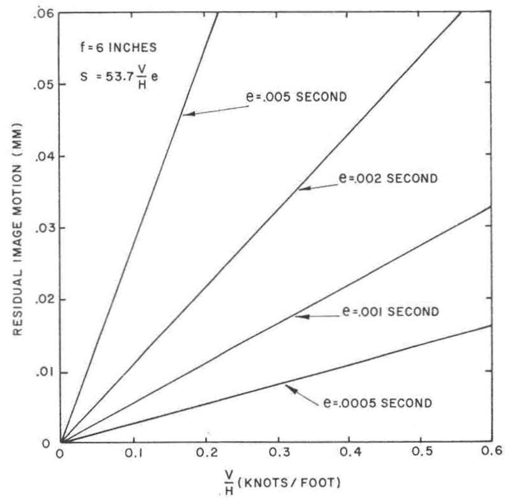

FIG. 3

The tilt angle for this type of camera is normally about 85 degrees, so the above equations may be written

$$S_x = (V/H)\left(0.99\frac{x_f y_f}{f} - 0.087x_f\right) \quad (59)$$

and

$$S_y = (V/H)\left(0.99\frac{y_f^2}{f} - 0.174y_f\right). \quad (60)$$

Our previous example of a 12-inch focal length is not typical of this type of camera, which does not usually have a focal length greater than six inches. At the top and bottom of the format, $x_f=0$, and the residual motion is in the Y direction only. At the sides, $y_f=0$, and the motion is in the X direction only and is relatively small, since the first term of (59) vanishes here. If we exclude consideration of the extreme corners of the format, then the worst case is the Y motion at top and bottom. For this, with a six-inch focal length and a $4\frac{1}{2}\times 4\frac{1}{2}$-inch format, (60) reduces to

$$S_y = 31.3\,(V/H)\,e. \quad (61)$$

This residual is plotted in Figure 3 as a function of V/H for various exposure times. These cameras are used at low altitude and at high speed, so that V/H values of 0.5 knot/ft would be typical. The need for short exposure times for this application is evident.

Frame Cameras Pointed at Compound Angles

Many special applications make it desirable to point a camera at a compound oblique angle in order to photograph a specific target area. Such pointing may be accomplished in various ways by two or three axis rotations. Equations 32 through 55 give the image motions resulting from all possible permutations of rotation about two or three orthogonal axes. Evaluation of these equations for a particular situation will determine whether there is an optimum choice and what the values of residual image motion will be. All cases involve motions in both X and Y directions over the format and hence cannot be completely compensated. In general, by introducing X and Y translational motions in the focal plane, or by "tracking" the camera during exposure, it is possible to compensate fully at any selected point in the field of view. This point will normally be the center of the format. Evaluation of the equations involved will determine whether, in a particular case, the residual uncorrected motion elsewhere in the photograph will be tolerable. In most cases where compound angle pointing is of interest, it is also desirable to have the highest possible resolution, so that even very small residuals can severely limit performance.

The following qualitative discussion will indicate the general nature of the problem.

All of the image motion rates are proportional to the quantity (V/Hf). For a typical case of high-altitude, high-resolution photography, V/H will be in the neighborhood of 0.03, and the focal length f will be of the order of 1,000 mm. so that the coefficient (V/Hf) will have a value of about 3×10^{-5} mm.$^{-1}$ sec^{-1}.

Nearly every equation contains a first term in f^2 which is independent of x_f and y_f; this represents the motion at the center of the format, which can be compensated by lens or film motion. The remaining terms in the equations represent the residual motions.

If we make the assumption that residual image motion of 0.2 of an image element is tolerable, then, for resolution in the 100 line/mm. region and for an exposure time of 0.005 second, we can consider a tolerable image motion residual to be

$$\delta S = 0.2 \times 0.01 \quad 200 = 0.4 \text{ mm./sec.}$$

With this kind of preliminary criterion, we can make a tentative analysis of the residual motions. As an example, let us take the X motion residual for XY pointing (Equation 32). For a $4\frac{1}{2}\times 4\frac{1}{2}$-inch format, the maximum values for x_f and y_f are 60 mm. Using this value and the adopted values for V/H and f of 0.03 and 1,000 mm. Equation 32, after correction for image motion at the center of the format, becomes

$$\begin{aligned}\delta S_x = {} & 1.8[\sin\phi\cos\phi(\sin^2\beta - \cos^2\beta) - \sin\beta\sin^2\phi] \\ & - .108[\sin\beta\cos\beta\sin\phi\cos\phi - \cos\beta\sin^2\phi]. \qquad (62)\end{aligned}$$

If we make the further simplification $\beta=\theta=A$ and substitute trigonometric identities, we obtain

$$\delta S_x = -0.45 \sin 4A - 1.8 \sin^3 - 0.027 \sin^2 2A + 0.108 \sin^2 A \cos A. \qquad (63)$$

From this point, it is easy to determine that δS_x reaches the marginal value of 0.4 mm./sec for $A=12$ degrees.

A similar technique of approximate analysis may be carried out for S_y and for the residual image motions for other pointings.

The Panoramic Camera

THE VERTICAL PANORAMIC

The vertical panoramic camera is mounted with its optical axis directed at nadir in nominal position, and scans by mechanical motion about the Y axis. The scan may be ± 90 degrees or less. Case 3 is the applicable case, and Equations 28 and 29 indicate that the only image motion is in the Y direction and that when correction is made for motion at the center of the format, only the second term of Equation 29 remains. This camera has a relatively narrow slit parallel to the Y axis, hence x_f never assumes large values and except for unusual cases, the term in x_f is negligibly small. In fact, the term in x_f represents what has been termed "parabolic" image motion. If we note that because of the motion of the film across the slit during the exposure time x_f varies with time—in fact

$$x_f = \dot{s}t \tag{64}$$

where $\dot{s}$ is the rate of film motion, then the movement (in the Y direction) during an exposure time is the integral of the second term of Equation 29 over the exposure time:

$$\delta S = \int_{t=0}^{t=e} (V/H)\ \dot{s}t \sin \beta dt \tag{65}$$

and we obtain Rosenau's expression* for parabolic image motion

$$\delta S = 1/2\ (V/H)\ \dot{s}e^2 \sin \beta. \tag{66}$$

The image degradation resulting from this type of image motion is much less severe than from other types, and a value of δS much greater than a resolution element can be tolerated.

THE TILTED PANORAMIC CAMERA

In order to obtain an increased stereo base, panoramic cameras are occasionally pointed fore and aft along a flight path. This corresponds to Case 5, and Equations 32 and 33 are applicable.

The first term in each equation represents the image motion at the center of the slit, and may be compensated by lens and/or focal plane motion. For most cases, $x_f \approx 0$, so that the residual uncorrected image motions become

$$\delta S_x = -\ (V/H)\ y_f \sin \beta \sin^2 \phi e, \tag{67}$$

and

$$\delta S_y = -\ (V/H)\ y_f \sin \phi \left[\cos \phi (1 + \cos^2 \beta) \quad \frac{y_f}{f} \sin \phi \cos \beta \right] e. \tag{68}$$

Analysis shows that the Y motion is by far the most severe, and that it is a maximum at $\beta = 0$. Figure 4 is a plot of the residual motion for a focal length of 12 inches and a $4\frac{1}{2}$-inch format, as a function of the pitch angle, ϕ, for several values of (V/H). The values do not change significantly for different focal lengths.

THE ROTATED PANORAMIC CAMERA

Another arrangement which has been suggested to obtain pointing at oblique targets fore and aft of the flight is rotation of a panoramic camera about the vertical axis, followed by conventional transverse scan about the rotated roll axis. This gives us Case 10, for which Equations 42 and 43 are used.

* M. D. Rosenau, Jr. "Parabolic Image Motion," Photogrammetric Engineering, June, 1961.

The first term in each equation can be compensated by focal plane motion, and for $x_f = 0$, we have the residual image motion

$$\delta S_x = 0, \tag{69}$$

and

$$\delta S_y = -\ (V/H)\ y_f \sin\theta \sin\beta \cos\beta e. \tag{70}$$

This motion is opposite in direction at the two ends of the slit and hence is not amenable to compensation. It is maximum for $\beta = 45$ degrees. Figure 5 is a plot of the motion as a function of θ for the case of a $4\frac{1}{2}$-inch format and an exposure of 1/1,000 second.

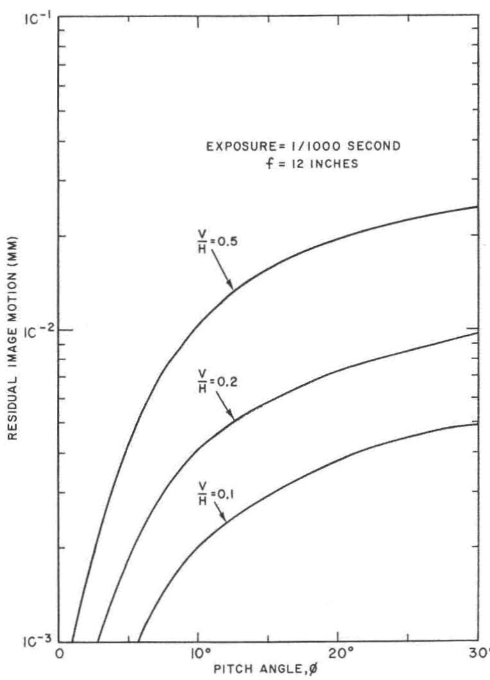

FIG. 4

OTHER PANORAMIC CONFIGURATIONS

For other pointings of panoramic cameras, Cases 12 and 15 apply. There are other possibilities, such as YXY' and YZY', but there is nothing to recommend these cases in preference to those already discussed.

THE STRIP CAMERA

THE VERTICAL STRIP CAMERA

The vertical strip camera contains a slit transverse to the flight direction, and film is transported normally to the slit, in accordance with Equation 25. The motion is constant over the slit, and hence is completely corrected by an appropriate film drive rate. Complete freedom exists as to the size and location of the slit, as long as the optical axis is vertical, so that displacement either fore and aft or transverse can be obtained in this manner without introducing image motion error—up to the available field of view of the lens. In the fore and aft direction, which is the narrow direction of the slit, the lens will have some freedom of field of view, so that this displacement of the slit can be used to obtain stereo coverage, using two slits (and in some cameras, two reels of film) with a single lens.

THE TILTED STRIP CAMERA

The strip camera may be tilted in either pitch or roll to obtain oblique coverage without placing the slit distant from the optical axis. Cases 2 and 3 apply for pitch and roll tilting, respectively, and Cases 5 and 7 apply to two-axis tilting.

Case 2, which covers fore-and-aft pointing, gives Equations 26 and 27.

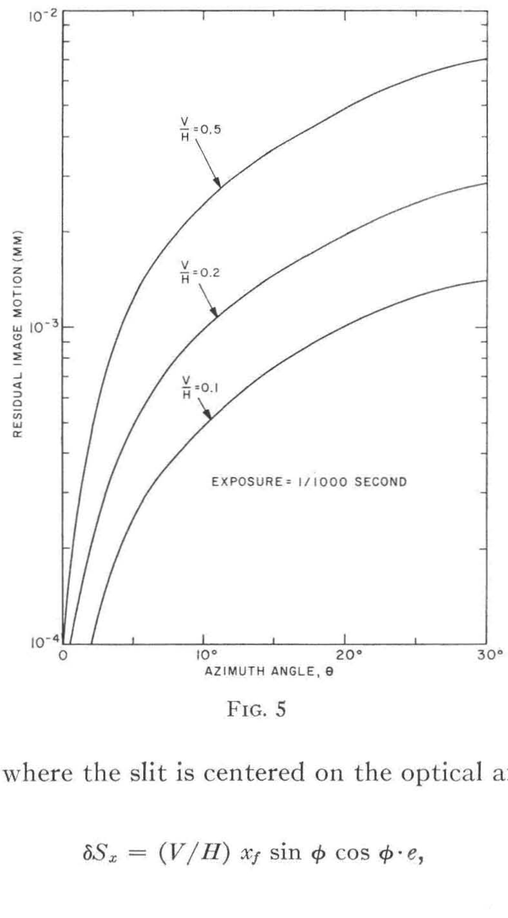

FIG. 5

For the situation where the slit is centered on the optical axis, $y_f = 0$, essentially, and we have

$$\delta S_x = (V/H)\, x_f \sin \phi \cos \phi \cdot e, \tag{71}$$

and

$$\delta S_y = -\,(V/H)\, f \cos^2 \phi \cdot e. \tag{72}$$

The Y motion is constant and is compensated by the film drive. The X motion is zero at the slit center and is opposite in direction at the two ends of the slit and hence cannot be directly compensated. For an example ($V/H = 0.03$, $x_f = 60$ mm., $e = 0.005$ sec, tolerable image motion $= 0.002$ mm.) the allowable value for ϕ is 13 degrees.

If the slit is displaced in the focal plane in the X direction, so that $x_f = x^0 \pm x_f'$, the X motion becomes symmetrical about the value $(V/H)x_0 \sin \phi \cos \phi$, and this value can be compensated by focal plane motion, so that, with this compensation, the residual error can be held to the same value as for the case of the slit on the optical axis. This is a means of obtaining effective transverse pointing, up to the field of view limit of the lens, without introducing additional image motion error.

Nothing would be gained by displacing the slit from the optical axis in the Y direction.

For a strip camera tilted in roll, Case 3 applies, and we have Equations 28 and 29.

The first term of Equation 29 is constant, and is compensated by the film drive motion, so that the only residual image motion to be tolerated is

$$\delta S_y = -(V/H)\, x_f \sin\beta \cdot e \tag{73}$$

which describes a motion perpendicular to the slit and opposite in direction at the two ends. Further, it is proportional to the distance from the center of the slit, which immediately suggests its correction by a rotary motion of the film in the focal plane about an axis bisecting the slit. The rate of motion to obtain complete correction will be

$$\omega = (V/H) \sin\beta \text{ (radians/second)}. \tag{74}$$

This means of compensation, of course, precludes continuous strip operation.

With a strip camera pointed in roll, the effect of two-axis pointing can be achieved without additional image motion error by displacing the slit in the Y direction. This can also be a means for obtaining stereo coverage by using two slits displaced fore and aft, as was pointed out for the vertical case.

If rotary compensation is not applied to the roll-pointed strip camera, Equation 73 gives a maximum tolerance for β of 13 degrees, using the conditions of the example for the pitched strip camera.

Second-order Image Motions

The previous sections have treated the cases of image motion introduced by the pointing of cameras in specified directions. The second-order image motions which are introduced when tolerances on the pointing angles have to be taken into account, and when the camera platform and/or stabilized mounts are subject to attitude and angular rate errors, are often of importance, especially in the case where the first-order motions have been compensated by appropriate techniques.

These second-order motions are obtained by forming differentials of the Equations 24 through 55. A completely general treatment for all of the cases would be hopelessly complex and not of great value. A simplified treatment of certain cases of interest will be the subject of a later paper.

Peace River Project

West Coast Division of Hunting Survey Corporation Limited has taken on a variety of photogrammetric and mapping assignments in connection with the British Columbia government's power development project in the Peace River area.

Largest single portion of the Hunting contract involved use of Wild P-30 photo-theodolite equipment on canyon walls, abutments, grout curtain areas as well as the canyon floor after the original course of the river had been diverted through three diversion tunnels. While primarily used in areas where ground survey was either impossible or impracticable, the P-30 equipment also had clear economy in local areas where low level air photography would normally have been employed.

The completed dam will be some 600 feet high, nearly a mile long and half a mile wide at the base. It will contain in all nearly 60 million cubic yards of materials. With such a large amount of fill to be moved, the conveyor belt system offers substantial economy and increased efficiency over the traditional trucking methods.

The entire field mission for the initial canyon project taken on by Hunting, including the necessary horizontal and vertical control, took about 21 days with a two-man party. (*Reprinted from* CANADIAN HUNTING NEWS-SURVEY.)