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Interpolation of a Function of Many Variables

ABSTRACT: *A solution containing arbitrary elements refers to the problem of interpolating the value of a function of several variables. The solution applies to the removal of the residual errors of an air survey block.*

IN THE aerial block the problem often arises of residuals of a non-random nature at the ground controls. This is usually solved by graphical methods, that is, the reversed residuals, regarded as corrections, are plotted against the planimetric positions of the controls and contours are constructed which are consistent with these. The corrections at positions intermediate to the controls are then interpolated between the contours.

This procedure is limited to corrections which are functions of two variables only. Furthermore, these days, a graphical operation of this type may be extremely inconvenient if the remainder of the operations are strictly digital. The method described below is entirely numerical and easily adapted for high-speed computations.

The problem stated above may be given a more abstract and general form as follows. The function $f(x, y, z, \cdots)$ has known values at the "points" $(x_1, y_1, z_1, \cdots)$, $(x_2, y_2, z_2, \cdots)$, $\cdots (x_n, y_n, z_n, \cdots)$; to determine its values at other points. There is no unique solution to this problem in the absence of specific information about the nature of the function f, so the solution given here contains arbitrary elements. First, we introduce the notion of distance or separation between the points, or sets of values, In many cases, and certainly in photogrammetric applications, we may use the Euclidean distance

$$r = + (\Delta x^2 + \Delta y^2 + \Delta z^2 + \cdots)^{1/2} \tag{1}$$

but in other applications it may be better to define r in another manner.

We now assume that f may be represented by the expression

$$f = k_1\phi(r_1) + k_2\phi(r_2) + \cdots + k_n\phi(r_n) \tag{2}$$

where r_i is the distance of the point $(x, y, z \cdots)$ from the control point $(x_i, y_i, z_i \cdots)$, while $k_1, k_2, \cdots k_n$ are unknown coefficients. $\phi(r)$ is a function which is continuous and a maximum at $r=0$ and positive for $r<a$, where a is the maximum distance separation between the control points. Many functional forms will fit these specifications and for simplicity we select

$$\phi(r) = 1 - r^2/a^2 \tag{3}$$

Applying (2) and (3) to the controls themselves and denoting the distance between the ith and jth controls by r_{ij} we get, since $r_{ii}=0$,

$$\begin{aligned}
&k_1 + k_2(1 - r_{12}^2/a^2) + \cdots + k_n(1 - r_{1n}^2/a^2) = f_1\\
&k_1(1 - r_{12}^2/a^2) + k_2 + \cdots + k_n(1 - r_{2n}/a^2) = f_2\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
&k_1(1 - r_{1n}^2/a^2) + k_2(1 - r_{2n}^2/a^2) + \cdots + k_n = f_n
\end{aligned} \tag{4}$$

These equations have normal symmetry and may be solved by any of the methods used for normal equations. With known values for the coefficients $k_1, k_2 \cdots k_n$ the value of f at any other point may be computed from (3). A test application of this method to the planimetric adjustment of an air survey block gave interpolated values quite close to those obtained by graphical methods.

The application of the above numerical procedure to an internally consistent block of aerial triangulation is fairly obvious. In this case f represents the reversed value of the outstanding discrepancy in any one of coordinates, so that for the complete adjustment (4) must be solved three times, each time with a different right-hand side. It is sufficient to compute r as the planimetric value, i.e.,

$$r^2 = x^2 + y^2.$$

The above procedure is most suitable for the case of a block which has been distorted during the internal adjustments, so that the errors are almost entirely a function of position within the block. The arbitrary constant a should then be larger than any distance in the block and it is advisable to make it at least twice this maximum distance.

NAS-NRC Panel Proposes Studies of Solid Earth

Washington, January 21—A panel of the National Academy of Sciences—National Research Council today outlined a vigorous program of fundamental research for the decade ahead to chart the invisible and still largely unknown interior of our own planet.

The scientific program carries forward efforts to understand crustal processes and the depths, far below the range of direct penetration, where the huge forces that shape the face of the earth are born.

The enlarged research effort proposed by the panel to trace the flow of matter and energy from the mantle to the crust, and thus to develop a coherent picture of the inner workings of our planet, is described in a 198-page report, *Solid-Earth Geophysics: Survey and Outlook.*

A program of deep drilling for geophysical purposes at selected land sites—to depths of two to five miles—is recommended in addition to the Project Mohole effort to drill through the ocean floor to the mantle. Such deep holes both permit the recovery of materials for laboratory study and provide a means for introducing instruments to measure heat flow, seismic activity, magnetism, radioactivity, and other properties of the interior.

As an outgrowth of the Mohole, the panel notes, techniques are being perfected for drilling in deep water that will also make it possible to drill many shallow holes in the sediments of the ocean floor, through which it is hoped to trace the evolution of the oceans and the life contained in them.

Triple-Access Discfile

Data Products Corporation delivered a large capacity, high-speed-transfer, triple-access DISCfILE (Trademark) to Scientific Data Systems. Announcement of the delivery and acceptance by the Santa Monica, California, computer manufacturer was made by Raymond Stuart-Williams, vice president and general manager of Data Products' DISCfILE Division.

The DISCfILE, designated Model 5025, interfaces with an SDS 930 computer. It features, in addition to the normal simultaneous dual-access channels, a third channel which accesses data by fixed read/write heads. Data access time is nominally equal to one-half a disc revolution. This fast access storage may be used for address tables, or function tables, frequently-used subroutines, monitor print-out for diagnostic purposes or buffer interface for high-speed data transfers between two computers.

Following acceptance of the triple-access DISCfILE, Scientific Data Systems placed an order for a dual-access 5025 unit for use with another 930 computer.