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Roll Film Mensuration

All measured distances greater than three inches fell within two parts in 10^4 of the theoretical value.

ABSTRACT: *The measurement of* x-y-*coordinate distances on roll film is one of the important factors in the design of high speed photographic data reduction viewers. High measuring accuracies are required for measuring distances on the film in both the* x *and* y *direction. Space limitations may require that the measurement in the* x-*direction be performed while transporting the film from spool to spool. An x-y-measuring system with digital readout in a film drive assembly with constant tensile force per film width is described and the measuring error sources pointed out. Equations evaluate measuring error contributions due to tensile force, mechanical instability of film, measuring roller diameter, gearing, and shaft encoder accuracy.*

INTRODUCTION

GLASS DIAPOSITIVES are commonly used to obtain distance measurement data for photogrammetric purposes. For photointerpretation purposes, however, it is often necessary to measure directly on film.

Measurement, while significant for interpretation, is by no means the most important function of high speed photographic data reduction equipment. Almost always rear projection is employed with high densities of visible energy passing through large film gates in order to obtain optimum brightness at high magnifications. Consequently large amounts of energy will be absorbed by the film resulting in film dimension changes, a fact not quite compatible with precision measurement.

Distances to be measured on roll film, in a lengthwise direction, may be so long that the X-Y-table concept is rendered impractical. A rather crude measurement of distance may be required over a full 1,000 foot film roll. Relatively accurate measurements may be required over distances of more than two feet along the film roll. This calls for accurate measurement of film movement in order to obtain distances in this direction, most often without timing or fiducial marks, sprocket holes or other such distance indicators. The only criterion for the magnitude of the distance between two points on the film is the movement of the film as the two points are successively referenced to the same viewing or projection point. Since the movement of the film is translational through the viewing area, it can be imagined that orthogonal scales are clamped to the film and distances read directly from the scales. However, because of the scale lengths required this form of measurement is ruled out as impractical.

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Assuming that the film is contained on spools and mounted in a film drive mechanism which allows projection or direct viewing, it is feasible and practical to translate the entire film drive mechanism to obtain the Y-coordinate measurement. The maximum distance to be measured in the Y-direction does not exceed the width of the widest film which in the case under investigation is 9.5 inches.

A display of the Y-coordinate measurement can be easily obtained by coupling a shaft encoder to the Y-direction drive mechanism through the proper gearing, and transmitting the electrical pulses from the shaft encoder to an electronic counter with digital readout.

The magnitude of the measurement in the X-direction can be much larger and may amount to more than two feet. If film footage or frame measurement is included, the magnitude may equal the total length of the film, for instance 1,000 feet. Displacing the entire film drive mechanism over such long distances is obviously impractical.

The translational motion of the film in the X-direction must be converted into a form best suited for measurement. Conversion into rotary motion with a revolution counter or shaft encoder in conjunction with an electronic pulse counter and digital readout on the control panel seems to be the most logical solution. A measuring roller around which the film is wrapped can serve as the converter from translational to rotational motion. Such a scheme requires not only a sophistication of the film drive but also an intricate knowledge of all the factors affecting the choice of the roller diameter and of the errors affecting the accuracy of the measurement.

The Film Drive

When film is required to drive a highly polished measuring roller, with gears and a shaft encoder coupled to it, no slippage can be allowed to occur between roller and film. The film is in contact with the roller over a certain angle. The torque which can be imparted to the roller is not only a function of the contact angle and the friction coefficient (film-steel) but is directly proportional to the film tensile force. A film drive (see Figure 1) should therefore not only transport film from one spool to the other but provide a certain constant amount of film tension independent of the film speed or film spool diameter.

Two systems are combined, a balanced film tension system consisting of two controlled torque motors coupled to the film spools providing constant film tension, and a drive system consisting of a variable speed motor driving a roller which is in contact with the film. This combination was found to assure successful measuring system performance.

X-Coordinate Measurement

The nominal diameter of the measuring roller is calculated by dividing the desired film distance/roller revolution factor by π. This diameter, however, assumes that the film has zero thickness and is not under tension. The roller radius has to be made smaller by an amount corresponding to the distance of the film's neutral plane from the roller surface. This distance is a function of the type of film used, the environmental conditions, and the amount of film tension.

The neutral plane of the film is that plane in the interior of the film in which neither elongation nor contraction due to bending or the application of tensile force occurs.

X-Coordinate Measuring Error

The X-coordinate measuring error can be subdivided into three components:

1. Dimensional error component incurred along the length of the film due to film parameter changes: $\Delta s/s$.

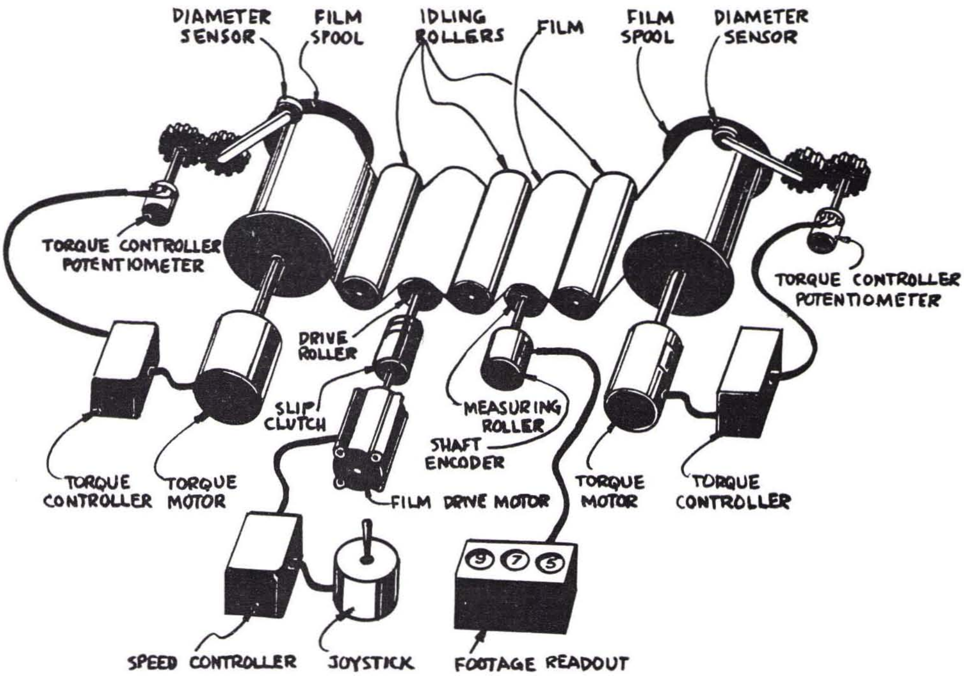

FIG. 1. Schematic view of the film drive.

2. Conversion error component associated with the effective measuring roller radius: $\Delta r/r$.
3. Encoding error component inherent in the gearing, encoder and readout: $\Delta\psi$

Concentrating on the first two error components it is shown that according to Figure 2 the relationship between translational and rotational motion is

$$\theta = s/r \tag{1}$$

s = length of film element on roller circumference
r = effective roller radius
θ = angle subtended by s.

Differentiating Equation 1 and dividing by Equation 1 results in the expression relating the error components

$$\frac{\Delta\theta}{\theta} = \frac{\Delta s}{s} - \frac{\Delta r}{r} \tag{2}$$

in which $\Delta\theta$, Δs, Δr are small changes of the above parameters. The amount of error is always expressed in parts per 10^4 because all measuring errors are related quantities having this order of magnitude.

Dimensional Error Component

Three factors enter into the dimensional error component: elongations (contractions) caused by changes in the film tension, the temperature and the relative humidity:

$$\frac{\Delta s}{s} = \epsilon_{TF} + \epsilon_T + \epsilon_{RH}.$$

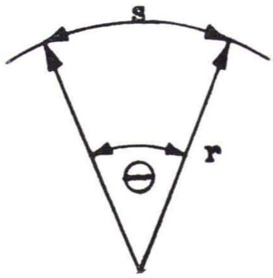

FIG. 2. Conversion into Rotational Motion.

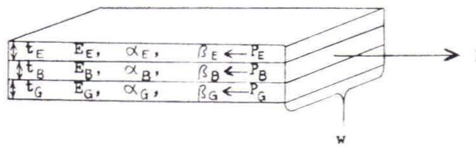

FIG. 3. Film strip under tension P and subject to changes in temperature and relative humidity. P = tensile force; w = film width; t = thickness; E = modulus of elasticity; α = temperature coefficient of expansion; β = humidity coefficient of expansion. Subscripts B, G, E = base, gel backing, emulsion.

Film consists of three layers: the base, the gel backing, and the emulsion (the gel backing may be absent in some films). It must be regarded as a laminated structure for which the laws of Strength of Materials apply (Figure 3). The elongations (relative length changes) due to tensile force, temperature, and relative humidity changes thus become

$$\epsilon_{TKF} = \frac{P}{w(t_B E_B + t_G E_G + t_E E_E)} \tag{3}$$

$$\epsilon_T = \frac{t_B E_B \alpha_B + t_G E_G \alpha_G + t_E E_E \alpha_E}{t_B E_B + t_G E_G + t_E E_E} \Delta T \tag{4}$$

$$\epsilon_{RH} = \frac{t_B E_B \beta_B + t_G E_G \beta_G + t_E E_E \beta_E}{t_B E_B + t_G E_G + t_E E_E} \Delta RH. \tag{5}$$

The evaluation of these equations requires a knowledge of the numerical values of the film parameters which have been published (see Appendix and References 1, 2, 3, 4, 5). Differentiating Equations 3, 4 and 5 and dividing by the equations results in the related errors as functions of film parameter changes. From this weighting functions of the form

$$\frac{\Delta \epsilon_{TF}}{\nu \epsilon_{TF}} = K_1 \frac{\Delta E_B}{E_B} + K_2 \frac{\Delta t_B}{t_B} + \cdots$$

are developed. These indicate the accuracy to which a parameter has to be known and hence the amount of confidence that can be put in an equation. Equation 5 was found to be very unreliable because of its heavy dependence on the moduli of elasticity of the gel backing and emulsion which are both not clearly defined. The amount of elongation must therefore be taken directly from film measurements (see Reference 2).

THE CONVERSION ERROR COMPONENT

Three factors enter into the conversion error component: the position of the neutral plane of the film, the roller machining tolerance (including eccentricity), and the thermal expansion of the roller cylinder.

$$\frac{\Delta r}{r} = \frac{\Delta c}{r} + \frac{\Delta R}{r} + \frac{\Delta R_T}{r}.$$

The position of the Neutral Plane is found with reference to Figure 4.

$$\Delta C = \frac{t_B(t_B + 2t_G)E_B + t_G^2 E_G + t_E(2t_B + 2t_G + t_E)E_E}{2(t_B E_B + t_G E_G + t_E E_E)} - R\frac{\Delta s}{s}$$

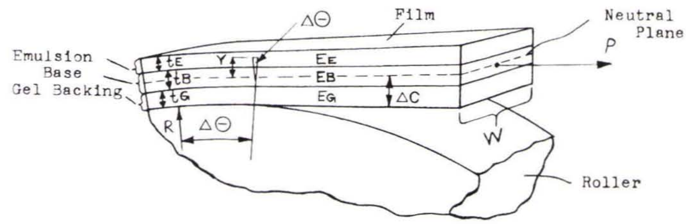

FIG. 4. Identification of the Neutral Plane.

Again a weighting function of the form

$$\frac{\Delta(\Delta C)}{r} = K_3 \frac{\Delta E_B}{E_B} + K_4 \frac{\Delta t_B}{t_B} + \cdots$$

can be derived showing the "weight" of the individual parameters.

The roller machining tolerance can be readily held to within 1.5 parts in 10^4. The maximum error caused by eccentricity will amount to twice the eccentricity value for the worst case of half a measuring roller revolution. The error is, of course, not cumulative.

The thermal expansion is based on the thermal coefficient of linear expansion of steel.

ENCODING ERROR COMPONENT

High precision gearing is used for the measuring application. The gear error represented by the total composite error and the backlash error cannot be expressed in a percentage but must be referred to the resolution increment (least significant bit) of the shaft encoder coupled to the output gear (see Figure 5). Assuming such a resolution increment to be 0.0005 inch, the gear error may amount to 20% to 40% of the encoder resolution. The encoder itself has an uncertainty of ± 1 least count.

NUMERICAL ERROR EVALUATION

The computation of error values for a typical polyester type thick base Topographic film results in

$$\epsilon_{TF} = 0.7 \times 10^{-4} \quad \text{for } P = 2 \text{ lb}; \ w = 9.5 \text{ in.}$$

$$\epsilon_T = 0.15 \times 10^{-4} \times \Delta T$$

$$\epsilon_{RH} = 0.16 \times 10^{-4} \times \Delta RH$$

$$\frac{\Delta C}{r} = 23.2 \times 10^{-4}$$

$$\frac{\Delta R}{r} = 1.5 \times 10^{-4}$$

$$\frac{\Delta R_T}{r} = 0.1 \times 10^{-4}.$$

The relatively high value related to the position of the neutral plane of the film c/r makes it imperative to allow for it in the determination of the measuring roller diameter. c/r is a film type constant. All measurement readings therefore are referenced to one particular film type, the most commonly used one. For measurements with a different film type and also at different values of temperature and relative humidity scale factors are determined and the readings modified accordingly.

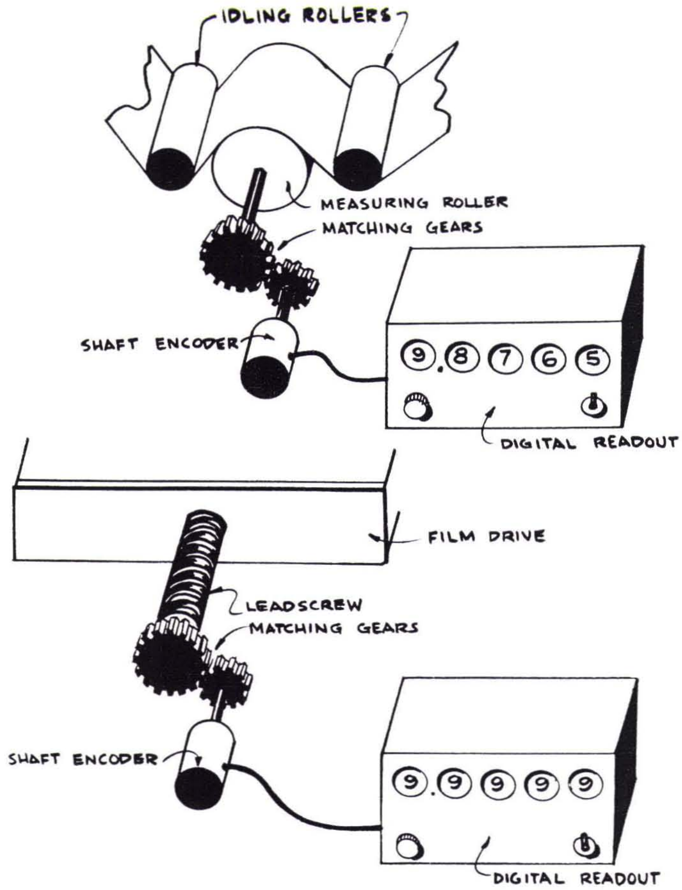

FIG. 5. Roller versus X-Y-table measurement.

Y-COORDINATE MEASUREMENT

It is assumed that the digital readout of the Y-coordinate measurement is also derived from a shaft encoder coupled through gearing to a lead screw in a conventional manner (see Figure 5). Then the dimensional and encoding error components correspond to those of the X-coordinate measurement. The conversion error component is replaced by the error component of the lead screw which has a machining tolerance of the same order as that of the measuring roller tolerance. Also the thermal coefficients of linear expansion of the roller and the lead screw are the same.

The only difference between measurements based on the lead screw and those based on the roller is that the former does not require a scale factor for the film type used if different from the reference film. Scale factors for different values of temperature and relative humidity have to be used in both cases for accurate measurements unless the environment is controlled.

EXPERIMENTAL RESULTS

Grooves of 25 micron width, $\frac{1}{4}$ inch length were ruled into the emulsion of film every 3 inches over 18 inches of film. The ruling was performed on a high accuracy ruling machine. The film was then loaded into the film drive assembly and the hairline in the eyepiece of a 100-power microscope focused on the edge of a groove. The digital readout was reset to zero and the film moved to the other lines and readings taken.

All readings of distances over 3 inches fell within 2 parts in 10^4 of the theoretical value. Eccentricity effects and the readout resolution permitted the 3-inch readings to fall just outside the tolerance limit. The reproducibility stayed within the tolerance limits independent of film speed and acceleration to speeds in a range of from 150 feet per minute down to 0.0003 inch per second. It is obvious therefore that no slippage had occurred.

References

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