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A Triangulation Technique for Linear Objects in Space

A scheme is devised to overcome the difficulty of identifying point-images on a nebulous chemical cloud in the study of winds aloft.

Abstract: *Corresponding points on films, taken from two different sites, of a linear object in space cannot be paired due to unknown foreshortening. The usual triangulation techniques must be replaced by an iterative scheme to solve the problem. The particular methods developed in this paper are most useful when graphical checks of analysis are desired. Furthermore, the methods are tailored for easy computer programing and the closure technique of the iteration allows easy and quick convergence in pairing points. These methods have been used on chemical cloud releases in space to determine winds and wind shears.*

Introduction

The release of a chemical cloud that can be photographed from the ground constitutes one of the tools most often used in the study of the upper atmosphere. If the cloud is released as a point, then the height-range relation can be obtained simply by double camera triangulation. If, on the other hand, the release is as a trail the problem is no longer as simple because equivalent points on the separate films cannot be determined by visual inspection.

In most cases the height-range relation is the information desired but in the diffusion studies being undertaken by the authors, correlation between trail points on the film from one site with trail points on the film from another site is needed. Selected pairs of points will then be scanned with a densitometer for diffusion data.

The problem of correlation is not indeterminant in most cases. This paper illustrates one method of solving the problem. The method is quite general, though of necessity it is usually tailored to fit the particular experimental equipment used in conjunction with the application of the method.

Experimental Arrangement of Apparatus

Normal station arrangement is three separate stations at least 150 kms. apart. Nominal height of the clouds is from 90 to 200 km. The lower part of the trail is usually too turbulent to permit good position determination but above about 105 km. the center line of the trail can usually be determined. The clouds consist of a chemiluminescent material such as sodium and the trail is fairly well defined. The cameras photographing the cloud place a grid on the film so that film position can be accurately determined. The photographs are made when star background will register on the film. From the star background the azimuth and elevation of the camera axis can be calculated very accurately. In addition, the station longitude and latitude are precisely defined.

Description of the Point Correlation Method

Certain simplifying assumptions will be made. The first is that there is no refraction due to the atmosphere.[1] This may or may not be used by other investigators at their own discretion. Usually the correction is quite small. Secondly, it is assumed that there are no camera orientation corrections necessary. And finally, there is the assumption which is not apparent at the outset but will become obvious later on, i.e., there are no so-called "double points."

Looking at Figure 1, assume we are interested in the cloud at trail Point 1. From the stars α, β, and γ, the camera center line orientation through $\Gamma(A)$ is given so that the elevation and azimuth of $\Gamma(A)$ are known. Point 1 is positioned on the film by

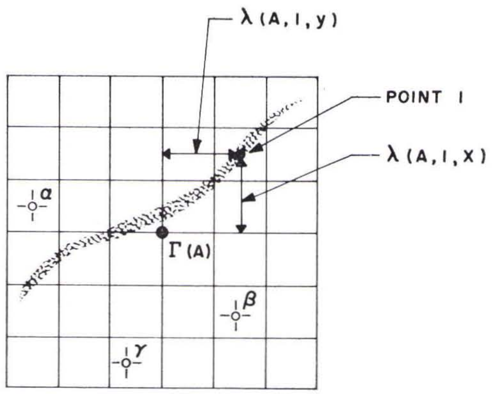

FIG. 1

the coordinates $\lambda(A, l, x)$ and $\lambda(A, l, y)$. This is called a pair point with the ith pair point of film A given by $\lambda(A, i, x)$ and $\lambda(A, i, y)$.

The film coordinates of the pair point are transformed into a correction to the center line elevation $\epsilon(A, 0)$ and azimuth $\zeta(A, 0)$. (Figures 2 and 3) Physically, the camera film is behind station point A (center of lens). Inspection of Figure 2 indicates the film and its geometrically inverted twin, with all the pertinent parameters. Numerical values are usually obtained from the twin. The twin is fictitious, of course, but will be used and simply referred to as the twin. Figure 3 shows the twin and the corrections in more detail. Right angles are indicated by the customary corner symbol at the proper location. The correction to the azimuth $\zeta(A, 0)$ is given by $\Delta\zeta(A, i)$, i.e.,

$$\Delta\zeta(A, i) = \arctan \left\{ \frac{\lambda(A, i, x)}{F \cos \epsilon(A, 0)} \right\}.$$

The change in elevation is more complicated and requires two corrections. The elevation of a trail point as depicted on the twin film is composed of an angle to the x''-axis of the film called $\epsilon(A, i, 0)$ and an incremental angle $\Delta\epsilon(A, i)$ formed by the point of the x''-axis where a line passing thru the desired trail point and perpendicular to the x''-axis, cuts the x''-axis.

The angle $\epsilon(A, i, 0)$ is given by

[1] For corrections due to atmosphere and fiducial plate refraction see: C. G. Justus, H. D. Edwards, and R. N. Fuller, "A Method Employing Star Backgrounds for Improving the Accuracy of the Location of Clouds or Objects in Space," Photogrammetric Engineering, Vol. XXX, No. 4 (July, 1964), p. 594.

$$\epsilon(A, i, o) = \arccos)\kappa/\mu$$
$$= \arccos \sqrt{\frac{[F \cos \epsilon(A, o)]^2 + [\lambda(A, i, x)]^2}{F^2 + [\lambda(A, i, x)]^2}} \tag{1}$$

and $\Delta\epsilon(A, i)$ by

$$\Delta\epsilon(A, i) = \arctan \left\{\frac{\lambda(A, i, x)}{\kappa}\right\}$$
$$= \arctan \left\{\frac{\lambda(A, i, x)}{\sqrt{F^2 + [\lambda(A, i, x)^2]}}\right\}. \tag{2}$$

Thus, trail point i, as seen by the camera of the film giving the pair values $\lambda(A, i, x)$ and $\lambda(A, i, y)$ has the elevation and azimuth of

$$\zeta(A, i) = \zeta(A, o) + \arctan \left\{\frac{\lambda(A, i, x)}{F \cos \epsilon(A, o)}\right\} \tag{3}$$

$$\epsilon(A, i) = \epsilon(A, i, o) + \arctan \left\{\frac{\lambda(A, i, y)}{\sqrt{F^2 + [\lambda(A, i, x)]^2}}\right\}. \tag{4}$$

Referring to Figure 2, the line in space $R(A, i)$ connecting the trail point to point A, the station point, is called a range line. For every point there is a range line.

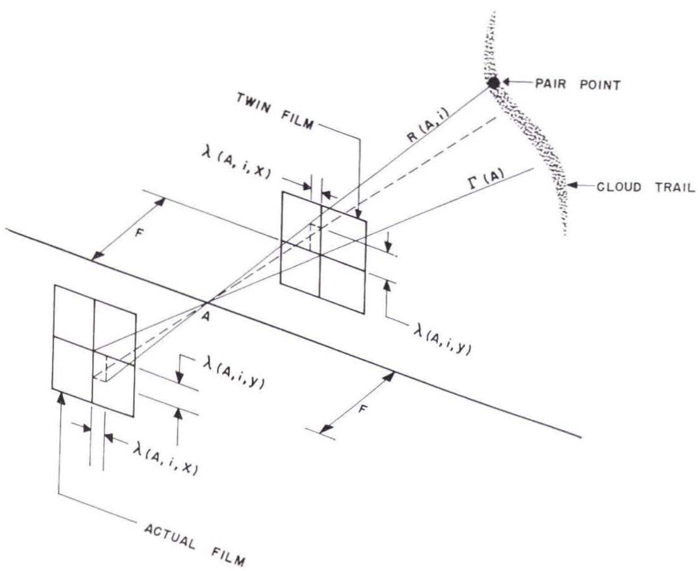

FIG. 2

This range line intersects at least one trail point. If it intersects two trail points on Film B, it is called a double point range line. Obviously, under peculiar conditions, it may intersect any number of trail points. However, it will be assumed in this paper that all range lines are single point range lines.

If it were possible to see a range line in space, then from Station B the film would be as seen in Figure 4. Call the range line of the ith pair point of Film A, $R(A, i)$, and its appearance on Film B, $R(A, i, B)$.

$R(A, i, B)$ can be mathematically described on Film B because on Film A the

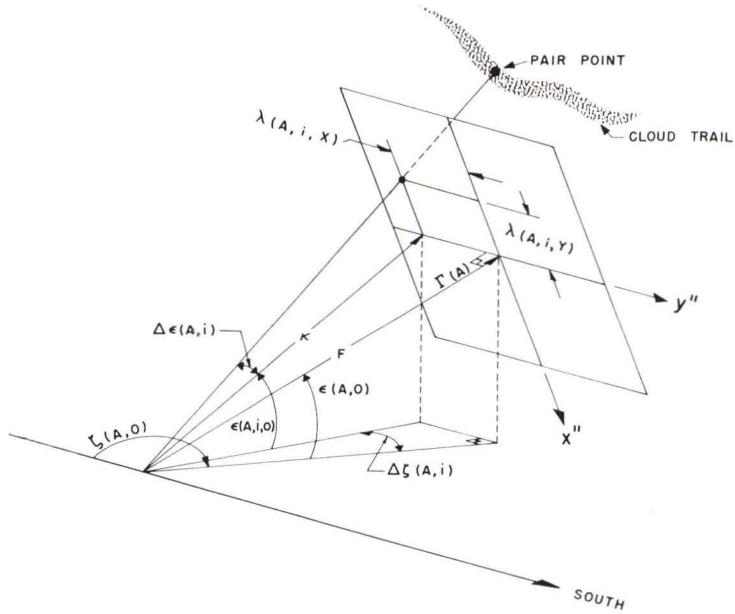

FIG. 3

range line parameters are precisely determined. On Film *B* it must intersect the trail at the same point. Numerous methods exist for determining this intersection. The range line can be drawn on Film *B* or the whole thing transferred to another setting such as graph paper. Essentially, the methods fall into two groups, graphic and analytical. The amount of work required in describing a range line from site *A* on Film *B* is quite laborious arithmetically and algebraically and, if many points are desired, the use of a computer may be required.

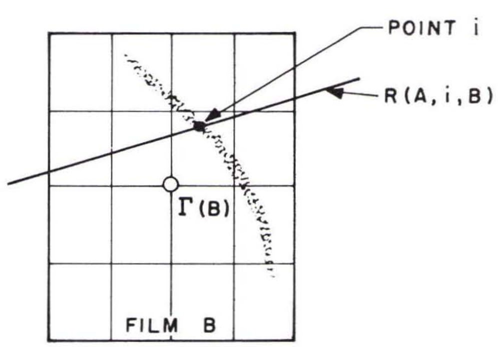

FIG. 4

GRAPHIC METHOD

If a graphic method is used then the solution is as exact as the drawing can be read and/or drawn. If an analytical method is used, then there must be some means of describing the trail location on the film mathematically. The films used by the authors were measured for density on a densitometer equipped with a stepping plate which was capable of steps of 1.25 microns. On a 5-inch film this gave more than enough points to describe the trail. This easily dictated the method, which was:

1. Obtain n points on Film A describing the trail.
2. From 1, select one point of interest.
3. From 2 describe analytically on Film B the corresponding range line, i.e., give its linear equation in terms of x'' and y'' coordinates on B.
4. Describe the trail on B by m pair points. m and n need not be equal.
5. In some systematic manner substitute the points on B into the Equation in 3 above. One such pair will render the equation nearly null in comparison to the other pair points on B

It would indeed be fortuitous if the pair points matched exactly. This is neither necessary or to be searched for. All that is required is that points match within the accuracy desired for further analysis, such as wind velocity at a given altitude, etc.

Analytical Method

If great accuracy is desired, then a variant of the method can be used in that two separate sets of pair points may be used, one in a coarse grid and one in a fine grid. Since the fiducial grid lines record on the densitometer trace, the position and angle of

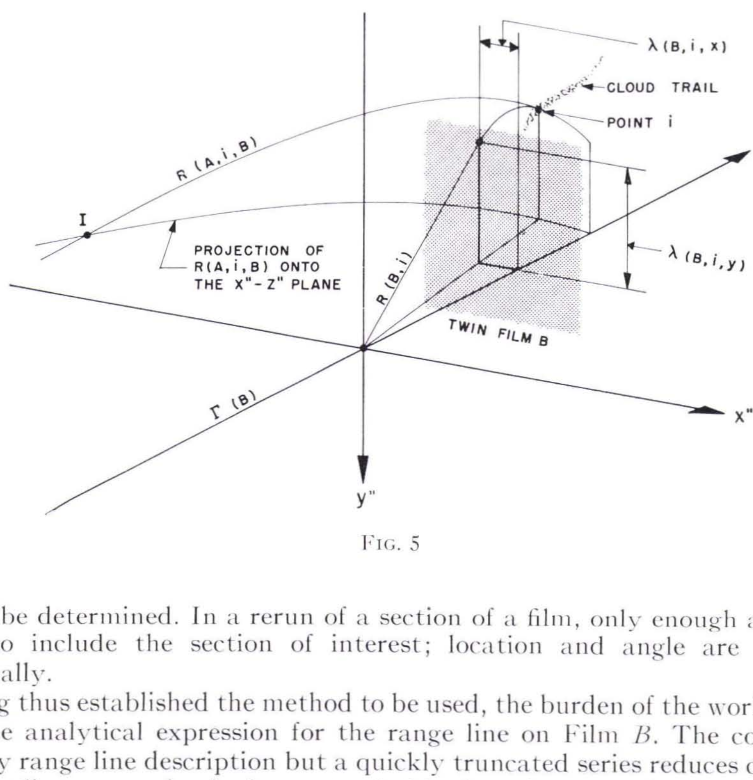

Fig. 5

trace can be determined. In a rerun of a section of a film, only enough area need be covered to include the section of interest; location and angle are recomputed automatically.

Having thus established the method to be used, the burden of the work is in determining the analytical expression for the range line on Film B. The computer can handle any range line description but a quickly truncated series reduces computation time; and a linear equation is the most suitable. In order to emphasize that the introduction of refraction does not alter the technique qualitatively (however it will increase computation time) the method will be illustrated with nonlinear range lines as shown in Figure 5, but will be described analytically using linear range lines.

The range line from a trail point in space to either Station A or Station B is unique, even though it be curved. Furthermore, for any given description of the refraction the type of curve in space is uniquely defined. A transcendental function might confound the computer but regardless of this, if $R(A, i, B)$ can be given functionally it can be given functionally in any defined coordinate system. The system chosen consists of the camera axis, z'', and the film coordinates. Note should be made of the fact that y'' is positive downward in order to preserve right hand systems. Furthermore, some exaggeration of the range line to B must be made in the drawing to insure the range being linear within a distance of the order of the focal length of the camera within the immediate vicinity of the camera. $R(A, i, B)$ is a range line from A as seen from B. $R(B, i)$ is an arbitrary range line from B intersecting both the film and the cloud. The point where $R(B, i)$ intersects the film is determined by an iterative scheme which follows.

Neglecting refraction and using linear range lines, $R(A, i, B)$ is obtained by a few straightforward coordinate transformations.

At station A there is a camera system called the A'' system which has been previously described. There is also the topocentric system at A called the A'-system. In the A'-system, in order to preserve right handedness,

$+x_A'$—westward

$+y_A'$—southward

$+z_A'$—direction of zenith (upward).

A similar pair of coordinates occurs at station B. In addition the geocentric system gives the station location by the longitude θ and the latitude ϕ. For the purposes of this paper the earth was assumed to be a sphere although in actual calculation made by the authors, the earth was treated as an oblate spheroid and the appropriate corrections made.

Using linear range lines the following items are all that is required:

1. Direction cosines of a range line from A in the B''-system.
2. Direction cosines of the base line D in the B''-system. D is the straight line connecting station A and station B.
3. Length of D.
4. Intercept in the B''-system of range line from A and the base line D.

From Items 2 and 3 the position of station point A in the B'' system is found. Since any range line from A passes through this station point, the range line intercepts in the B''-system can be determined. Denoting the matrix transformation from system C to system D by $T(C/D)$, then the transformation $T(A''/A')$, $T(A'/G)$, where G denotes the geocentric system, $T(G/B')$ and $T(B'/B'')$ are required. Ultimately, the transformation $T(A'/B'')$ is required but its actual calculation is best accomplished by the computer since it involves matrix multiplication.

The direction cosines of a range line from A to the ith pair point as seen from A' are denoted by $\cos(A, i, A'(j))j=x, y$, or z. $T(A'/G)$ then is given by

$$\cos(A, i, G(j)) = (A'/G, ji)\cos(A, i, A'(i)). \tag{5}$$

The $(A'/G, ji)$ are given in Table 1 are the inversion, say for $(G/B', jk)$ in Table 2. The Transformation $T(B'/B'')$ is analogous to $T(A'/A'')$ and is given in Table 3. In actual practice

$$(A'/B'', jn) = (A'/G, ji)(G/B', ik)(B'/B'', kn). \tag{6}$$

TABLE 1

$\cos\theta$	$\sin\phi\sin\theta$	$\cos\phi\sin\theta$
$-\sin\theta$	$\sin\phi\cos\theta$	$\cos\phi\cos\theta$
0	$-\cos\phi$	$\sin\phi$

TABLE 2

$\cos\theta$	$-\sin\theta$	0
$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$-\cos\phi$
$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$\sin\phi$

The transformation from the A''-system to the B''-system can be done with no more information about the oblate spheroid earth than angle relation between the A'' and B''-coordinate systems and the geocentric system. To obtain the base line distance D, additional information is required but can be found by the usual techniques. Assuming that the length and direction cosines in the B''-system of the base line D, are known, the equation of the range line projection on the film at B is determined.

Inasmuch as both a range line and the base line pass through the station point A,

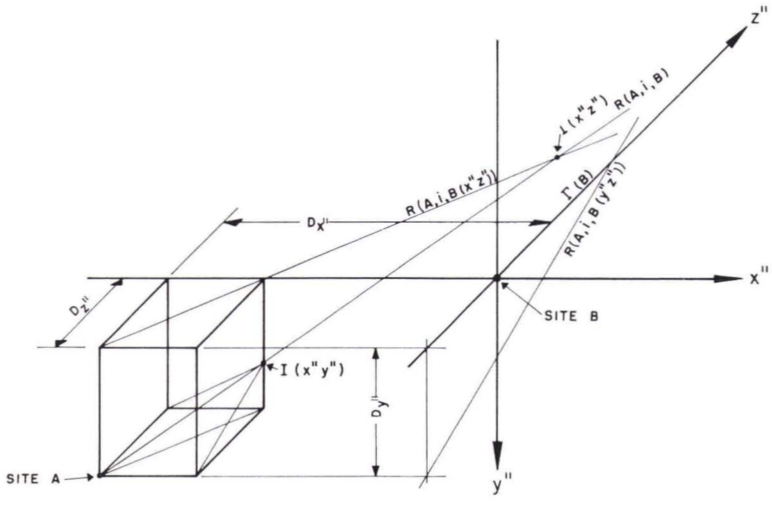

FIG. 6

this gives a known point on the range line. From Figure 6 the quantities $D_{x''}$ and $D_{z''}$ are given by:

$$D_{x''} = D\cos(D_{x''}B'') \tag{7}$$

$$D_{z''} = D\cos(D_{z''}B'') \tag{8}$$

where $\cos(D_{x''}B'')$ and $\cos(D_{z''}B'')$ are the respective direction cosines of D in the B''-system.

The equation of the (x'', z'') projection is given by the equation

TABLE 3

$-\cos\xi$ (A, i)	$\sin\xi$ (A, i)	0
$-\sin\epsilon(A, i)\sin\zeta(A, i)$	$-\sin\epsilon(A, i)\cos\zeta(A. i)$	$-\cos\epsilon(A, i)$
$-\cos\epsilon(A, i)\sin\zeta(A, i)$	$-\cos\epsilon(A, i)\cos\zeta(A, i)$	$\sin\epsilon(A, i)$

$$\frac{x'' - D_{x''}}{\cos(A, i, B''(x))} = \frac{z'' - D_{z''}}{\cos(A, i, B''(z))} \tag{9}$$

where $D_{x''}$ and $D_{z''}$ are the x''-axis and z''-axis projections of the distance to station point A from B as given above. Similarly for the (y'', z'') projection,

$$\frac{y'' - D_{y''}}{\cos(A, i, B''(y))} = \frac{z'' - D_{z''}}{\cos(A, i, B''(z))}\,. \tag{10}$$

The cloud point T is projected on the film at B by the range line $R(B, i)$. (See Figure

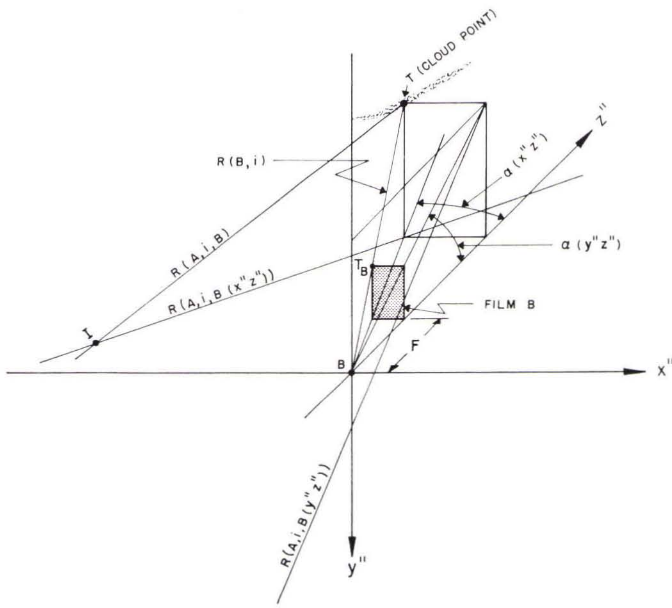

FIG. 7

7.) Its x''-location on film B is obtained through the range line projections of $R(B, i)$ which have the equations

$$x'' = z'' \tan \alpha(x''z'') \tag{11}$$

and

$$y'' = z'' \tan \alpha(y''z'') \tag{12}$$

with the tangents being defined by

$$\lambda(A, i, B''(x)) = F \tan \alpha(x''z'') \tag{13}$$

$$\lambda(A, i, B''(y)) = F \tan \alpha(y''z''). \tag{14}$$

Substitution from Equations (11) and (13) into Equation (9) and solving gives

$$\frac{1}{z''} = \frac{\dfrac{\lambda(A, i, B''(x))}{F \cos(A, i, B''(x))} - \dfrac{1}{\cos(A, i, B''(z))}}{\dfrac{D_{x''}}{\cos(A, i, B''(x))} - \dfrac{D_{z''}}{\cos(A, i, B''(z))}}\,, \tag{15}$$

and from Equations (12) and (14) into (10) fives

$$\frac{1}{z''} = \frac{\dfrac{\lambda(A,\, i,\, B''(y))}{F\cos(A,\, i,\, B''(y))} - \dfrac{1}{\cos(A,\, i,\, B''(z))}}{\dfrac{D_{y''}}{\cos(A,\, i,\, B''(y))} - \dfrac{D_{z''}}{\cos(A,\, i,\, B''(z))}}. \tag{16}$$

Equating (15) and (16) one obtains

$$a\lambda(A,\, i,\, B''(x)) + b\lambda(A,\, i.\, B''(y)) + C = 0 \tag{17}$$

where

$$a = \frac{\dfrac{D_{y''}}{\cos(A,\, i,\, B''(y))} - \dfrac{D_{z''}}{\cos(A,\, i,\, B''(z))}}{F\cos(A,\, i,\, B''(x))} \tag{18}$$

$$b = \frac{\dfrac{D_{x''}}{\cos(A,\, i,\, B''(x))} - \dfrac{D_{y''}}{\cos(A,\, i,\, B''(z))}}{\cos(A,\, i,\, B''(z))} \tag{19}$$

$$c = \frac{\dfrac{D_{x''}}{\cos(A,\, i,\, B''(x))} - \dfrac{D_{y''}}{\cos(A,\, i,\, B''(y))}}{\cos(A,\, i,\, B''(z))} \tag{20}$$

a, b, and c being constants.

From here on the technique is routine. The computer program determines a, b, and c for a given range line and the points on Film B are searched for a best fit. The machine logic would consist of substituting the values for a pair point on B in Equation (17).

For an incorrect point the equation will not be null but will have a residual. As a better point is substituted the residual will decrease. If the residual increases a point in the opposite direction should be picked next. The best two points on B will be those just before and just after the residual changes sign. The correct point is somewhere between these. If more accuracy is required, then a finer grid on either Film A or Film B will suffice. In this manner the trail can be described in space by both range and altitude.

Solution of the double point problem is accomplished by a "piggy-back" computer program.

Camera tilt angle and refraction in the fiducial plate have been accounted for by the authors but were not discussed here since they do not alter the technique.

Conclusion

It is not apparent from the analytical development that the residuals are of any particular use. In actuality they turn out to be most helpful, and plots of the residuals versus trail points indicate the location of the true corresponding point quite accurately.

The plane formed by the base line between two stations and a range line from one of the stations should be coplanar to the like plane from the other station. If the range lines are not to the same point, then there will be some angle between these two planes. Other iterative schemes try to reduce this angle to the smallest value.

These methods are in general hard to visualize and do not lend themselves readily to graphical techniques. Furthermore, such methods become quite difficult when double points are encountered.

The method herein described has allowed the authors to display the film on a ground glass projector and plot graphically where the range line from Station *A* appears on the film from Station *B*. This has saved considerable time by eliminating analysis of unnecessary points.

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