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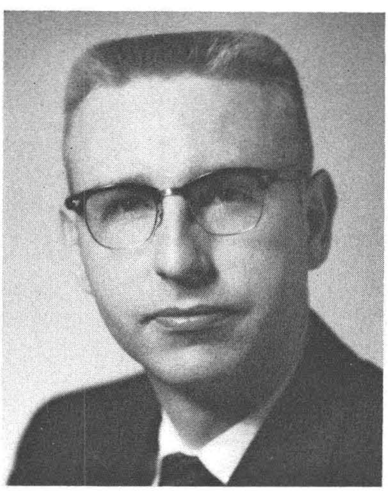
Contour Accuracy versus Spot Heights

Contouring accuracy turned out to be substantially nearer to spot-height accuracy than has generally been expected



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The accuracy of elevations obtained from contouring versus the accuracy of spot-height readings has been the subject of considerable discussion for several years. Many times the question has been asked, "What is the least contour interval that can be accurately plotted, given an accuracy value for spot heighting?" To answer the question, a series of tests was conducted at the U. S. Army Engineer Geodesy, Intelligence and Mapping Research and Development Agency (GIMRADA) in which both contouring and spot-height readings were independently accomplished on several models.

Five models, flown at 15,000 feet over Nebraska, were chosen for the test. Four of the models had terrain differences up to 300 feet and the other a difference of only 100 feet. Each model was compiled on the Wild Autographs A-7 and A-9, the Military High Precision Plotter, and the Multiplex. Seven different operators, ranging in experience from beginners to 15 years, took part in compiling the models. Some operators compiled only one model while others compiled as many as six models. All together, there were a total of 20 models compiled.

Control for each model was obtained from 1:25,000 U. S. Geological Survey maps of the Nebraska area. Elevations were given on the maps at every quarter section,* and horizontal control had to be scaled from the map. Map control was considered to be adequate for the test since a relative comparison was being made between elevations obtained by contouring and elevations obtained by spot heights.

The plottable contour interval for each instrument was found from the following formula:

$$\text{Control Interval} = H/(C\text{-factor}) \qquad (1)$$

where H is the average altitude above terrain.

Since the altitude was fixed in the test conducted, the C-factor chosen for substitution in Formula 1 determined the contour interval. In order to make certain that each instrument was exploited to the utmost, the following optimistic C-factors were chosen:

Instrument	*C-factor*
Multiplex	1,000
High Precision	1,500
A-7	1,800
A-9	1,500

Two horizontal and three vertical control

* Presented at the Annual Convention of the American Society of Photogrammetry, Washington, D. C., March 1965.

* Corners and mid-sides of each one-mile square —*Editor.*

points per model were identified on photographs for the operator to use in absolutely orienting each model. There was no attempt made to dictate the scale used on any of the models by operators. Each operator chose a convenient scale; however, it turned out that almost identical scales were used by various operators on any one instrument.

Each operator performed relative and absolute orientation on models that he compiled.

ABSTRACT: *Contouring accuracy turned out to be substantially nearer to spot-height accuracy than has generally been expected. From statistical values, empirical numbers were calculated for deriving contour intervals directly from spot-height errors. Five models were compiled and spot-height readings were made in each model with each of the Multiplex, Military High Precision, Wild Autograph A-7 and A-9 stereoplotters. Elevations obtained on checkpoints from contours and spot heights were compared to true elevations, and analyzed statistically.*

After orienting a model, the operator chose an index point and contoured the model with just enough planimetry to allow identification and interpolation on check points. As soon as the operator finished contouring, he was given a photograph with the check points identified on it to make sure that enough planimetry had been drawn around each point. Compiled sheets were then removed from the plotting surface of the instrument. While the model was still oriented from compilation, spot heights were read on 25 check points. The operator used the same index point for spot heighting as he had used for contouring.

Evaluation of the tests consisted predominately of relative comparisons rather than absolute accuracies; therefore, extra care in calibration of various parts of the instruments to ensure that proper geometrical relationships existed was not necessary. However, it was believed that principal distance determinations and grid model flatness tests should be performed on each instrument to provide a reasonable amount of confidence in the results. Results from these two tests indicated that each instrument was in satisfactory condition for compilation.

A root-mean-square-error ($RMSE$) and a standard deviation (σ) for both contouring and spot height readings were computed for each model from the following formulas:

$$RMSE = \sqrt{\frac{\sum (DD)}{N}} \qquad (2)$$

$$\sigma = \sqrt{\frac{\sum (EE)}{N}} \qquad (3)$$

where D was the difference between "true" and instrumental elevation, E was the difference between the mean error and the error, and N was the number of points. The root-mean-square-error is considered to express more nearly the *accuracy* of the measurements, whereas the standard deviation expresses a measure of the *precision*.

By making a relative comparison between the results, some of the errors which occur in actual practice do not affect the results of this test. For example, earth curvature, lens distortion, and film shrinkage will all have the same effect on contouring as on spot-height readings. No attempt was made to reduce the errors by correcting for these effects.

Table 1 is a summary of the test results. An examination of Table 1 shows that spot height values are generally better than contouring values. One reason for this is due to having one degree of freedom when measuring spot heights versus two degrees of freedom when contouring a model. When reading spot-heights, only one movement, Z-direction, is required, while movements are made in the X- and Y-directions when contouring. This does not, however, make spot-height readings twice as good as contour values because contours have a tendency to correct themselves. An operator uses his *contouring license* to shape contours when compiling a model. Shaping of contours is legitimate because an operator has first hand knowledge of the terrain he is working on; that is, whether the terrain rises gently or sharply to a peak.

A figure commonly "tossed around" in relating spot-height accuracies to contouring elevations is four. This number is derived in part from the criteria for establishing the vertical accuracy for contours which is stated in the MANUAL OF PHOTOGRAMMETRY as

TABLE 1
AVERAGE VALUES OF TEST RESULTS

Instruments	*Contouring*		*Spot Heights*		*Ratio* $=\frac{Contouring}{Spot\ Heights}$	
	Avg. (*Ft.*)	*RMSE Avg.* (*Ft.*)	*Avg.* (*Ft.*)	*RMSE Avg.* (*Ft.*)	σ	*RMSE*
Multiplex	10.3	13.5	12.9	18.0	0.8	0.8
High Precision	8.3	8.8	7.1	8.9	1.3	1.0
A-7	7.0	7.7	4.9	6.4	1.6	1.2
A-9	9.1	9.5	6.4	9.0	1.6	1.2

follows: "90 per cent of the contours or elevations of points interpolated from contours shall show the true elevations of the ground surface within half a contour interval." In addition to this criteria, it is then arbitrarily stated that elevations obtained from spot heights are twice as accurate as those from contours. Combining one-half from the contour criteria and one-half from the spot-height criteria gives the indication that spot heights are four times as reliable as contours. The results, however, obtained from the test refute this reasoning. The ratios of Table 1 show that standard deviations of spot heights are at best only 1.6 times better than contours.

The mathematical statement for expressing the MANUAL OF PHOTOGRAMMETRY criteria for the least contour interval is:

$$c.i. = 3.3\overline{RMSE_c} \tag{4}$$

where $RMSE_c$ is the root-mean-square-error of contour elevations.

There is some question as to whether Formula 4 should contain the standard deviation or the root-mean-square-error of contour elevations. The pure statistician may disagree with using the root-mean-square-error; however, it does take into account indexing and leveling error whereas the standard deviation would reflect the error about the mean.

By using an empirical constant, the least contour interval can also be expressed in terms of the root-mean-square-error from spot heights as follows:

$$K\,\overline{RMSE_{s.h.}} = c.i. \tag{5}$$

TABLE 2
EMPIRICAL NUMBERS FROM TEST RESULTS

Instrument	*Empirical Number* (*K*)
Multiplex	2.6
High Precision	3.3
A-7	4.0
A-9	4.0

where K is an empirical constant, and $RMSE_{s.h.}$ is the spot-height root-mean-square-error.

Formulas 4 and 5 can be combined and solved for K as follows:

$$K = 3.3(\overline{RMSE_c}/\overline{RMSE_{s.h.}}). \tag{6}$$

Using the results obtained from the test, values of K for each instrument were computed, and are shown in Table 2.

The average empirical value for optical-mechanical plotters is 4.0, and for projection-type plotters is 3.0. These numbers are not to be confused with the value 4 mentioned earlier. The empirical numbers are to be used in finding the least contour interval by multiplying them with the root-mean-square-error from spot heights.

As tests are conducted on other instruments, empirical numbers for other plotters can be added to Table 2. It may be possible then to develop a trend for the empirical number for various groups of plotters.