**Lead screw type comparators are calibrated so as to reduce errors to one micron on each axis for ballistic camera plate measurements.**

RALPH A. GUGEL
*RCA Service Co.—Missile Test Project*
*Patrick Air Force Base, Florida*

# Comparator Calibration

(*Abstract on page 857*)

## INTRODUCTION

THIS PAPER extracts major items of information from RCA Data Reduction TR-63-1, Photogrammetric Data Reduction Analysis—Calibration of Comparators for Ballistic Camera Data, by Ralph Gugel, December, 1963. The paper is a sequel to the previous paper published by Rosenfield (2), in that it presents the statistical analysis necessary to validate the results obtained from Rosenfield's mathematical model.

The author wishes to express his appreciation to Dr. L. Lasman and Mr. George H. Rosenfield for their assistance in preparing this report, and to Mr. James Duncan who programmed the calibrations for an automatic computer reduction. Also, he wishes to acknowledge the efforts of the film readers who contributed to the calibration of the comparators described in the appendices.

## NONPERPENDICULARITY OF AXES

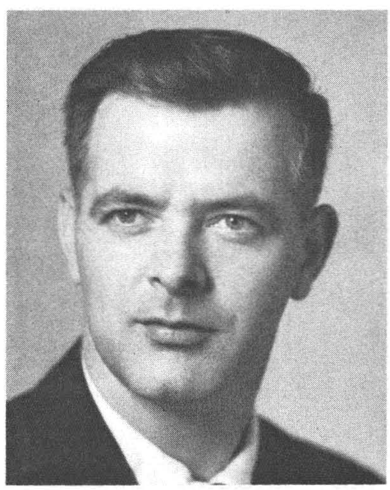
Rosenfield (2) describes nonperpendicularity as "the correction angle $\epsilon$ through which the secondary guide way must be rotated to be perpendicular to the principle guide way." The development presented below is based on the method of inversion so that a calibrated standard is not necessary. This method makes use of the principle that, in the absence of nonperpendicularity error, the $X$-coordinates relative to a fixed coordinate system on the plate will be equal in magnitude but opposite in sign when the plate is inverted about the $Y$-axis of the comparator.

Two points whose $X$-coordinates are widely separated are selected as reference points to establish the grid coordinate system. These points are used to align the plate with the $X$-(principle) axis of the comparator. It is suggested that a minimum of five vertical lines of 11 points each equally spaced over the majority of the comparator measuring format should be selected to read as calibration points to give sufficient redundancy and strength to the solution. A minimum of two independent sets of observations are made on each grid point, including the reference points, in order to determine the setting error. The plate is then inverted about the secondary ($Y$) axis, realigned and the readings repeated.

Each set of readings is corrected for temperature fluctuations and the comparator errors previously calibrated. The average value of the observations and the setting error are computed independently for the direct and inverted positions. The average values of the observations in each position are then translated to the grid origin by the equations

$$X_i' = \frac{2x - x_r - x_1}{2}$$

$$Y_i' = \frac{2y - y_r - y_1}{2}. \qquad (1.1)$$



RALPH A. GUGEL

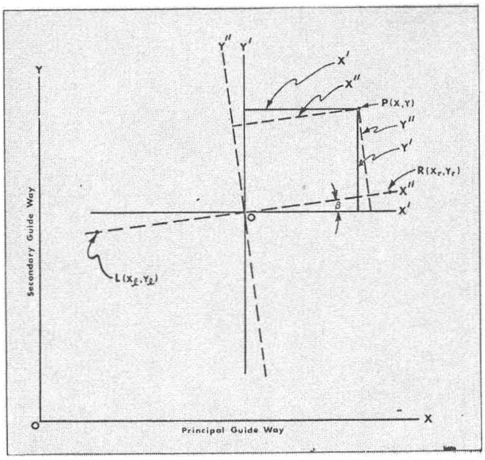



FIG. 1. Relationship between comparator coordinates $(X, Y)$, unrotated grid coordinates $(X', Y')$, and rotated grid coordinates $(X'', Y')$.

The coordinates for each point are then rotated to a rectangular system parallel to the comparator $X$-axis by the equations (Figure 1)

$$X_1'' = x' \cos\beta - y' \sin\beta$$
$$Y_1'' = -x' \sin\beta + y' \cos\beta. \qquad (1.2)$$

in which

$$\sin\beta = \frac{y_r - y_l}{[(x_r - x_l)^2 + (y_r - y_l)^2]^{1/2}}$$
$$\cos\beta = [1 - \sin^2\beta]^{1/2} \qquad (1.3)$$

It is obvious that, in the absence of nonperpendicularity error, the rectangular coordinates $X''$, $Y''$ for the image $i$ will be unaltered by the inversion of the plate about the $Y$-axis except for the sign of the $X''$-coordinate. The average of the $X''$-coordinates of a point for the direct and inverted positions should then be zero for each point. Any deviation from zero is due to nonperpendicularity of the measuring axes of the comparator. If the comparator coordinates in the direct position are denoted by $X_{1i}''$, $Y_{1i}''$ and the coordinates in the inverted position are denoted by $X_{2i}''$, $Y_{2i}''$ then

$$\overline{X}_i = \frac{X_{1i}'' + X_{2i}''}{2}$$
$$\overline{Y}_i = \frac{Y_{1i}'' + Y_{2i}''}{2} \qquad (1.4)$$

where $\overline{X}_i$ represents the error due to nonperpendicularity of the measuring axes of the comparator at a distance $\overline{Y}_i$ from the center of the plate. (See Figure 2.)

The errors may then be fitted to an error model of the form

$$\delta_x = a_0 + a_1 y. \qquad (1.5)$$

The constant term represents the average error in the uncorrected data. The linear coefficient represents the sine of the correction angle $\epsilon$ through which the secondary guide way must be rotated to be perpendicular to the principle guide way. Actually the linear coefficient represents the tangent of the correction angle $\epsilon$, but for the small angles associated with nonperpendicularity of axes it may be assumed that tan $\epsilon$ equals sine $\epsilon$ equals $\epsilon$ in radians.

The coefficients of the error model may be computed using a least squares adjustment on all the points read by

$$a_0 = \frac{\sum_{i=1}^{n} \overline{X}_i}{n}$$

$$a_1 = \text{sine}\,\epsilon = -\frac{\sum_{i=1}^{n} \overline{X}_i \overline{Y}_i}{\sum_{i=1}^{n} \overline{Y}_i^2}. \qquad (1.6)$$

The value for cos $\epsilon$ may then be computed by

$$\cos\epsilon = [1 - \sin^2\epsilon]^{1/2}. \qquad (1.7)$$

With the values for sine $\epsilon$ and cos $\epsilon$ computed, the coordinates of the calibration points corrected for nonperpendicularity error may be computed by

$$V_x = X'' + Y'' \sin\epsilon$$
$$V_y = Y'' \cos\epsilon \qquad (1.8)$$

when the angle $\epsilon$ is very small.

If the method of calibrating a comparator using a grid as given by Hallert (5), is reduced to only the portion concerning perpendicularity, it can be seen that it is equivalent to the method described above. It has been found from experience that either of these two methods provides better results than the method previously used at AMR. The previous method is described by Rosenfield (2) and was derived from a method developed by Zug (3). This method, as employed at AMR, had two basic weaknesses. Firstly the computation of the angle of nonperpendicularity is not based upon a least squares adjustment of the observations on all the data points, but instead is based upon an averaging technique using a number of individually computed angles of nonperpendicularity. Secondly, the error propagation is weak since the reading variance is automatically increased by the double differencing method of the analysis. This earlier method has therefore been replaced by the method described above to calibrate the comparators at AMR.

## Methods of Analyzing the Results

The quality of the data used to determine the coefficients of the error models is determined by the precision of the readings and the quality of the standards used. The quality of calibrations cannot be expected to be any better than the quality of the data fitted to the error models. Therefore, a satisfactory calibration has been achieved when there is no significant difference between the variance of the data fitted to the error model and the variance of the residuals remaining after the adjustment. An $F$-test can be used to determine whether a satisfactory calibration has been achieved.

Some of the error models used for the calibrations are capable of producing zero residuals. If the adjustment is allowed to proceed beyond the point where the two variances are approximately equal, then the error model could be correcting for errors that are due only to reading error. This could actually introduce errors in the corrected coordinates. Although the magnitude of these errors would not exceed the standard error of the data fitted to the error model, it definitely is not desirable to introduce them. Consequently, it is advisable to check the variance ratio prior to the adjustment, and during each step in the adjustment, so that this undesirable overadjustment does not occur.

The $F$-test to determine the proper stopping point is constructed as shown below. The following notation is introduced:

$S_v^2$ = the variance of the residuals.
$S_r^2$ = the variance of the data fitted to the error model.

If $(S_v^2/S_r^5) > F(\alpha,\beta)(0.95)$, one continues with adjustment; otherwise he stops the adjustment.

$F(\alpha,\beta)(0.95)$ is a tabular value from the statistical $F$ distribution and is a function of the degrees of freedom associated with the variance of the residuals ($\alpha$), the variance of the data fitted to the error model ($\beta$), and the probability level (0.95). The degrees of freedom associated with these variances are a function of the error model, the number of points involved, the number of sets of readings, and the operations performed on the readings to provide the data fitted to the error models. The following sections provide the details on computing the necessary values for the $F$-test for each type of calibration.

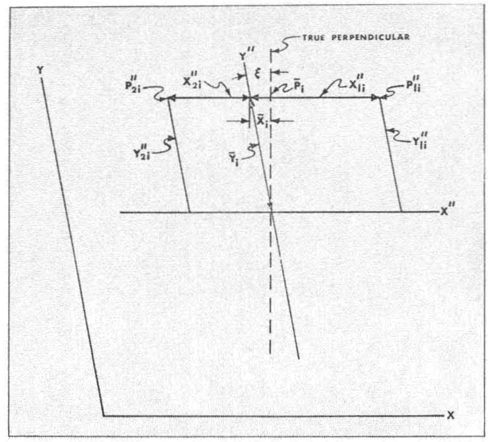



FIG. 2. Determination of Nonperpendicularity.

### PERIODIC LEADSCREW ERROR

The data to be fitted to the error model are the differences between the average measured coordinates of points in the initial position and their respective average measured coordinates in the reset position. If there were no error in the measurements or the leadscrew, these differences would be a constant equal to the distance the scale had been moved for the readings in the reset position. In the calibration procedure it is first necessary to determine whether the variance of these differences from a constant reflect anything other than reading error. The computations for this analysis are as follows.

The variances of the readings in each position are given by

$$\sigma x_1{}^2 = \frac{\sum_i \sum_j (d_{1ij})^2}{n(N_1 - 1)}$$

$$\sigma x_2{}^2 = \frac{\sum_j \sum_k (d_{2ik})^2}{n(N_2 - 1)} \cdot \qquad (2.1)$$

where the subscript 1 denotes the initial position, the subscript 2 denotes the reset position, $d$ is the deviation of each reading from its respective average coordinate,

$i = 1, 2, \cdots, n$; $n$ is the number of points read,
$j = 1, 2, \cdots, N_1$; $N_1$ is the number of sets of readings on each point in the initial position, and
$k = 1, 2, \cdots, N_2$; $N_2$ is the number of sets of readings on each point in the reset position.

The variance of the differences in the average coordinates is

$$S_r^2 = \frac{\sigma x_1^2}{N_1} + \frac{\sigma x_2^2}{N_2}. \tag{2.2}$$

To determine whether any error other than reading error is reflected in the uncorrected differences compute

$$S_{vo}^2 = \frac{\sum_{i=1}^{n} (\bar{D} - D_i)^2}{n - 1} \tag{2.3}$$

where $D$ is the difference between the average coordinate in the reset and initial position for each of the n points and $\bar{D}$ is the average difference between points.

If

$$(S_{vo}^2/S_r^2) > F\{(n-1), [n(N_1 + N_2 - 2)]\}(0.95),$$

then the data reflect period leadscrew error over and above the reading error and the data should be fitted to the error model. If the ratio of the two variances is not greater than the F value then no significant periodic error is present in the leadscrew and the adjustment is not necessary.

If the data are fitted to the sine wave error model the variance of the residuals becomes

$$S_{v1}^2 = \frac{\sum_{i=1}^{n} V_{xi}^2}{n - 3}, \tag{2.4}$$

where $V_{xi}$ represents the residual for each point resulting from the least squares adjustment and the $F$-test becomes

$$(S_{v1}^2/S_r^2) > F\{(n-3)\ [n(N_1 + N_2 - 2)]\}(0.95).$$

The residuals should be tested after each iteration during the least squares adjustment.

### SCALE ERROR AND SECULAR LEADSCREW ERROR

The data to be fitted to the error model are the deviations of the average measured coordinates of the scale graduations from the given calibrated coordinates of the scale. If there were no errors in the measurements, the leadscrew, or the given coordinates of the scale graduations, these deviations would all be zero. In the calibration procedure it is first necessary to determine whether the variance of these deviations reflects anything other than reading error and error in the given coordinates of the scale graduations. The computations necessary for this analysis are as follows.

The variance of the readings is computed by

$$\sigma_x^2 = \frac{\sum_i \sum_j (d_{ij})^2}{n(N - 1)} \tag{2.5}$$

where $d$ is the deviation of each measured coordinate from the average measured coordinate for each point,

$i = 1\ 2, \cdots\ n$; $n$ is the number of points read and

$= 1\ 2, \cdots\ N$; $N$ is the number of sets of readings.

The variance of the data fitted to the error model is then

$$S_r^2 = \frac{\sigma_x^2}{N} + \sigma^2 S_c \tag{2.6}$$

where $\delta S_c$ is the standard error of the scale calibration.

The constant error in the measured coordinates is computed by

$$A_o = \frac{\sum_{i=1}^{n} D_i}{n} \tag{2.7}$$

where $D$ is the deviation of the average measured coordinate from the given calibrated coordinate for each scale graduation.

The variance of the data about the constant error is then

$$S_{vo}^2 = \frac{\sum_{i=1}^{n} V_{xi}^2}{n - 1} \tag{2.8}$$

where $V_x$ is the difference between the average measured coordinate for each point and the constant error $A_o$.

If

$$(S_{vo}^2/S_r^2) > F\{(n-1), [n(N-1)]\}(0.95)$$

then the residuals reflect scale error and/or secular leadscrew error and the data should be fitted to a polynomial. If the ratio of the two variances is not greater than the tabular $F$-value then no significant error other than reading error and error in the calibrated scale is present in the data and the adjustment is not necessary.

If the data are fitted to the polynomial the variance of the residuals becomes

$$S_{v1}^2 = \frac{\sum_{i=1}^{n} V_{xi}^2}{n - p} \tag{2.9}$$

where $p$ is the number of coefficients computed for the polynomial and $V_x$ is the residual for each point resulting from the least squares adjustment.

The $F$-test for the polynomial fit becomes

$$(S_{v1}^2/S_r^2) > F\{(n-p), [n(N-1)]\}(0.95).$$

ABSTRACT: *The procedures for calibrating a precision leadscrew type comparator are described and the results of the calibration of the ballistic plate comparators in use at the Atlantic Missile Range are given. The errors to be calibrated are: (1) periodic leadscrew error, (2) scale error and secular leadscrew error, (3) weave and curvature of the ways and (4) nonperpendicularity of axes. The standard error of the plate coordinates obtained from Mann comparators 422C66 and 422D49 and Wild/Stereo Comparator STK-1 were reduced from 2.2, 4.9 and 5.2 microns to 1.2, 1.2 and 1.9 microns respectively as a result of the calibrations. Each calibration is described in detail giving the reading procedures, calibrated standards necessary, error models fit and methods of analyzing the results.*

The degree of the polynomial to which the data are fitted should be increased by steps of one and the residuals tested as above after each step until the $F$-test indicates no significant errors remain or until a second degree polynomial has been fitted. Experience has shown that the higher order terms of the polynomial do not significantly reduce the errors. If significant errors still remain after the data have been fitted to a second degree polynomial, the data should then be subjected to a harmonic analysis to correct the remaining errors.

When the data are subjected to the harmonic analysis it is necessary to start the adjustment using the first two harmonics because only the cosine coefficient of the last harmonic is used.

For the harmonic analysis the variance of the residuals becomes

$$S_{r2}^2 = \frac{\sum_{i=1}^{n-1} V_{xi}^2}{n - p - 2q + 1}, \tag{2.10}$$

where $q$ is the number of harmonics used in correcting the data.

The $F$-test becomes

$$(S_{r2}^2/S_r^2) > F[(n - p - 2q + 1), n(N - 1)](0.95).$$

Again, the harmonic analysis should be applied by adding one harmonic at a step and the residuals tested after each step to determine when to stop the adjustment.

WEAVE AND CURVATURE OF THE WAYS

The data to be fitted to the error model are the deviations of the average measured coordinates from a straight line. Since the calibration does not use a calibrated standard the procedures used employ direct and inverted measurements to remove errors in the standard. The data fitted to the error model are the result of averaging the readings in the direct and inverted positions.

If there was no error in the readings, the standard, or the comparator ways, the direct and inverted readings could both be expected to produce means of zero. Since these errors do exist, the direct and inverted readings will produce different means and it is necessary to compute the reading variance independently for the direct and inverted readings. The variance of the deviations fitted to the error model is then a function of the variances of the two subgroups.

In the calibration procedure it is first of all necessary to determine whether the variance of the data to be fitted to the error model reflects anything other than reading error. The computations necessary for this analysis are as follows.

The variances of the readings in each position are computed by

$$\sigma x_1^2 = \frac{\sum_i \sum_j (d_{1ij})^2}{n(N_1 - 1)}$$

$$\sigma x_2^2 = \frac{\sum_i \sum_k (d_{2ik})^2}{n(N_2 - 1)} \tag{2.11}$$

where the subscript 1 denotes the direct position, the subscript 2 denotes the inverted position, $d$ is the deviation of each reading from the average coordinate for that point,

- $i=1,2, \cdots, n$; $n$ is the number of points read,
- $j=1,2, \cdots, N_1$; $N_1$ is the number of sets of readings on each point in the direct position
- $k=1,2, \cdots, N_2$; $N_2$ is the number of sets of readings on each point in the inverted position.

The variance of the data to be fitted to the error model is then

$$S_r^2 = \frac{1}{4}\left(\frac{\sigma x_1^2}{N_1} + \frac{\sigma x_2^2}{N_2}\right). \tag{2.12}$$

Compute the constant error in the data to be fitted to the error model by

$$A_0 = \frac{\sum_{i=1}^{n} \overline{X}_i}{n} \tag{2.13}$$

where $\overline{X}$ is the average measured coordinate for each point.

The variance of the data about $A_o$ is then

$$S_{vo}^2 = \frac{\sum_{i=1}^{n} (\overline{X}_i - A_o)^2}{n-1}. \tag{2.14}$$

To determine if the variance of the data reflects anything other than reading error compute the ratio of the two variances.

If

$$(S_{vo}^2/S_r^2) > F\{(n-1), [n(N_1+N_2-2)]\}(0.95)$$

then the residuals reflect error due to weave and curvature of the ways and the data should be subjected to the harmonic analysis. If the ratio of the two variances is not greater than the tabular $F$-value then no significant error other than reading error is present in the data and no adjustment is necessary.

When the data are subjected to the harmonic analysis the variance of the residuals becomes

$$S_{v1}^2 = \frac{\sum_{i=1}^{n} V_{xi}^2}{n - 2q + 1} \tag{2.15}$$

where $V_x$ represents the residual for each point resulting from the adjustment and $q$ is the number of harmonics used in correcting the data.

The $F$-test for the harmonic analysis becomes

$$(S_{v1}^2/S_r^2) > F\{(n-2q+1), [n(N_1+N_2-2)]\}(0.95).$$

If this $F$-test shows significant errors over and above the reading error then the data should be subject to the harmonic analysis.

Again, the analysis should be started by evaluating the first two harmonics in the first step and adding one harmonic at a time until the $F$-test indicates a satisfactory calibration has been achieved.

## NONPERPENDICULARITY OF AXES

The data to be fitted to the error model are the deviations of the average measured $X$-coordinates from a straight line perpendicular to the $X$-axis. Since the calibration does not use a calibrated standard, the procedures used employ direct and inverted measurements to remove errors in the standard. The data fitted to the error model are the result of averaging the readings in the direct and inverted positions.

Since the individual means of the direct and inverted readings for a point are opposite in sign, it is necessary to compute the reading variance independently for the direct and inverted readings. The variance of the deviations fitted to the error model is then a function of the variance of the two subgroups.

In the calibration procedure it is first of all necessary to determine whether the variance of the data to be fitted to the error model reflects anything other than reading error. The computations necessary for this analysis are as follows.

The variances of the readings in each position are computed by

$$\sigma x_1^2 = \frac{\sum_i \sum_j (d_{1ij})^2}{n(N_1 - 1)}$$

$$\sigma x_2^2 = \frac{\sum_i \sum_k (d_{2ik})^2}{n(N_2 - 1)} \tag{2.16}$$

where the subscript 1 denotes the direct position, the subscript 2 denotes the inverted position, $d$ is the deviation of each reading from the average coordinate for that point,

$i = 1, 2, \cdots, n$; $n$ is the number of points read,

$j = 1, 2, \cdots, N_1$; $N_1$ is the number of sets of readings on each point in the direct position,

$k = 1, 2, \cdots, N_2$; $N_2$ is the number of sets of readings on each point in the inverted position.

The variance of the data due to reading is then

$$S_{ro}^2 = \frac{1}{4}\left(\frac{\sigma x_1^2}{N_1} + \frac{\sigma x_2^2}{N_2}\right). \tag{2.17}$$

The total variance in the data to be fitted to the error model is then

$$S_{r1}^2 = S_{ro}^2 + S_p^2 + S_s^2 + S_w^2 \qquad (2.18$$

where $S_p^2$, $S_s^2$ and $S_w^2$ are the standard errors of the periodic error, secular error and weave of the ways calibrations respectively.

The error in the comparator coordinates due to nonperpendicularity of the measuring axes is the average of the direct and inverted $X$-coordinates for each point. These errors are computed by

$$\bar{x}_i = \frac{X_{1i}'' + X_{2i}''}{2} \tag{2.19}$$

where $X_1''$ is the average coordinate in the direct position and $X_2''$ is the average coordinate in the inverted position for each point.

The variance of the uncorrected errors is then

$$S_{vo}^2 = \frac{\sum_{i=1}^{n} \overline{X}_i^2}{n}. \tag{2.20}$$

To determine if the variance of the uncorrected data reflects anything other than reading error compute the ratio of the two variances.

If

$$(S_{vo}^2/S_{r1}^2) > F[n, n(N_1 + N_2 - 2)](0.95)$$

then the residuals reflect error due to nonperpendicularity of the axes and the data should be fitted to the error model. If the ratio of the two variances is not greater than the tabular $F$-value then no significant error other than reading error is present in the data and no adjustment is necessary.

When the data are subjected to the rotation to correct for nonperpendicularity error the variance of the residuals becomes

$$S_{v1}^2 = \frac{\sum_{i=1}^{n} V_{xi}^2}{n - 2} \tag{2.21}$$

where $V_x$ represents the residual for each point resulting from the adjustment.

The $F$-test for the error model fit becomes

$$(S_{v1}^2/S_{r1}^2) > F[(n - 2), n(N_1 + N_2 - 2)](0.95).$$

If this $F$-test shows significant errors over and above the reading error then the calibration is unsatisfactory and must be repeated.

The standard error of the correction angle $\epsilon$ is then

$$s\epsilon = \left[\frac{S_{v1}^2}{\sum_{i=1}^{n} \overline{Y}_i^2}\right]^{1/2}. \tag{2.22}$$

## Summary

The calibration procedures and the statistical analysis presented here will produce calibration coefficients for correcting readings made on any leadscrew type comparator. Although some better methods may exist, these methods will produce calibrations whose quality is limited only by the mechanical limits of the comparator, the reading precision, and the standard error of the calibrated scale, with a minimum of expense required for calibrated standards. The number of sets of readings to be made depends on the setting precision that can be achieved and the desired quality of the calibration.

The methods described have been used at the Atlantic Missile Range to reduce comparator errors to one micron in each axis of our ballistic plate comparators.

## References

1. Bennet, J. M., "Method for Determining Comparator Screw Errors with Precision," *Journal of the Optical Society of America*, Volume 51, No. 10, October 1961, pp. 1133–1138.
2. Rosenfield, G. H., "Calibration of a Precision Coordinate Comparator," PHOTOGRAMMETRIC ENGINEERING, Volume 29, No. 1, January 1963, pp. 161–174.
3. Zug, R. S., "High Altitude Range Bombing by the Aberdeen Bombing Mission, Using Ballistic Cameras," Research Services Division, Report No. 1, Ordnance Research and Development Center, Aberdeen, Md., 10 December 1945, pp. 112–117.
4. Collen, F. C., Appendix A, "Calibration of a Precision Coordinate Comparator," PHOTOGRAMMETRIC ENGINEERING, Volume 29, No. 1, January 1963, pp. 161–174.
5. Hallert, Bertil, P., "Determination of the Geometrical Quality of Comparators for Image Coordinate Measurement," GIMRADA Research Note No. 3, 1 August 1962. Geodesy, Intelligence, and Mapping, Research and Development Agency, U. S. Army, Corps of Engineers, Fort Belvoir, Virginia.