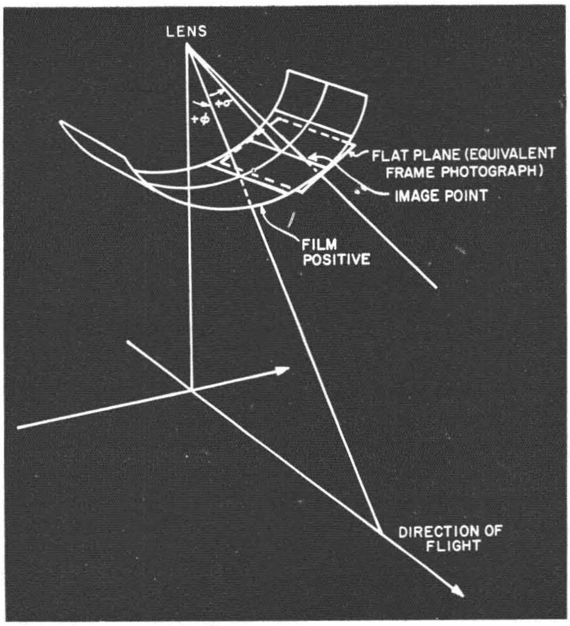

FRONTISPIECE. Illustration of the geometric equivalence of a panoramic camera to a double oblique frame camera.

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Image Motion Due to Camera Rotation

A set of formulas yields a realistic estimate of the degradation of image resolution based on expected rotational velocities and camera parameters

(*Abstract on page 863*)

INTRODUCTION

THE PRESENCE of random air currents and varying air densities in the atmosphere prevents airborne vehicles from remaining absolutely stable in flight, especially at the very low altitudes. Instead, these aircraft are subject to random pitch and roll motions. This is an undesirable condition for aerial photography, since any motions during exposure cause blurring of the image and loss of resolution.

To lessen the effects of aircraft motions, the camera may be installed in a stabilized mount which remains relatively steady despite motions of the vehicle, or it may be given a very fast shutter which "stops" the image motion. Since velocities sometimes reach as high as 10° per second for low altitude aircraft, one of these two means should be included to avoid considerable loss in resolution. To evaluate the necessity or ability of any means, the effect on the photographic resolution must be investigated.

Only the rotational components of the

random motions have an effect on resolution. The translational components generally are of no concern since they are so small compared with the forward velocity of the aircraft.

This article examines the image motion and loss of resolution from the *rotational* motions of the camera.

Determination of the Effect of Rotational Motions

A measure of the effect of an aircraft's random motion upon photographic quality is the degradation of resolution. This degradation is directly influenced by the distance of image motion blur and the stationary lens-film resolution. The final resolution is determined by the product of the modulation transfer functions for the target, atmosphere, lens, film, image motion and any other factors which contribute to the reduction of resolution. For an analysis of the relationships, the references (1, 2, 3) listed at the end of this article should be consulted. However, an approximate value of the degradation can be obtained for small blue distances ($DR_0<1$) from the reciprocal square law.[1]

$$\frac{1}{R^2}=\frac{1}{R_0^2}+D^2 \qquad (1)$$

or

$$\Delta R\% = 100\{1-[1+(DR_0)^2]^{-1/2}\} \qquad (2)$$

where

R = dynamic resolution
R_0 = static lens-film resolution
D = total distance of blur
$\Delta R\%$ = percentage loss in resolution.

The degradation computed from this formula is plotted against blur distance for different static resolutions in Figure 1. Note that R is the resolution in the direction of the blur since the image motion is usually in a straight line. In the perpendicular direction, the resolution is close to the static resolution, which also depends on the direction due to aberrations of the lens.

The required inputs to Equation 1 are the static lens-film resolution and the total distance of blur. The resolutions of the lens and of the film for specified target contrasts are frequently furnished separately, and from these the lens-film resolution must be computed. This relationship may also be represented approximately by the reciprocal square law:

$$\frac{1}{R_0^2}=\frac{1}{R_f^2}+\frac{1}{R_L^2} \qquad (3)$$

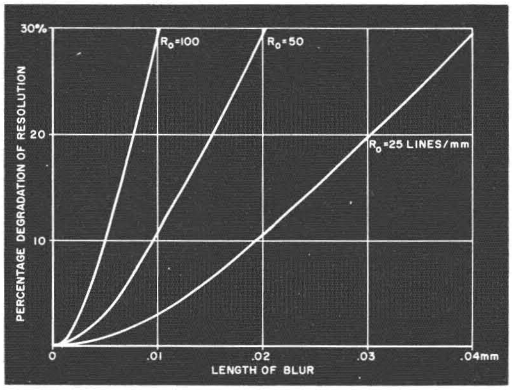

FIG. 1. Relationship of degradation of resolution to uniform image motion and static resolution according to inverse square law.

where

R_0 = static lens-film resolution
R_f = film resolution
R_L = lens resolution.

This simple formula does not take into account blurring due to improper focussing and other static sources of resolution degradation.

The other input parameter to Equation 1 is the total distance of blur considering all sources of image motion. It is through this parameter that the rotational motions enter into the analysis. However, it is incorrect to set the total blur distance equal to the blur distance from rotational motions since there are other sources of image motion. The effect of a given rotational velocity is quite dependent on the magnitudes of the other sources of blur, as substitution into Equation 1 would show.

Let us first suppose that the image motion from each significant source is known as a function of the probability of occurrence; more specifically, that the velocity which is exceeded a given percentage of the time is known. The most common probability description for this type of variable is the Gaussian or normal probability distribution. Then the problem is to find the total blur distance. The individual values cannot be added since the probability of each source contributing its specified sigma (σ)* value at the same time is very remote. The correct procedure is to take the root-sum-square of all the values, which yields a resultant blur distance based on the same probability of occurrence,

$$D=\left(\sum D_i^2\right)^{1/2}$$

* A 1σ value of the velocity is exceeded 32% of the time, a 2σ value (twice the 1σ value) is exceeded 5% of the time, and a 3σ value, 0.3% of the time.

The D_i are the probable distances of the motion blur (same sigma for all distances) contributed by all sources such as the aircraft velocity, vibration, mechanical errors, and the rotational motions. This relationship is not very useful if the probability distribution of any source is not Gaussian or if the probability associated with any blur distance is not known.

Unfortunately, the image motion from the other sources is often not readily known. For this situation, the loss of resolution, assuming that rotational motions are the only source of blur, is the single bit of information which may be computed. Very often, this information is sufficient, for it may show that rotational motions have either an excessively large or a very small effect on resolution. For example, the values in Table I (which will be described in more detail later) indicate that for some cases the rotational motion has a negligible effect on the resolution, while in other cases its effect is devastating and some provision must be made to compensate for it.

ABSTRACT: *As aerial cameras are used at higher and higher altitudes for reconnaissance missions, better photographic quality is demanded to resolve ground targets. An important factor influencing the photographic quality is image motion. This article examines the image motion and corresponding loss of resolution from one source of image motion, the rotational motions of the airborne vehicle.*

The method of computing the image motion from a pitch, roll or yaw velocity is explained. The equations for the image velocity apply to vertical and oblique frame photography, and vertical panoramic photography. The equations for the equivalent distance of blur on the ground and the errors in image distance from an angular offset are included.

COMPUTATION OF THE IMAGE MOTION

The blur distance is computed by multiplying the image velocity by the exposure time. This presupposes that the velocity is constant during the period of exposure, which is approximately true. The equations for the velocity are given in this article for both the frame and panoramic cameras in the vertical and oblique positions.

The remaining problem is the specification of the rotational rates. Since the motions are random, their values do not follow any regular pattern and hence are unpredictable.

A typical example of the variation in rates is shown in Figure 2. The rotational velocity for a modern aircraft is plotted on the ordinate and the time on the abscissa. The three uppermost graphs refer to the three rotation axes of pitch (around an axis through the wings), roll (around the longitudinal axis of the aircraft) and yaw (about the vertical), and the fourth graph shows the angular offset in the roll direction. Note that the roll rate is the largest while the yaw velocity is almost negligible. In regard to offsets, the pitch offset (not shown) is generally the largest because of shifts in the center of gravity as fuel is expended.

Usually the pattern of rotational motions for the aircraft is not described by a test curve. Only a figure is given which is supposed to represent some sort of maximum rate. Sometimes the percentage of time that this rate is exceeded may also be furnished by specifying a sigma probability value. Therefore, in substituting these values into the resolution equations it must be realized that the computed degradation is exceeded for just a small percentage of the photographs, and that the others are degraded less.

METHOD OF DERIVATION OF THE IMAGE VELOCITY EQUATION

The equations for the image velocity from rotational motions were derived by a geometrical analysis. Although the equations and derivations are relatively simple for the forward and side oblique orientations, they are not simple for the double oblique orientation, in which the camera is pointed in some skew direction. For this reason the exact equations and derivations are not included in this article for the general case of camera tilt. However, approximate equations for the vector velocity are given.

The velocity equations for the special cases listed in Table II were obtained from the equations for the general case. The latter were determined by deriving the equation for the path of an image point in the film plane as the aircraft rotates about any of its axes, then finding the velocity along this path in both the x and y-directions for any camera orienta-

TABLE I

RESOLUTION DEGRADATION FROM ROTATIONAL MOTION FOR VERTICAL CAMERA

Focal Length	*Static Resolution*	*Exposure Time*	*Rotational Velocity*	x	y	*Percentage Degradation**
Inches	Lines per mm.	Sec	Deg. per sec.	Inches	Inches	Per cent
3	20	$\frac{1}{500}$	1	0	0	0.1
			10	0	0	11
			10	$2\frac{1}{4}$	$2\frac{1}{4}$	25
6	20	$\frac{1}{500}$	1	0	0	0.6
			10	0	0	0.6
			10	$4\frac{1}{2}$	$4\frac{1}{2}$	~ 50
		$\frac{1}{2,000}$	1	0	0	0.0
			10	0	0	4
24	100	$\frac{1}{100}$	0.01	0	0	0.5
			0.1	0	0	30
			1.0	0	0	~ 90
		$\frac{1}{500}$	0.01	0	0	0.0
			0.1	0	0	3
			1.0	0	0	~ 60
		$\frac{1}{2,000}$	0.01	0	0	0.0
			0.1	0	0	0.1
			1.0	0	0	11
			1.0	$2\frac{1}{4}$	$2\frac{1}{4}$	11

* As determined by reciprocal square equation.

tion. The path of the image point is the intersection of a cone and a plane (which is a conic section), since a light ray from an object point to the film traces out a cone about the axis of rotation.

An analytical derivation involving matrices was also considered since a rotation matrix transforms the coordinates of a point in one system to its coordinates in a rotated coordinate system. It appears logical, therefore, that matrices can be applied to obtain the image velocity, since the aircraft motions under consideration are rotational. However, the matrix approach is not appropriate for two reasons: (1) matrices do not describe the actual movement of the image point (unless additional translational terms are introduced to account for the changes in the radius vector), because the image remains in the film plane and hence its motion is not equivalent to holding the image point fixed and rotating the coordinate system; and (2) the matrix approach is not simpler than the geometrical approach but actually involves more equations. Multiplication of the matrices would show this.

IMAGE VELOCITY EQUATIONS

The equations for the image velocity from rotational motions are given in Table II for several special cases of camera tilt. These are:

- Vertical frame camera
- Forward oblique (pitched) frame
- Side oblique (rolled) frame
- Vertical panoramic
- Tilted frame, approximate velocity at origin.

The velocity equations for the panoramic camera are included since this sensor is becoming more widely used in aerial photography. The derivation of parametric equations related to the panoramic is straightforward, because the panoramic photograph can be considered as a series of infinitely narrow frame photographs. Consider the geometry of the panoramic camera, shown in the Frontispiece. If a flat plane is placed tangent to the cylindrical format at a desired location, then the image velocity in both the x and y-directions at this point is the same for both the cylindrical surface and flat plane. In other words, the image velocity for the panoramic photograph is the same as for a frame photograph tangent to it at the point in question.

If the camera (frame) has a focal plane shutter, the x in the image velocity equations is not the actual film coordinate in the x-direc-

TABLE II

IMAGE VELOCITY FROM ROTATIONAL MOTIONS FOR SPECIAL CASES

Special Case	*Velocity from Aircraft*	*Velocity from Aircraft*	*Velocity from Aircraft*
	Pitch Rate	*Roll Rate*	*Yaw Rate*
1. Vertical Frame Camera	$\dot{x}_\phi=\left(\frac{f^2+x^2}{f}\right)\dot{\phi}$ $\dot{y}_\phi=\frac{xy}{f}\dot{\phi}$	$\dot{x}_\omega=\frac{xy}{f}\dot{\omega}$ $\dot{y}_\omega=\left(\frac{f^2+y^2}{f}\right)\dot{\omega}$	$\dot{x}_\kappa=y\dot{\kappa}$ $\dot{y}_\kappa=x\dot{\kappa}$
2. Forward Oblique Frame	$\dot{x}_\phi=\left(\frac{f^2+x^2}{f}\right)\dot{\phi}$ $\dot{y}_\phi=\frac{xy}{f}\dot{\phi}$	$\dot{x}_\omega=y\left(\sin\phi-\frac{x}{f}\cos\phi\right)\dot{\omega}$ $\dot{y}_\omega=\left[\left(\frac{f^2+y^2}{f}\right)\cos\phi + x\sin\phi\right]\dot{\omega}$	$\dot{x}_\kappa=y\left(\cos\phi+\frac{x}{f}\sin\phi\right)\dot{\kappa}$ $\dot{y}_\kappa=\left[\left(\frac{f^2+y^2}{f}\right)\sin\phi - x\cos\phi\right]\dot{\kappa}$
3. Side Oblique Frame	$\dot{x}_\phi=\left[\left(\frac{f^2+x^2}{f}\right)\cos\omega + y\sin\omega\right]\dot{\phi}$ $\dot{y}_\phi=x\left(\sin\omega-\frac{y}{f}\cos\omega\right)\dot{\phi}$	$\dot{x}_\omega=\frac{xy}{f}\dot{\omega}$ $\dot{y}_\omega=\left(\frac{f^2+y^2}{f}\right)\dot{\omega}$	$\dot{x}_\kappa=\left[\left(\frac{f^2+x^2}{f}\right)\sin\omega - y\cos\omega\right]\dot{\kappa}$ $\dot{y}_\kappa=x\left(\cos\omega+\frac{y}{f}\sin\omega\right)\dot{\kappa}$
4. Vertical Panoramic	$\dot{x}_\phi=\left(\frac{f^2+x^2}{f}\right)\cos\sigma\dot{\phi}$ $\dot{y}_\phi=x\sin\sigma\dot{\phi}$	$\dot{x}_\omega=0$ $\dot{y}_\omega=f\dot{\omega}$	$\dot{x}_\kappa=\left(\frac{f^2+x^2}{f}\right)\sin\sigma\dot{\kappa}$ $\dot{y}_\kappa=x\cos\dot{\sigma}$
5. Double Oblique Frame, Approximate Velocity at Principal Point	$v\approx f\cos\omega\dot{\phi}$	$v\approx f\cos\phi\dot{\omega}$	$v\approx f\sin\iota\dot{\kappa}$ $\approx f(1-\cos^2\phi\cos^2\omega)^{1/2}\dot{\kappa}$

tion, but should be the film coordinate adjusted for the image motion compensation correction. However, it is a good approximation to use the actual x-coordinate.

The symbols appearing in the velocity equations are defined as follows:

f = focal length
x = photocoordinate in flight direction
y = photocoordinate perpendicular to flight direction
ϕ = pitch angle, around an axis through the wings, positive in forward direction
ω = roll angle, around the longitudinal axis of the aircraft (direction of flight), positive to the right
κ = yaw angle, around the vertical
$\dot{\phi}$, $\dot{\omega}$, $\dot{\kappa}$ = angular velocities of pitch, roll and yaw, in radians/sec.
σ = sweep angle of panoramic camera, positive when scanning to the left.

A rough value of the image motion for the vertical camera orientation is obtained by computing the product of angular velocity and focal length. If the relationship is so simple, then why derive the more complex exact velocity equations? If rotational motions are found to exert a measurable influence on the resolution, the blur distances computed from the above formulas may be grossly in error for some situations. The main inaccuracy of these formulas is their failure to take the off-axis points into account. This leads to a large error if there is a yaw rotation with a vertical camera, or if the focal length is comparable with the film width. Therefore, exact equations are given (in Table II). Figure 3 shows the variation of velocity over the

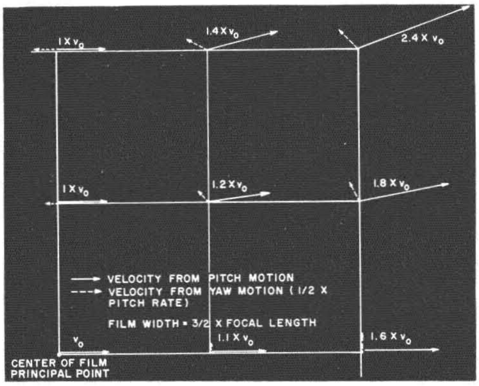

FIG. 2. Variation of image motion from vehicle rotation for vertical camera. Only one quadrant of the film is shown. The effect of roll is similar to the effect of pitch.

film from pitch, roll and yaw motions. For this case, the film width is one and a half times the focal length.

Computed values of the resolution degradation for the vertical frame camera are listed in Table I. The chosen examples are a low altitude mapping camera with a three or six inch focal length lens and a high altitude reconnaissance camera, operating at altitudes of around 70,000 feet. The smallest angular velocity for the 24 inch camera pertains to a stabilized mount, while the one degree per second rate is a typical value of the aircraft motions at this altitude.

Equations for the Ground Distance of Blur

A useful parameter in judging the effect of a source of image motion is the ground distance equivalent to an image blur. The equations for this ground distance in terms of the ground coordinates are simply the equations for the image motion with the vertical frame camera in Table II, since the ground plane is also perpendicular to the vertical. The altitude replaces the focal length and the ground coordinates are substituted for the photocoordinates. Hence, the ground blur distance is independent of focal length and the type of camera as long as the ground coordinates are specified.

The ground coordinates are obtained from the film coordinates by the appropriate transformation equations.[5] It is worth mentioning that the transformation equations for an oblique camera with a focal plane shutter are not the same as the standard equations with an intralens shutter since there are two additional image displacements, the sweep positional displacement, due to the finite time of sweep of the shutter, and the image motion compensation displacement. Equations for these displacements are given in an earlier article by the author[6] for the oblique frame camera.

The ground distance of blur should not be confused with the smallest resolvable ground distance, since the latter is also dependent upon the lens and film resolution and includes other sources of image motion. The ground resolution or smallest resolvable ground distance is computed directly from the dynamic film resolution:

$$R_G \text{ (feet)} = 1/305 \times R \text{ (lines/mm.)} \times \text{Scale} \quad (4)$$

where R_G is the ground resolution and R the film resolution. For any oblique camera orientation the scale depends upon the location on the photograph and the direction.

Equations for the Location Error Due to an Angular Offset

In situations where the random aircraft motions are a factor, the effect of an offset from the vertical may also be of interest. If the camera is used for mapping or locating a target, an angular offset is undesirable because it leads to errors of location. Even if the nadir point is properly located, calculated distances will be in error since the scale of a non-vertical photograph is variable.

The displacement of an image due to pitch, roll or yaw can be determined from the equations for the image velocity on a vertical

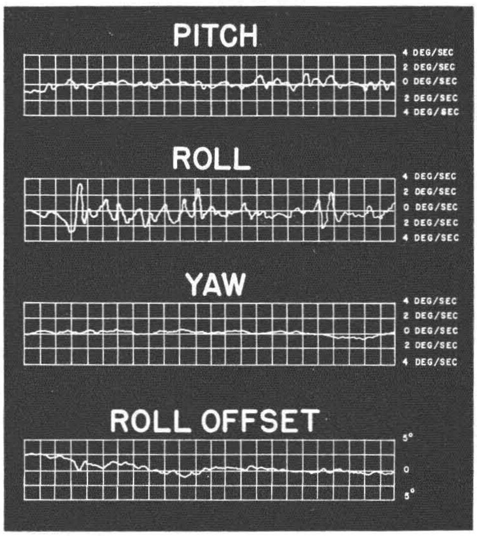

FIG. 3. Rotational velocities of a typical reconnaissance aircraft. Time is plotted as the abscissa; each division represents five seconds.

frame photograph, listed in Table II. The image displacement replaces the image velocity, the angular offset replaces the angular velocity, and the ground coordinates replace the film coordinates in the equations. These equations apply for differential tilts regardless of the nominal orientation of the camera.

In general, the error in the distance between two image points is of interest rather than the absolute displacement of an image. This error is just the difference of the absolute displacements. For the worst case, taking one point at the origin and the other at the edge of the photograph, the error from a pitch offset of the camera is

$$\delta x = \left(\frac{f^2 + w^2/4}{f}\right)\Delta\phi - \left(\frac{f^2}{f}\right)\Delta\phi = \frac{w^2\Delta\phi}{4f}.$$

Taking a case in which the film width w is equal to the focal length and the offset is 0.2°, the maximum percentage error is

$$\delta x\% = 100\cdot\frac{\delta x}{w/2} = 100\cdot\frac{1}{2}\cdot 0.00347 = 0.17\%$$

This error is erased if the attitude of the camera is accurately known at the moment of exposure, and it is attenuated if the points appear on other photographs.

Summary

Any movements of the airborne vehicle during exposure affect the photographic resolution. The forward movement of the vehicle causes a movement of the image which is compensated by a translational movement of the lens or film or by a rotation of the camera (image motion compensation). The image motion arises from inaccuracies in the mechanism and velocity (V/H) sensor, and from the ground topography. For an oblique camera, the image velocity cannot be entirely compensated since it is variable over the film. Grading the IMC helps to reduce the residual image motion. The image velocity and the optimum type of image motion compensation for the oblique camera is analyzed in an earlier article by the author.[6]

Another type of vehicle motion is the random motion caused by varying air densities and turbulence in the atmosphere. Only the rotational component of these motions has a significant effect on the resolution (the translational component is attenuated by a factor of focal length/altitude). If the magnitude of the rotational velocity could be estimated based upon the type of aircraft and the atmospheric conditions it is likely to encounter, or upon the capabilities of a stabilized mount, the loss of resolution can be computed from the equations in this article. The procedure is summarized as follows:

1. Determine the rotational velocities from available data.
2. Substitute these values into the appropriate image velocity equations in Table II.
3. Multiply the image velocity by the exposure time to get the approximate distance of blur from rotational motions.
4. Substitute the blur distance into the resolution equation (Equation 2). The result is the approximate percentage loss in resolution in the direction of blur.

It must be emphasized that this procedure assumes there are no other sources of image motion. The effect of a rotational motion does depend to some extent upon the magnitudes of the other sources.

Additional information is readily available from the image velocity equations. The distance on the ground equivalent to the image smear from the random motions can be computed from equations in this article, and the determination of the location error due to a tilt of the aircraft is also explained.

A third type of vehicle motion affecting resolution is vibration which is not analyzed in this series. There are, of course, other sources of resolution degradation for an aerial camera, other than those associated with the vehicle itself, such as atmospheric scattering and the camera vibration from operation of the moving parts.

The third and final part (which may appear in a later issue) of this series is concerned with the image motion from the aircraft velocity and the proper image motion compensation velocity for the vertical and forward oblique panoramic cameras.

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