A conformal-like transformation facilitates an effective compensation of film distortion based on eight fiducial marks for analytic aerotriangulation.

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Film Distortion Compensation

THE DIMENSIONAL STABILITY of aerial photographic film material has received much attention from manufacturers with the happy result that greatly improved films have become available in the past few years. The precision of aerial photogrammetric mapping has shown marked improvement as a result of the use of these materials.

The stability of both the new and the older film bases has been studied by a number of investigators, sometimes in great detail, by means of measurements and the use of moire distortion patterns.[1,2,3,4] In the Coast and Geodetic Survey, research has been aimed at developing the optimum mathematical compensation of the inevitable distortions for use in a production analytic aerotriangulation system.

In general, distortion is determined by the measurement of certain points of known position, such as fiducial marks or reseau points, on a photographic print, and compensated by distributing corrections based on the observed distortions. The minimum practical number of such points is four, with one in each corner of the photograph. The ultimate correction would seemingly be obtained through the use of a reseau, with many control points distributed throughout the photograph. However, there are at least three practical considerations that must be taken into account when a reseau is expected to effect an appreciable improvement over simpler methods which have proven capable of reducing distortion to the order of five microns.

First, the comparator pointing error should be reduced to less than one micron in order that the total measuring error will be less than two microns. This would require at least three pointings of each reseau point. The number of reseau measurements on a typical photograph with nine pass points and two control stations would be 33 if only the nearest reseau points were measured, or 132 if it were desired to bracket each photogrammetric point with four reseau points.

Second, the reseau images are produced as shadows which vary in dimension and contrast, depending on the brightness of the terrain background, in a manner which can cause actual displacement of the image. This could be remedied by pre- or post-printing on the film a pattern consisting of crosses within small clear areas which would print as light against a uniform dark background. This would, of course, obscure small portions of the terrain. It could be objected that this would record only changes caused by processing and aging rather than the conditions caused by the temperature, humidity, and

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stresses at the time of photography; however, it must be remembered that a reseau within a camera does not record this ideally because of changes in the reseau itself with conditions.

Third, the introduction of a glass reseau in the camera degrades the image resolution sufficiently to contribute measurably to the error of photogrammetric point identification. This can also be alleviated by pre- or post-printing a control pattern. Some degradation is also possible here because of double exposure unless the dark areas of the pattern can be made completely opaque.

ABSTRACT: *The system of analytic aerotriangulation in use in the Coast & Geodetic Survey offers the opportunity to compensate mathematically for systematic errors caused by film distortion. The effectiveness of compensations based on the fiducial system of an aerial camera has been examined by the measurement of a precise grid printed on film samples. Formulas are presented for a conformal fitting to eight fiducial marks as well as four. A standard residual error of four microns over the entire photograph area is reported using the eight marks, where the pass points ordinarily employed for triangulation lie in more favorable locations. A small unexpected residual radial error was revealed by the tests.*

IN VIEW OF the problems involved in the use of a reseau for the precise determination of film distortion, a major research effort in the Coast and Geodetic Survey has been in the study of the effectiveness of points on the perimeter of a photograph in determining compensation. These studies are now being extended to ascertain the effectiveness of points at a 2-cm. interval in improving the results. If the improvement is sufficient to offset the possible degradations, the adoption of some type of a reseau system will be seriously considered.

Data for the present film distortion studies were obtained by contact printing a ruled grid on various film samples and in turn, contact printing the film at intervals on glass diapositives for measurement. Treatment of the data thus far has included the computation of errors due to distortion in (1) uncompensated film, (2) film compensated for scale only, (3) film adjusted to fit four controlling points near the corners of the grid, and (4) film adjusted to fit eight points, one near each corner and one in the center of each side.

The grid used was of Zeiss manufacture, 22.5 cm. square with 22.5 mm. spacing and 20-micron line weight. The coordinates of 53 grid intersections were established by three sets of measurements with a Mann 422F comparator equipped with a binocular microscope.

The grid was then contact printed on three different kinds of aerial film. Three prints were made on each, with one near the beginning, middle, and end of standard length rolls. Processing and drying were by standard laboratory methods, and the film was stored along with aerial film intended for mapping purposes.

Two of the films tested used modern, polyester, stable bases. One cellulose acetate butyrate base film was also tested to permit observation of the improvement to be expected from the use of polyester bases.

After five weeks, each sample was printed on a 9-by-9-inch glass diapositive. In addition, one sample of each film was printed after 1, 9, 13, 19, 26, 39, and 52 weeks to provide information on the effects of aging.

As the diapositives became available, the 53 known grid intersections were measured on each, using the Mann comparator. Each intersection was measured three times and the measurements meaned.

AS CAN be imagined, a large amount of data was involved, and several computer programs were required to analyze the results of the tests. The first was a least square rotation and translation to the known positions of the grid intersections. The chief value of this computation is in the determination of the average dimensional change of the film sample.

All samples at all ages showed a dimensional decrease from the calibrated size of the grid plate. Although there was an apparent trend toward continued shrinkage with age, this effect was almost masked by changes which appear to be random with respect to time. However, there was a remarkable correlation between various samples printed onto diapositives at the same time, indicating that environmental conditions probably have a greater effect than age on the absolute size of developed film.

It should be mentioned that while the film was stored under controlled temperature and humidity conditions, the controls are not precise, and the observed variations are in accordance with those expected from manufacturers' data. In addition, aging was certainly retarded by the fact that the film was stored in closed cans.

The greatest dimensional change observed in polyester base film was .07 per cent and in acetate base film .23 per cent.

As most photogrammetric plotting instruments automatically compensate for difference in scale with at most a small uncompensated error in the camera focal length, a transformation of the film samples which permit scale change is of greater interest. The errors of position of points after the transformation represent values which must be reduced by any practical method of distortion compensation.

Algebraic formulas have been developed which permit the compensation of film distortion by means of the determination of the distortion existing at certain points of known position. The derivation of the formulas is given in the appendix.

The aerial cameras used by the Coast and Geodetic Survey have four fiducial marks placed in the corners of the photographs. Two cameras have, in addition, four fiducial marks placed at the midpoints of the sides.

The effectiveness of adjustments to points in these positions was then examined by adjusting the measurements of the film samples to the known positions of, first, four grid intersections near the corners and, second, to the same intersections plus the midpoints of the sides. Some results of the tests are given in Table 1.

Table 1

Standard Errors of Positions of 53 Grid Intersections after Three Adjustments, in Microns

Film No.	*Scale*	*4 Fiducial Marks*	*8 Fiducial Marks*
1	15	13μ	10μ
2	15	12	11
3	18	17	15

Table 2

Standard Errors of Positions of 9 Pass Points Simulated by Grid Intersections, after Three Adjustments, in Microns

Film No.	*Scale*	*4 Fiducial Marks*	*8 Fiducial Marks*
1	11	9μ	4μ
2	11	5	4
3	13	11	8

The standard errors shown in the table are based on the measurements of three samples of each type film after aging five weeks. Films numbers 1 and 2 are on polyester bases. Film number 3 is on an acetate base.

It should be noted that the values for film adjusted to scale only are unrealistically small, as the scale adjustment was based on a best fit to 53 points; whereas, in any type of bridging without distortion compensation, scale is normally controlled through nine pass points.

Of primary importance in the accuracy of aerotriangulation is the positioning of pass points. Therefore nine grid intersections were selected as representing the most probable positions of pass points, and their standard errors computed. The results are shown in Table 2. It will be noted that the error after compensation to eight simulated fiducial marks is of the order of 4 microns in the case of polyester base film.

Tables 3 and 4 are based on the same data, and show the percentage of the original error of the scaled film samples remaining after the two degrees of compensation. It is noteworthy that the acetate base film, in addition to having larger original errors, is also less amenable to distortion compensation.

A graphic display of the errors at the various stages portrays the effects of compensation somewhat more dramatically than the tables.

Table 3

Percentage of Error Remaining in 53 Grid Intersections after Distortion Compensation

Film No.	*4 Fiducial Marks*	*8 Fiducial Marks*
1	86%	67%
2	79	72
3	92	81

TABLE 4

PERCENTAGE OF ERROR REMAINING IN 9 PASS POINTS AFTER DISTORTION COMPENSATION

Film No.	*4 Fiducial Marks*	*8 Fiducial Marks*
1	86%	36%
2	46	38
3	88	63

Figure 1 shows the relative positions of the grid intersections used in the study with the simulated fiducial marks indicated by small squares, and the other 44 measured intersections indicated by circles. Figures 2, 3, and 4 show the distortions of a typical sample of acetate base film after compensation for scale only, and after adjustment to 4 and 8 fiducial marks respectively. The magnitude of the distortions have been exaggerated by a factor of 5000.

As expected, the adjustment to four fiducial marks compensates for deviations from a square format but does little or nothing to correct for curvature. The additional information provided by eight fiducial marks permits the correction of systematic curvature and provides the improvement noticeable particularly along the lower edge of the film sample.

Figures 5, 6, and 7 portray the same treatment of a typical sample of polyester base film. Because of the greater inherent stability of the film, it was necessary to exaggerate the distortions by a factor of 10,000 for the illustrations.

In both samples it will be noted that extrapolation of compensation beyond the area bound by the fiducial marks is comparatively ineffective.

An additional advantage in the use of eight fiducial marks is the fact that eight of the nine normal pass point locations on a photograph fall near fiducial marks and will receive the best determined compensation.

The Coast and Geodetic Survey has recently equipped two of its aerial cameras with eight fiducial marks and aerotriangulation test projects are now in progress to determine their effectiveness in actual use. Simultaneous photography with a glass plate camera will provide an additional basis of comparison. At the present writing, it is too early to report any results of the tests.

Many graphic displays of the type shown in Figures 2–7 have been prepared for the

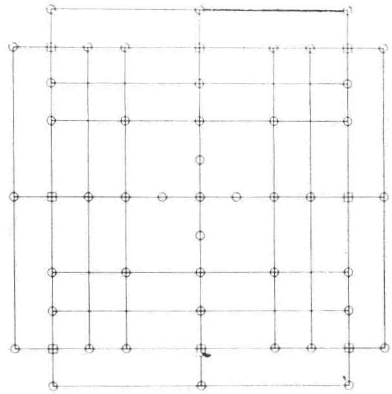


FIG. 1. Design of the master grid.

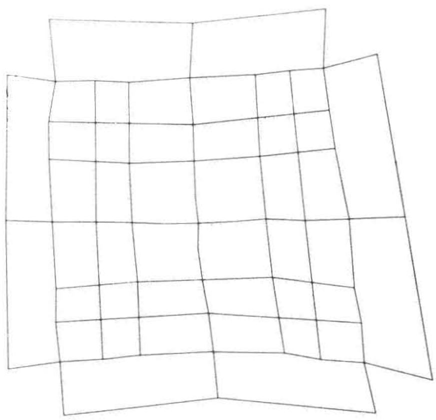


FIG. 2. Uncompensated distortion in acetate base film. (5,000:1 exaggeration)

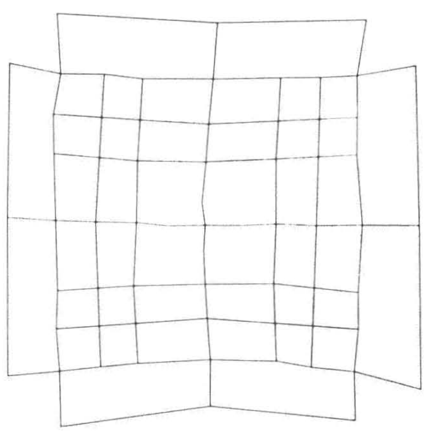


FIG. 3. Residual distortion in acetate base film after compensation to four fiducial marks. (5,000:1 exaggeration)

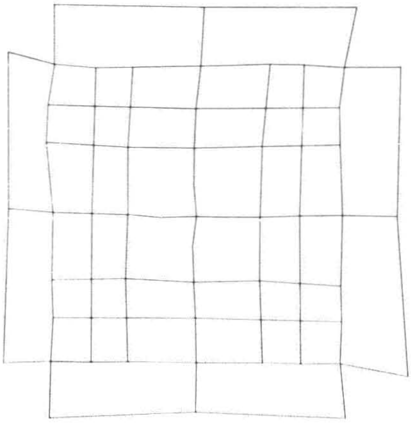


FIG. 4. Residual distortion in acetate base film after compensation to eight fiducial marks. (5,000:1 exaggeration)

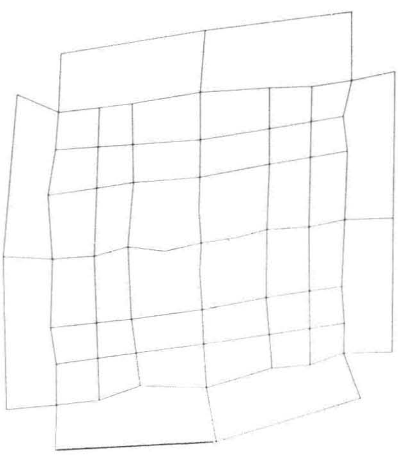
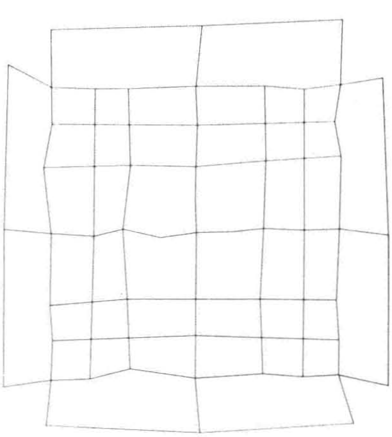


FIG. 5. Uncompensated distortion in polyester base film. (10,000:1 exaggeration)



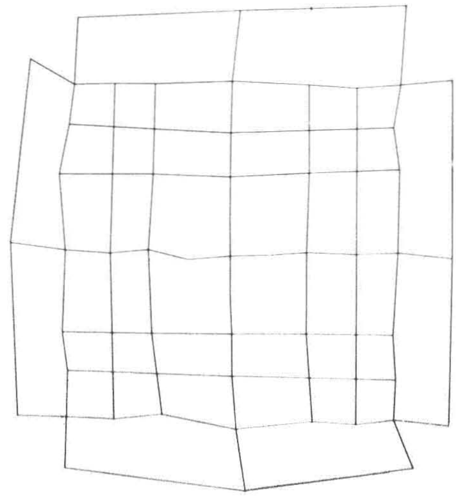


FIG. 7. Residual distortion in polyester base film after compensation to eight fiducial marks. (10,000:1 exaggeration)

various measurements of the film samples and studied with the aim of ascertaining the characteristics of film distortion. In particular, the errors remaining after compensation to eight fiducial marks were examined for systematic residual distortions. One such distortion common to all film samples was the tendency of points within the area bounded by the fiducial marks to be displaced toward the center and of points outside the area to be displaced away from the center.

THE MATHEMATICAL compensation of distortion of this nature is straightforward and equations for the purpose are derived in the appendix. The determination of the amount of correction is more difficult. If it can be demonstrated that the error is systematic in a large number of film samples, then a correction should obviously be applied. Since there is no way to determine the error for an individual photograph, the magnitude of the correction can only be determined empirically, through the measurement of many test film samples, and will probably vary from one type of film to another.

An additional difficulty is that the determination of the coefficients of the compensating equations requires that the positions of

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FIG. 6. Residual distortion in polyester base film after compensation to four fiducial marks. (10,000:1 exaggeration)

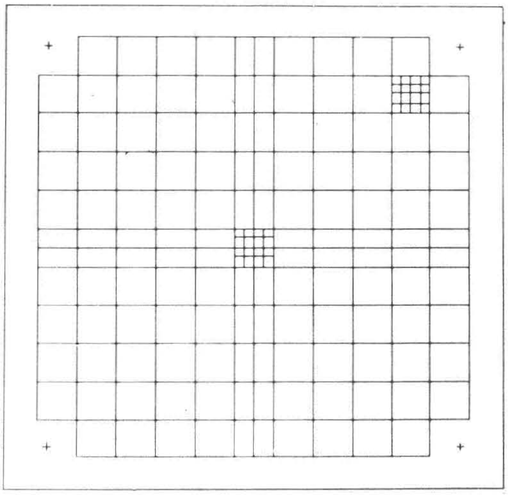

FIG. 8. New grid design that includes intersections near the fiducial marks and near the usual locations of the pass points, in addition to two small areas containing a high density of points.

the simulated fiducial marks of the grid used in the tests must approximate the positions of the fiducial marks in an actual camera. The design of the grid and the number of film samples in the tests thus far described were obviously unsuitable for further investigation of this type of distortion.

A new grid of the design shown in Figure 8 has now been obtained. Grid intersections have been provided in the same positions as the eight camera fiducial marks, as well as in the most probable positions of pass points. A much larger number of points are available for distortion analysis and the design is better adapted to the study of reseau methods. In addition, two small areas have a high density of points to provide information on the random distortions to be expected in very short distances.

No results of the new tests are available yet; however, the new grid has been calibrated and printed onto three kinds of polyester base film a large number of times. We are currently printing six samples of each film onto glass diapositives at intervals starting at one week and up to three months at the present. New computer programming is required for the additional information expected, and this is now in progress.

IN SUMMARY, while it is apparent that improvement of photographic measurements can be expected by means of compensation to fiducial marks, the tests described have demonstrated the degree of improvement to be expected when four and eight fiducial marks are used. In addition, a systematic radial error has been discovered, the compensation of which will no doubt contribute additional improvement after a more intensive investigation has been made. It is even possible that less obvious systematic errors could be uncovered after the masking effects of the radial error have been removed.

Another current research project is the calibration of aerial cameras through stellar photography, utilizing glass plates in the focal plane, a procedure already thoroughly tested by ballistic cameras. As a by-product, the effects of flatness of glass on distortion will be examined by comparing plates exposed under similar conditions, thus providing a basis of comparison between the use of distortion compensated film and glass plates for photogrammetry.

APPENDIX

THE COMPENSATION of film distortion by means of the measurement of fiducial marks is based on the comparison of the true and the measured positions of the marks, the determination of equations which will restore the measured marks to their true positions and the use of the same equations to compensate the positions of measured points throughout a photograph.

The Coast and Geodetic Survey has adopted certain conventions in the numbering of fiducial marks which are illustrated in Figure 9. The orientation of the photograph relative to the coordinate system is determined by the direction of flight being in the direction of the increasing x-coordinate.

It is convenient in developing compensa-

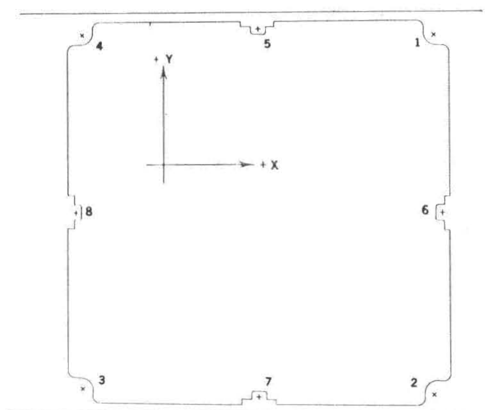

FIG. 9. Arrangement of fiducial marks in the aerial camera and the numbering system.

tion equations to consider corrections to the x and y-coordinates independently. Once such an equation is developed for one coordinate, a similar equation can be obtained for the other by simply exchanging x and y terms through the recognition of symmetry.

To approach the problem of compensation based on four fiducial marks, one may begin by determining that the x-coordinates of fiducial marks 1 and 2, for example, require certain corrections to restore them to their true position. Points falling between fiducial marks 1 and 2 probably require corrections varying in magnitude between the two values according to their distances from the fiducial marks. From the available information, only a straight-line interpolation can be assumed; in other words, the x-correction is a linear function of y. The mathematical expression for the correction is $\Delta x = Ay + B$. When the values of Δx and y are known for two points, the coefficients A and B may be determined.

A similar equation may be determined using fiducial marks 3 and 4, obtaining different values for the coefficients A and B. Thus may be obtained corrections for points with the same x-coordinates as the fiducial marks and of varying y coordinates. It is now desired to determine corrections for points whose x-coordinates also vary. This may be approached by assuming that the coefficients A and B are linear functions of x and are controlled by the two sets of values a ready determined. The mathematical expression for this is:

$$A = Cx + D$$
$$B = Ex + F.$$

Substituting in the original expression gives:

$$\Delta x = (Cx + D)y + Ex + F.$$

Simplifying and renaming the coefficients gives:

$$\Delta x = Axy + By + Cx + D.$$

Following the same approach for y-corrections will give:

$$\Delta y = A'xy + B'x + C'y + D'.$$

Since Δx, Δy, x, and y are known for four fiducial marks, the coefficients may be determined by two systems of four equations in four unknowns. The resulting equations may be used to compensate the position of any point on the same photograph.

The approach described here is somewhat different from that described in previous publications of the Coast and Geodetic Survey,[5,6] and result in equations which do not appear to be the same. The results, however, are identical. The changed approach, on the other hand, permits a systematic extension to the problem of compensation for eight fiducial marks.

REFERRING again to the arrangement of fiducial marks in Figure 9, it will be seen that the presence of control points at the midpoints of the sides provide sufficient information to determine a curvilinear correction along the sides. Confining attention to the x-correction and assuming that the correction is a quadratic function of the y-coordinate, this may be expressed for fiducial marks 1, 2, and 6, for example, as:

$$\Delta x = Ay^2 + By + C.$$

Again referring to Figure 9, it is seen that another such curve may be determined from fiducial marks 3, 4, and 8. If, as in the case of four fiducial marks, it is again desired to provide a smooth transition between the two curves, it will be found necessary to provide means for applying the proper corrections at fiducial marks 5 and 7. No new information as to the curvature of the correction is provided by the introduction of the two points, however, a tilt or lateral shift and a translation of the correction curve is required in order to achieve a fit.

Analysis of the basic compensation equation by methods of the calculus shows that the curvature of the correction is determined by the A-coefficient, lateral shift by B, and translation by C. Curvature is determined twice, and the transition can only be assumed as linear. Lateral shift and translation are determined at three positions and can therefore be assumed quadratic. Since a change in the x-coordinate provides the transition, A, B, and C may be described as linear and quadratic functions of x, as follows:

$$A = Dx + E$$
$$B = Fx^2 + Gx + H$$
$$C = Jx^2 + Kx + L.$$

Substituting in the original expression gives:

$$\Delta x = (Dx + E)y^2 + (Fx^2 + Gx + H)y + Jx^2 + Kx + L.$$

Simplifying and renaming the coefficients gives:

$$\Delta x = Axy^2 + By^2 + Cx^2y + Dxy + Ey + Fx^2 + Gx + H.$$

A similar approach to the y correction will give:

$$\Delta y = A'x^2y + B'x^2 + C'xy^2 + D'xy + E'x + F'y^2 + G'y + H'.$$

Eight unknown coefficients occur in each equation, and the values of Δx, Δy, x, and y for the eight fiducial marks provide two sets of eight simultaneous equations for their solution. It is noted that the pair of equations constitute a conformal transformation, and they could also have been derived by those classical techniques.

The compensation equations for both four and eight fiducial marks provide greater versatility than may be supposed from their manner of derivation. Any degree of rotation or translation between the measured and the true positions of the fiducial marks may be accommodated. A mirror reversal of the coordinates will be accommodated correctly. The fiducial marks do not need to conform to a particular pattern; for example, in the case of eight fiducial marks, the marks along a side need not lie in a straight line.

THE FINAL FORM of the equations permits unexpected economies in computing and programming. The equations for the use of four fiducial marks may be rearranged to the following form, keeping the arbitrary coefficient designations in alphabetical order:

$$\Delta x = A + Bx + Cy + Dxy$$
$$\Delta y = A' + B'x + C'y + D'xy.$$

The equations for eight fiducial marks may be similarly rearranged as:

$$\Delta x = A + Bx + Cy + Dxy + Ex^2 + Fy^2 + Gx^2y + Hxy^2$$
$$\Delta y = A' + B'x + C'y + D'xy + E'x^2 + F'y^2 + G'x^2y + H'xy^2.$$

It will be noted that the x and y terms needed for the determination of the coefficients are the same for both x and y corrections in both cases. In addition, the terms used in the four fiducial mark problem are the same as the first four terms of the eight fiducial mark problem. A single matrix can serve in a dual purpose program for the solution of all required coefficients, as shown in Table 5. The subscripts refer to the numbers of the fiducial marks. The equations may be solved by any of the standard methods for non-symmetrical equations, using the Δx and the Δy columns for two independent back solutions, thus determining the $A \ldots H$ and the $A' \ldots H'$ coefficients, respectively. For the case of four fiducial marks, only the first four rows and columns plus the first four rows of the Δx and Δy columns need be used.

The presence of ones in the first column also allows a small programming economy in some methods of solution.

IN THE Coast and Geodetic Survey, the true coordinates of the fiducial marks of a camera are determined by placing a sensitized glass plate in the focal plane of the camera in a darkroom and exposing the fiducial marks. At least three plates are made and then measured on a comparator that has been calibrated for differential x and y scale and lack of perpendicularity of the ways. The measurements are individually rotated and translated to a system suitable for use in further computations and meaned. The comparator calibration corrections are then applied.

The coordinate system normally chosen is one which gives two of the fiducial marks on one side the same y-coordinate, the origin of coordinates at the principal point as determined by the intersection of the diagonals through the corner fiducial marks, and the direction of increasing x in the direction of flight.

The use of these coordinates as the basis of compensation makes it unnecessary to apply comparator calibration corrections for differential scale and lack of perpendicularity of the ways as a separate step. The restoration of the fiducial marks to their true position automatically provides the proper correction even though the comparator may change and even though measurements are made on a different comparator. In addition, with the use of eight fiducial marks, if the original measurements of the true fiducial mark co-

TABLE 5

DUE TO SYMMETRY, A SINGLE COEFFICIENT MATRIX APPLIES TO BOTH COORDINATES, AS WELL AS TO BOTH EIGHT AND FOUR FIDUCIAL MARKS

1	x_1	y_1	y_1x_1	x_1^2	y_1^2	$x_1^2y_1$	$x_1y_1^2$	Δx_1	Δy_1
1	x_2	y_2	x_2y_2	x_2^2	y_2^2	$x_2^2y_2$	$x_2y_2^2$	Δx_2	Δy_2
1	x_3	y_3	x_3y_3	x_3^2	y_3^2	$x_3^2y_3$	$x_3y_3^2$	Δy_3	Δy_3
1	x_4	y_4	x_4y_4	x_4^2	y_4^2	$x_4^2y_4$	$x_4y_4^2$	Δx_4	Δy_4
1	x_5	y_5	x_5y_5	x_5^2	y_5^2	$x_5^2y_5$	$x_5y_5^2$	Δx_5	Δy_5
1	x_6	y_6	x_6y_6	x_6^2	y_6^2	$x_6^2y_6$	$x_6y_6^2$	Δx_6	Δy_6
1	x_7	y_7	x_7y_7	x_7^2	y_7^2	$x_7^2y_7$	$x_7y_7^2$	Δx_7	Δy_7
1	x_8	y_8	x_8y_8	x_8^2	y_8^2	$x_8^2y_8$	$x_8y_8^2$	Δx_8	Δy_8

ordinates are free of harmonic errors, measurements of photographs will be compensated for leadscrew errors or curvature of the ways arising from changes in the comparator or the use of other comparators to the extent that the errors can be approximated by a quadratic curve.

THE SYSTEMATIC ERROR observed in all film samples after compensation using eight simulated fiducial marks took the form of radial displacements, as illustrated in Figure 10, with somewhat larger displacements outside the area bounded by the fiducial marks. Pursuing the independent correction of x and y-displacements, the x-components may be shown as illustrated in Figure 11. An empirical curve representing the displacements will take the form shown in Figure 12, with the curve becoming zero at points where the error is either zero or random rather than systematic, specifically, at the lines joining fiducial marks. A cubic equation will obviously provide a close approximation to the empirical curve. The general form of the cubic equation is:

$$\Delta x = Ax^3 + Bx^2 + Cx + D.$$

The magnitude of Δx is quite small and the x-coordinate used as the argument is noncritical, therefore, certain approximations are permissible in the derivation of equations. The line joining the center fiducial marks is assumed to have an x-coordinate of zero. Since the corner fiducial marks form an almost perfect square, the x-coordinate of any one of the marks will adequately represent all, and will be indicated by the quantity P in the derivation.

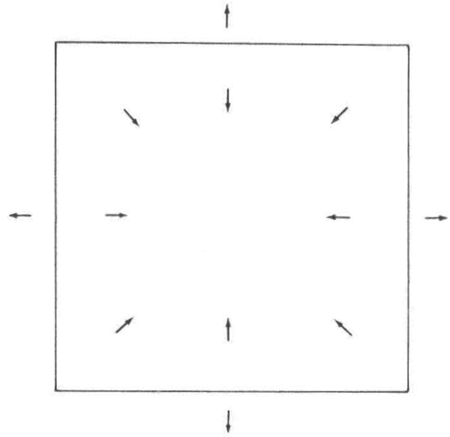

FIG. 10. The form of the residual systematic error observed in all film samples after compensation using eight fiducial marks.

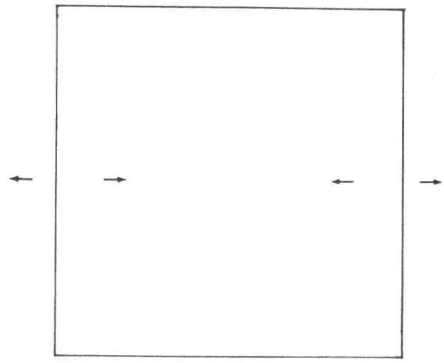

FIG. 11. The form of the independent x-components of the residual systematic error plotted in Figure 12.

As Δx is zero when x is zero, the coefficient D is necessarily zero. Since Δx is zero when $x = \pm P$, substitution gives:

$$AP^3 + BP^2 + CP = 0$$
$$-AP^3 + BP^2 - CP = 0.$$

Solving simultaneously gives:

$$B = 0$$
$$C = -AP^2$$

Modification of the general cubic equation results in:

$$\Delta x = Ax^3 - AP^2x \text{ or}$$
$$\Delta x = A(x^3 - P^2x).$$

The single coefficient, A, determines the degree of curvature of the correction. The symmetry of the aerial photograph leads one to expect that the optimum curvature on the axis may differ somewhat from the optimum near the edges, with no reason to expect

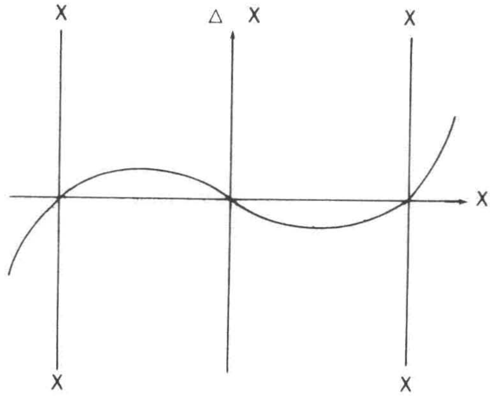

FIG. 12. The empirical curve representing the independent x-components of the residual systematic error.

different systematic behavior for opposite edges in a large number of samples. It seems desirable, then, to allow the coefficient A to vary symmetrically above and below the axis. A quadratic variation appears justified in order to provide the desired symmetry and a smooth transition. This can be expressed in terms of the general quadratic form as:

$$A = By^2 + Cy + D.$$

As it is desired to enforce that A has the same value for positive and negative values of y:

$$By^2 + Cy + D = By^2 - Cy + D.$$

Therefore C must equal zero, leaving:

$$A = By^2 + D.$$

Substituting in the previously derived equation, and renaming the arbitrary coefficient designations in alphabetical order gives:

$$\Delta x = (Ay^2 + B)(x^3 - P^2 x) \quad \text{or}$$
$$\Delta x = Ay^2(x^3 - P^2 x) + B(x^3 - P^2 x).$$

A COMPLICATION arises in an actual aerial camera equipped with eight fiducial marks. The x-coordinates of the fiducial marks at the midpoints of the sides differ from those at the corners by approximately four millimeters and the assumption that the value P is constant is not valid. If the coordinates of the fiducial marks are assigned designations as shown in Figure 13, the x-coordinate, or P, at any point of the line joining the fiducial marks can be found from the equation of the line. On the right side of the photograph, this is:

$$P = P_2 + |y|(1 - P_2/P_1)$$

Since symmetry of the equation has been enforced, the substitution of this expression for P will accommodate the requirements of both sides of the photograph:

$$\Delta x = Ay^2[x^3 - x(P_2 + |y|[1 - P_2/P_1])^2]$$
$$+ B[x^3 - x(P_2 + |y| \cdot [1 - P_2/P_1])^2]$$

This equation may be simplified by factoring, and by noting that the quantity $(1 - P_2/P_1$ is a constant for a particular camera. Denoting this constant by C gives:

$$\Delta x = x(Ay^2 + B)x^2 - (P_2 + C|y|)^2.$$

A parallel derivation for Δy gives:

$$\Delta y = y(A'x^2 + B')y^2 - (P_2 + C|x|)^2.$$

Only two unknown coefficients occur in each equation. Since they cannot be determined from an individual aerial photograph, they can only represent typical values for a particular film. Before applying the correction, the coefficients should be determined by least-square curve fitting methods to many points subject to the systematic error on many film samples. In the derivation of the equations, it is implicit that the positions of the simulated fiducial marks of the film samples must closely approximate those of the actual camera in order to make the same coefficients applicable.

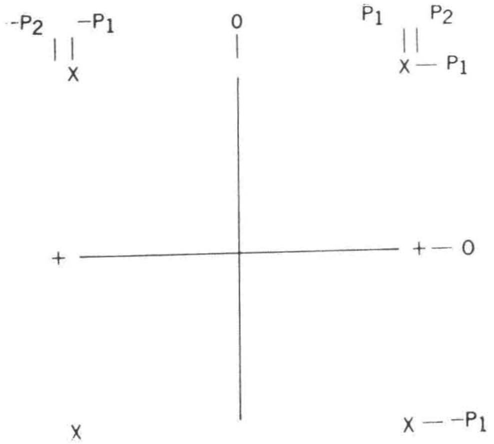

FIG. 13. Designations for x and P used in recognizing that the abscissas of the side fiducial marks differ from those at the corners.

Careful attention must be given to the orientation of the test points on the film. Different corrections are to be expected along and across the film and must be applied accordingly.

It should be emphasized that the testing of this correction is incomplete. Several assumptions were made in the formulation of the equations, and although they are probably correct in essence, it is possible that modifications may be desirable.

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