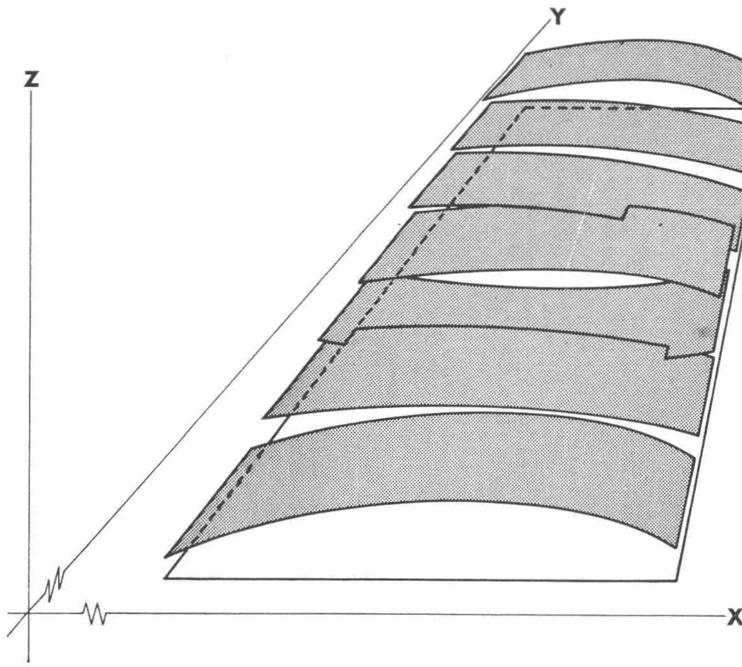

FRONTISPIECE. The error surfaces of seven bridged strips of aerial photographs.

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Computational Methods in the USGS

The U. S. Geological Survey tests fully analytical and semianalytical aerotriangulation techniques reporting remarkably small rms discrepancies of 0.5 feet vertically and 1.2 feet horizontally where $H = 10{,}000$ feet.

(*Abstract on page 886*)

INTRODUCTION

THE TOPOGRAPHIC DIVISION of the U. S. Geological Survey is investigating methods representing two general approaches to aerotriangulation: the fully analytical technique, in which both the triangulation and the adjustment procedures are accomplished computationally, beginning with the measured photocoordinates of pass and control points; and the semianalytical technique, in which the computational input consists of model or strip coordinates determined with

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plotting instruments. In both procedures, the end product is pass-point positions and/or elevations for use in orienting stereomodels in topographic map compilation.

The Fully Analytical Method

DESCRIPTION OF THE SYSTEM

The completely analytical system being developed by the Geological Survey may be described as an iterative, simultaneous, least squares adjustment of exposure-station space positions and camera orientations in compliance with constraints applied to the projective image rays. It is a modification and extension of the method originated by Herget at Ohio State University. Bundles of projective rays for each photograph, defined by the perspective center and the measured photocoordinates of pass and control points, are caused to assume optimum positions and attitudes in space by forcing common pass-point rays to intersect and control-point rays to pass through the ground locations at the control, or to intersect upon geodetically defined surfaces containing the control points. All constraints are enforced simultaneously and repetitively until no further worthwhile improvement results.

The system, which has been programed for the Burroughs 220 computer, is capable of solving problems in units of as many as 22 photographs. The photographs may be arranged either as a single strip or as a block. For larger problems, contiguous 22-photograph blocks can be fitted together after they have been solved and adjusted internally. Switches in the program permit a problem to be solved by either using the complete program or, when conditions permit, in a truncated version which provides a less rigorous solution. The complete program includes provision for modifying the shapes of the bundles of rays to permit correction of errors in photocoordinates and ground control. In the less rigorous solution, the shapes of the ray bundles are held fixed.

It is hoped that the fully analytical system will permit a sufficient reduction in the amount of required ground control to enable it to compete with current analog methods. Theoretically, the system can provide a perfect solution for a block of any size containing only two horizontal control points and, for vertical control, three elevations on any one strip and one elevation on each of the remaining strips. Although, in practice, several times these minimum amounts would always be specified (and all available points would always be used), these vertical control requirements represent, nevertheless, a considerable reduction from present requirements of two vertical points per photograph.

TESTS OF THE SYSTEM

The development of the system is nearly complete. The computer program has been written and is now being tested with both fictitious and real problems.

One series of three tests was made using a seven-photograph strip flown at 10,000 feet above the Coast and Geodetic Survey's McClure, Ohio, test area. In the portion of the test area covered by the strip were about 80 horizontal and vertical control points of first- and second-order accuracy, targeted on the ground by circular concrete platforms 6 or 8 feet in diameter. The photographs, borrowed from the Coast Survey, were exposed on Mylar-base film in a Wild RC8 camera. Photocoordinate measurements were obtained with a Wild stereocomparator.

In the first test, diagrammed in Figure 1, the strip was controlled with 2 horizontal and 6 vertical control points. As the figure shows, 17 horizontal and 13 vertical points were used as pass points in the solution to provide test points. The results of this test are given in Table 1 and are summarized in the first column of Table 2. The results reported in the latter table are based on errors at the test points. It is important to point out that the exact values after convergence depend upon the particular cycle or iteration at which the problem is terminated. Convergence usually occurs after four or five cycles; after this point is reached, the subsequent solution

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values oscillate a few hundredths of a foot around some value. Therefore, the reported results have been rounded to the nearest tenth of a foot to render the oscillations insignificant. It is also important to point out that in all three of these tests the shortened, less rigorous program was used.

The discrepancies in relative orientation for Test 1 are given in Table 3. Here, the middle column gives the amounts by which the two rays for each point, as defined by successive photographs, fail to intersect. To emphasize the extreme smallness of these discrepancies, the values are given in *inches* at ground scale. The right-hand column gives, for points appearing in two successive models, the separation in space between the positions defined by the two models.

The conditions for Test 2 were identical with those for Test 1 except that the solution was divided into two phases: first, the problem was solved to convergence with minimum control—i.e., two horizontal and three vertical points as shown in Figure 2—then the strip thus derived was adjusted vertically by a second-degree equation to the same six vertical control points as were used in Test 1. Eliminating redundant control in the initial solution allowed the photographs to come to a best possible relative orientation, free of the encumbering effects of the control. The results of both phases of Test 2 are given in Table 2. That Test 2 produced slightly better results than Test 1 is not considered significant. What *is* considered significant is that the tests indicate considerable promise for both of these approaches.

Test 3 was identical with Test 1 except that the number of test points was increased several-fold, as indicated in Table 2.

ABSTRACT: *The Geological Survey is investigating methods representing two general computational approaches to aerotriangulation: the fully analytical and the semianalytical techniques. The fully analytical system is an iterative, simultaneous, least squares block solution based on the Herget method. The development of this system is now virtually complete except for such changes as may be indicated by operational tests. In one series of tests just completed, involving a strip of seven photographs flown at 10,000 feet above a targeted test area and controlled by 2 horizontal and 6 vertical points, solutions were obtained in which the errors at withheld control used as test points were on the order of 0.5 foot (RMSE) for elevations and 1.2 feet (RMSE) for positions.*

Four different horizontal semianalytical methods are being developed or tested. In these, interior and relative orientations of overlapping photographs are performed with stereoplotters and absolute orientation and final adjustment of pass-point positions are performed on electronic computers. Each of the methods is designed for a different plotter-computer combination, and each represents a different level of refinement.

A method for adjusting the elevations of points in a block of bridged photographs is also being developed. Error surfaces determined for each strip define adjustments in which each strip is fitted to both the control and to adjoining strips in a repetitive process.

SEMIANALYTICAL METHODS

The development of highly accurate devices for converting mechanical motions to digital form has made it possible to combine instrumental and mathematical aerotriangulation. Thus, advantage can be taken of the expediencies to be found in projection-type stereoplotting instruments and the conveniences and accuracies associated with electronic computations. Interior and relative orientations of overlapping photographs are performed with stereoplotters whereas absolute orientation and the final adjustment of pass-point positions are performed on electronic computers.

The Topographic Division of the U. S. Geological Survey is presently investigating several of these semianalytical methods. Four of them are for establishing horizontal pass points; one is for vertical points. The studies are being made in several different offices of

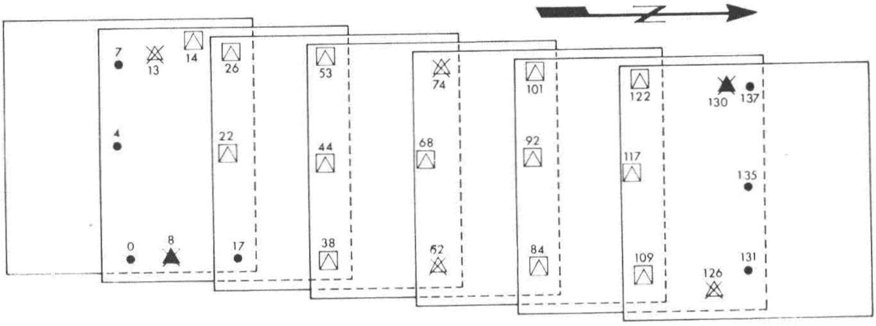

FIG. 1. Diagram showing control, test points, and pass points for McClure Test 1.

the Division, in each case adapting the method to the computer and the photogrammetric equipment available.

HORIZONTAL METHODS

Cross-strip method.—One of the horizontal procedures was specifically designed for application to Kelsh plotters and small computers. Stereomodels for the strip extension are formed with the Kelsh projectors, and the x- and y-coordinates of horizontal pass points are measured in each stereomodel with a tracing table fixed to a coordinatograph. Translational motions of the tracing table are converted to digital form through an electronic conversion unit coupled to the coordinatograph. The measured coordinates are automatically punched on paper tape and recorded on a typewriter.

TABLE 1

RESULTS OF TEST 1, MCCLURE, OHIO

Point	*Horizontal error (feet)*	*Vertical error (feet)*
8*	0.1	0.4
13†	0.3	0.1
14	0.4	0.2
22	0.8	−0.9
26	0.6	0.3
38	1.7	0.5
44	0.9	0.4
53	0.7	−0.5
62†	1.4	0.4
68	0.5	−0.8
74†	0.3	0.0
84	2.3	0.8
92	0.6	0.0
101	0.4	0.6
109	2.0	0.2
117	1.1	0.1
122	0.2	1.2
126†	2.0	0.0
130*	0.0	0.0

* Horizontal and vertical control.
† Vertical control.

After the coordinates of pass points are recorded for each model, the leading photograph is rotated 180° and the tilt angles are reinstated before the next stereomodel is formed. The new model is then scaled to the previous one, using a common vertical pass point (scale transfer point). Thus, a strip of disjointed models is formed, with alternate models rotated 180° (Figure 3) and with all models referred to a common vertical datum.

Computations in the adjustment phase are performed with an RPC 4000 computer. Models are joined analytically to form strips of pass points (Figure 4), and then cross-strips are constructed by the successive joining of two-model sections from adjoining strips. The cross-strips may, if desired, be skewed across the regular strips so as to include the maximum number of ground control points (Figure 5). The cross-strips are fitted to the ground control in a linear transformation in which the strip is held rigid in shape, but translated, rotated, and changed in size. This preliminary fit to the ground control serves two purposes: first, it effects a transformation from instrumental or strip coordinates to the ground coordinate system with a minimum loss of accuracy from roundoff, and, second, it furnishes a means of detecting blunders. Control points with gross disagreements between true ground coordinates and the transformed coordinates may be rejected.

After the linear transformation, the cross-strips are again fitted to ground control, but this time in a second-degree transformation. That is, they undergo bending and nonlinear

TABLE 2

SUMMARY OF RESULTS OF McCLURE, OHIO, TESTS

	Test 1	Test 2		Test 3
		Phase 1 (Before adjustment)	*Phase 2* (After adjustment)*	
Vertical RMSE (feet)	0.6	2.2	0.5	0.7
Max. vertical error (feet)	1.2	3.8	1.0	1.8
Horizontal RMSE (feet)	1.1	1.3	—*	1.3
Max. horizontal error (feet)	2.3	2.6	—*	2.7
No. of horizontal control points	2	2	2	2
No. of vertical control points	6	3	6	6
No. of horizontal test points	17	17	17	78
No. of vertical test points	13	16	13	75

* The Phase 2 adjustment in Test 2 involved only the vertical dimension, hence no horizontal error values are given.

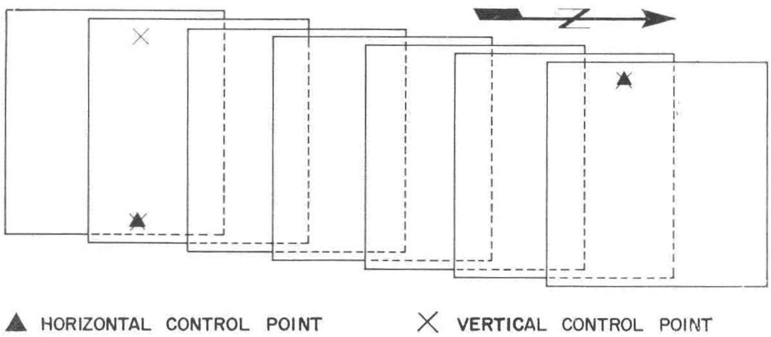

FIG. 2. Control used in McClure Test 2.

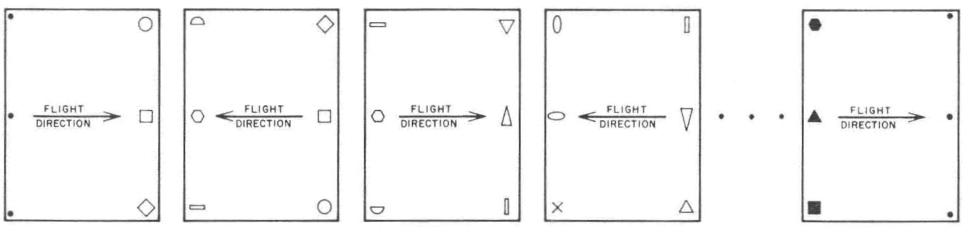

FIG. 3. A strip of models with alternate models rotated 180°. Corresponding pass points are shown with identical geometric figures.

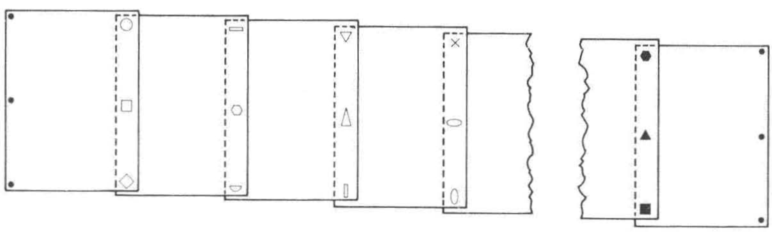

FIG. 4. The strip of models of Figure 3 joined analytically.

scale changes as well as overall translations and rotations in order to fit the control.

A block is formed by a second-degree transformation of the regular strips to the pass points of the cross-strips, and then the block is translated, rotated, and changed in scale in a linear transformation so as to fit ground control in a least squares solution. Discrepancies between tie points of adjoining strips are averaged.

Block adjustment by weighted-mean transformations.—In another horizontal semianalytical system, the x and y model coordinates of pass points are established with either ER-55 projectors or universal plotters. In either case, the instruments are equipped with analog-to-digital converters. If model coordinates are used, individual models are analytically joined to form strips prior to the main adjustment; otherwise, the adjustment begins with instrumentally-determined strip coordinates. All computations are performed on a Burroughs 220 computer.

In joining the strips to form a block, weighted-mean rotations, translations, and scale factors needed to fit the pass points of one strip to those of the adjoining strip are determined by computing these factors from the various combinations of corresponding line lengths and line azimuths that can be formed from as many as 20 tie points common to two adjoining strips (Figure 6).

The second strip is fitted to the first, the third to the second, and so forth, to form a block of strips. The entire block is then fitted to ground control in a linear transformation in which parameters are determined in the same way as for tie points—by computing factors to force the adjusted line lengths and azimuths defined by adjusted control-point positions to agree with the true line lengths and azimuths. In both the strip joins and the fit to control, provisions have been incorporated in the computer program for automatically rejecting points with anomalous discrepancies.

A test of the method showed it to be a very expedient and economical means of adjusting short strips of pass points. Less than 13 minutes of computer time was required to solve a 15-minute quadrangle problem. This method is now considered operational.

Schut-NRC method.—A method of adjusting pass-point coordinates developed by G. H. Schut of the National Research Council of Canada is also being evaluated. Although the procedure is designed to perform both vertical and horizontal adjustments using the same data decks, the Geological Survey is evaluating only the horizontal adjustment capability of the procedure.

The Schut system as applied by the Geological Survey consists of two parts: first, a linear join of models into strips using premarked pass points and a linear fit of the strips to an approximate ground coordinate system; and, second, a simultaneous block

TABLE 3

DISCREPANCIES IN RELATIVE ORIENTATION, TEST 1

Point No.	*Amounts by which rays fail to intersect (inches)*	*Amounts by which models fail to join (inches)*
1	0.216	
4	2.100	
7	3.216	
8	2.700	
13	2.508	
14	2.088	
17	0.156	
17′	2.628	2.424
22	5.928	
22′	2.220	6.576
26	2.532	
26′	2.580	3.864
38	2.280	
38′	3.696	4.716
44	2.148	
44′	1.404	8.424
53	2.004	
53′	5.640	4.728
62	3.216	
62′	0.084	1.692
68	0.504	
68′	0.924	1.140
74	4.668	
74′	6.708	0.636
84	0.696	
84′	0.120	0.192
92	1.344	
92′	5.436	0.468
101	4.800	
101′	4.188	0.252
109	3.240	
109′	1.836	4.740
117	1.176	
117′	1.872	7.224
122	2.304	
122′	2.448	4.212
126	3.792	
130	1.380	
131	1.740	
135	3.228	
137	0.624	

Units are inches on the ground, in all cases. Primes indicate the second model.

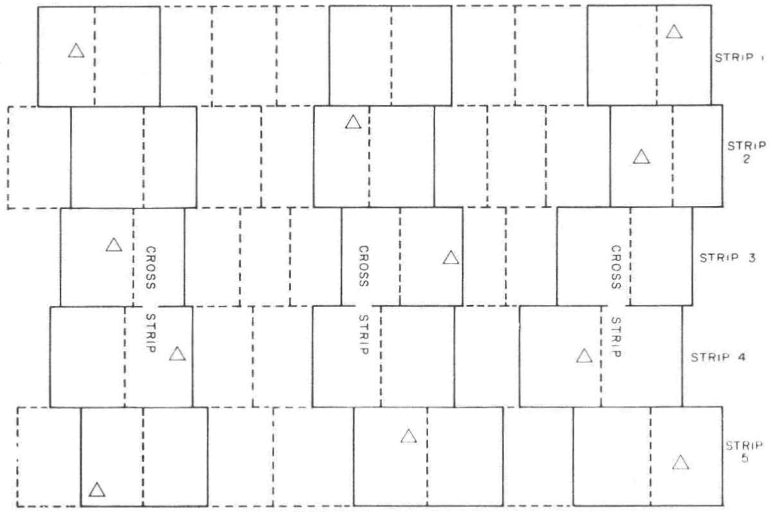

FIG. 5. Cross-strips (solid lines) formed from two-model sections of regular strips.

adjustment of strips to each other and to ground control. The degree of adjustment (linear or nonlinear) and the type of adjustment (vertical or horizontal or both) can be specified.

Block solution method.—A fourth horizontal adjustment procedure has been programed for the Burroughs 220 computer and is now being debugged. Although the method was conceived for adjusting blocks of stereomodels in a linear solution, the program is flexible in that models, sections, strips, or blocks of pass points can be adjusted in either a linear or nonlinear simultaneous solution in which transformation parameters are determined in a least square solution.

The number of units the program will accommodate in a single simultaneous solution is limited to 18 or 28, depending upon the degree of adjustment used. However, the program is capable of handling very large problems. A large area may be divided into blocks of 18 or 28 units each (the units may be models, sections, or strips), and after each block is adjusted to control, it becomes a unit for the final adjustment. The data for each unit is stored on magnetic tape until the final adjustment stage.

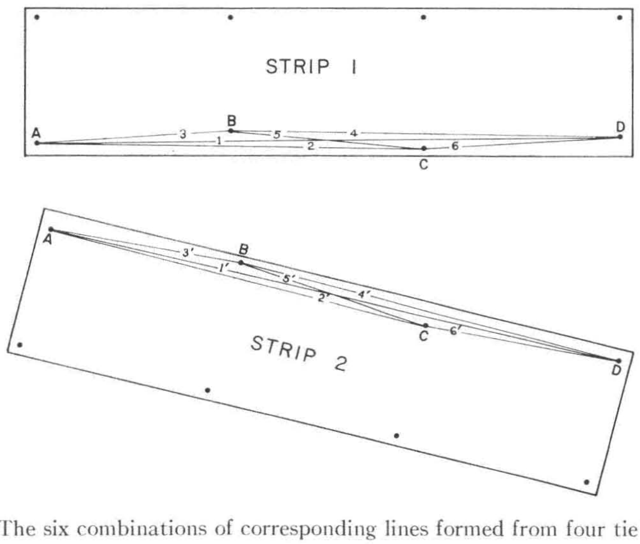

FIG. 6. The six combinations of corresponding lines formed from four tie points.

A VERTICAL BLOCK ADJUSTMENT METHOD

A method for adjusting the elevations of points in a block of bridged photographs has also been programed for the Burroughs 220 computer and tested. In this method, the elevations of pass points determined with bridging instruments are adjusted mathematically by determining elevation error surfaces for all strips in a block (Frontispiece) and correcting all the points in each strip in accordance with the error surface. Each strip is adjusted singly, but a block solution is effected by fitting each strip both to the control and to adjoining strips in a repetitive process, back and forth across the strips, until no further improvement in the adjustment can be made. The solution converges toward that solution which would be obtained if all the strips in a block were adjusted simultaneously. As compared with a simultaneous adjustment, it has the advantage of requiring much less internal computer storage space and, hence, of increasing the size of problems that can be solved with a given computer.

CONCLUSIONS

Although it is hoped that increased accuracy and expediency can be gained by employing one or all of these techniques, it is not anticipated that these methods will completely replace optical or mechanical analog techniques now in use. It is likely that some or all of these methods will be found to have particular advantages under certain operational conditions so that it will be desirable to add them to the growing stock of available techniques. The project planning engineers, then, would have at their command a wider selection of procedures, so that the method could be chosen, in each case, which best fits the project parameters.

ERRATA

Page 389, n. 3, 1965. The name of the Keynote Speaker should read Mr. James E. Webb.

Page 528, n. 3, 1965. The address for Prof. John O. Eichler, member of the Board of Direction of the American Society of Photogrammetry, should read Dept. of Civil Engineering, Georgia Institute of Technology, Atlanta, Georgia.

Page 571, n. 4, 1965. The name of Atlantic Aerial Surveys, Inc., should be added to the lists of Sustaining Members in all 1965 issues. See also page 31 herein.

Page 748, n. 4, 1965. Refer to page 938 herein for a correct listing of the officers of the American Society of Photogrammetry.