MORTON KELLER*
U. S. Coast & Geodetic Survey
Rockville, Maryland 20852

Documented Computer Programs

The first program (Strip Adjustment) of a series on aerotriangulation is now available on request. The complete system, which is in process of documentation, enables the analytic solution for a strip to be computed at the rate of about 4 seconds (costing about 30 cents) per photo.

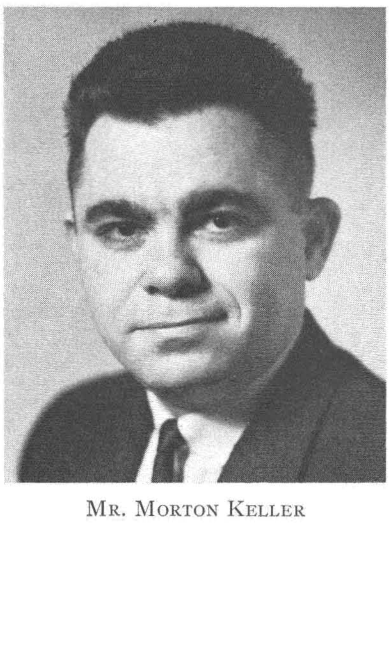
INTRODUCTION

ABOUT eight years ago, the Coast and Geodetic Survey began to explore the possibilities of analytic aerotriangulation in an effort to improve accuracy, to augment its productive capacity in this field, to automate some of the more tedious operations, to avoid the procurement of additional analog plotters, and to exploit the advantages of the Bureau's computer facilities. To this end it was necessary to formulate various mathematical relationships, procure some special instruments, make numerous tests, and devise new computer programs. In 1961 the analytic system of aerotriangulation was put into routine production alongside conventional first-order stereoscopic plotting instruments.

Last year, the Bureau began rewriting its computer programs with a three-fold purpose in mind: first, to convert programs written for obsolete computing equipment into programs for present-day computers; second, to increase their efficiency by combining programs where possible, and incorporating improvements discovered during the past four years of experience with them, and also eliminating all manual decisions; and third, to provide the public with published programs in the Fortran language in order to expedite the application by others of analytical aerotriangulation methods. The purpose of this paper is to report on the progress being made in this effort.

The Coast Survey method is an adaptation, for medium-size electronic computers, of the approach developed by Dr. Hellmut Schmid who is now with the Bureau[1,2]. The system permits the accurate determination of the ground positions of objects appearing on a strip or block of overlapping aerial photographs, using relatively few known ground positions.[3] The digital calculations involved depend on coordinate measurements of pertinent image positions on each photograph. This solution differs from the more conventional instrumental method based on measurements of a stereoscopic model which is solved through the use of an analog device. The analytic solution offers certain worthwhile advantages accruing from automation, digital accuracy, least-squares adjustment, and freedom from the mechanical discrepancies contributed by the plotting instruments.

The system now in operation at the Coast Survey has been referred to as "poor-man's analytics" because of the emphasis on the use of relatively simple instruments of moderate cost and ready availability. The instrumenta-



MR. MORTON KELLER

* Presented at the Annual Convention of the Society in Washington, D. C., March 1965.

tion consists primarily of three components. A stereoscopic point transfer device, the Wild PUG 2, is used first to select, mark and transfer suitable images to adjacent photographs. Next, the coordinates of pertinent images are measured to micron accuracy on a digitized Mann Monocular Comparator. Finally, the observed data are submitted for processing on a modest electronic computer of sufficient capacity and moderate cost located on the premises. Although it seems now to have been firmly established that accurate aerotriangulation *can* be performed with a *mono*comparator, and that a *stereo*comparator is not an absolute necessity, our instrumentation has recently been augmented by the acquisition of a Wild STK-1 Stereocomparator in order to evaluate its potential. Just comparison, however, is not feasible at this time because of our very limited experience with the device.

The mathematical solution was originally programmed for the IBM 650 electronic computer. Because of its limited 2,000 word magnetic drum memory, the 650 processor is not large enough to accept, process, and furnish output in a single operation for all of the data associated with the analytical problem. Consequently it was necessary to use a series of 12 programs to determine the final values. The language required for programming on the 650 precluded the ready adaptation of the program for use on other computers.

The ever-increasing demands for data processing related to scientific computing and fiscal and administrative operations within the Bureau emphasized the need for enlarging the available computer facilities. This led to the acquisition in 1962 of an IBM 1620 computer having a 100,000 digit core storage. However, the problem of computer capacity still persists, and the Coast and Geodetic Survey has negotiated for time on large outside computers such as the IBM 7030 (STRETCH) computer.

In addition to increased versatility, the 1620 has the capability of accepting programs written in the Fortran language. Fortran usage materially shortens the writing of a computer program in comparison to the relatively large amount of expert effort required to develop even a modest program in a machine-oriented language. The ease with which it can be mastered, and the relatively large number of computers that will accept it, have made Fortran notation most popular in the United States. The Coast Survey has trained several of its photogrammetric personnel in Fortran programming because it seemed easier for them to learn to program than to teach professional programmers the mathematics of photogrammetry. This training is now paying dividends as is shown in the consolidation of many small programs to minimize the handling of data. A good example of this is the analytic aerotriangulation solution.

ABSTRACT: *The U. S. Coast and Geodetic Survey is in the process of rewriting its computer programs for Analytic Aerotriangulation in order to increase their efficiency and to eliminate manual decisions wherever possible. The new programs will incorporate numerous improvements which have been uncovered during the past four years experience using the original programs. At the same time, the Bureau has undertaken to publish the programs in the Fortran language for the purpose of expediting the application by the public of analytical aerotriangulation methods. On completion these programs will include: systematic correction of observed image coordinates, three-photo orientation, horizontal and vertical strip adjustment, and block adjustment. It will be possible to separate the programs into smaller components as need be to function on computers of limited storage capacity. The purpose of this paper is to report on the progress being made in this effort.*

THE ANALYTIC AEROTRIANGULATION FORTRAN PROGRAMS

On completion, the analytic aerotriangulation Fortran programs will include: systematic correction of observed image coordinates, camera orientation, horizontal and vertical strip adjustment, and block adjustment. These programs are being written to facilitate their separation or condensation into smaller components as need be to function on computers of limited storage capacity such as a 40K IBM 1620 computer.

The Coast Survey version of the analytic solution calls for three separate programs on the 1620. The first program includes coordinate refinement and a three-photo orientation. However, it may be necessary for organizations having smaller storage capacity computers to break this program into the two individual program steps. The second program phase is a horizontal and vertical strip adjustment.[4,5,6] We are maintaining the strip adjustment as a separate program at this time because of its dual application for both analytical and analogical aerotriangulation. It is realized that coordinate refinement, camera orientation, and the strip adjustment can be combined into a single program by selecting various combinations of control before starting computer operations. This consolidation may be undertaken later if further study justifies the step. A block adjustment constitutes the third and final program in the analytic solution.

Following is a brief description of each program phase.

COORDINATE REFINEMENT PROGRAM

Analytic aerotriangulation computations begin with the refinement of image coordinate values that have been measured with a comparator. The refinement consists of corrections for the calibration of the comparator, the distortion of the aerial camera film, both symmetrical and asymmetrical radial distortion of the aerial camera lens, atmospheric refraction and, if desired, a correction for earth curvature. Also included is a computation of the average of multiple coordinate readings and a mathematical orientation of the measurements that places the X-axis parallel to the line of flight. The program is designed to process data from cameras having either four or eight fiducial marks.

The Coast and Geodetic Survey combines the coordinate refinement and camera orientation into a single program operation. Inasmuch as this program may be too large for small-memory computers, the coordinate refinement phase is being documented as a separate program in a soon-to-be-released technical bulletin by the Bureau. It requires approximately 30K storage as compared to the 54K memory necessary for this phase in the combined program. The reduction in program size has been achieved primarily by substituting a manual acceptability-check of the comparator readings in place of the automatic checking incorporated into the combined operation.

CAMERA ORIENTATION

After all of the coordinate observations have been refined, the program proceeds to the camera orientation phase. The Coast Survey method, at the present time, is built around the processing of the aerial photographs in triplets—the first triplet being photos 1, 2, and 3; the second triplet being photos 2, 3, and 4; etc. This permits a higher degree of automation and accuracy. The basic mathematical concepts for relatively orienting the photographs and attaching successive models are developed in Technical Bulletin No. 21, entitled "Analytic Aerotriangulation," published by the Bureau. A form of Newton's Method is employed to solve the projective transformation equations using least-squares techniques. This is an iterative method based on initial approximations for the unknown camera parameters.

Several novel programming techniques have been resorted to in an effort to reduce computer storage needs. Instead of storing all of the observation equations, the program is designed to take each equation as it becomes available and immediately compute its contribution to the normal equation matrix. Furthermore, the normal equation matrix has been compressed to avoid the storage waste that would arise because of the appearance of a large number of empty locations when this matrix is formed in the conventional manner.

This portion of the analytic aerotriangulation computer operation is now being optimized and will be documented in a technical bulletin as soon as the Fortran writing has been finalized. After we have had some experience with the program, the orientation phase may be enlarged to accommodate five or more photographs in a simultaneous orientation solution.

The card output from the camera orientation program will contain the positions of all pertinent objects in an arbitrary three-dimensional model coordinate system at the approximate scale of the photography. The mathematically-developed bridge is analogous to the product obtained from the conventional stereoscopic plotting instruments

STRIP ADJUSTMENT

No restraints have been imposed on the x, y, and z model coordinates during the computation of the prior orientation phase. As a consequence, the chain of models has been permitted to deviate freely under the influence of systematic and random errors. A small number of the objects have their true ground positions already known from field observations and therefore can serve as control stations for a horizontal and vertical strip adjustment of the bridge data. Such an adjustment requires the derivation of equations for the removal of the systematic errors from the chain of model coordinates and the transformation of the corrected values into true ground positions.

The Coast Survey strip adjustment program includes several features that are considered to be significant:

1. The strip adjustment problem is in three dimensions and of third degree. However, an ideal conformal adjustment of a three-dimensional field using higher degree cannot exist. Strip adjustment is therefore separated into a horizontal and vertical adjustment inasmuch as a conformal transformation in a plane is not limited as to degree. A practical solution to the dilemma is offered by a one-step iteration that results in a geometric integration of the two types of corrections. The integration is accomplished by a preliminary vertical adjustment applying slope corrections to the horizontal coordinates, and horizontal oriented scale corrections to vertical coordinates.
2. The program contains a variable-degree facility that allows the selection of a third, second, or first degree polynomial correction in the horizontal direction, and also in the vertical direction independent of the horizontal choice. The variable facility permits this same program to be applied to very short strips as well as long ones.
3. The strip adjustment program will accept analytic and also analog input data. The same program can therefore service aerotriangulated bridges developed either by computational methods or by conventional stereoscopic plotting instruments.
4. Finally, the program provides for an output display of information that aids in the diagnosis of control data validity.

The adjustment program has already been documented in Technical Bulletin No. 23, entitled "Aerotriangulation Strip Adjustment," and is available on request from the Bureau.

Block adjustment

The final stage in the solution of the analytic aerotriangulation problem is block adjustment. This is the simultaneous solution of the absolute orientation of all the photographs. A block adjustment program has been written for the IBM 7030 but it has not been documented. We anticipate rewriting the program to incorporate new thoughts on the subject, and to optimize the automation features. It will then be documented for distribution to interested organizations.

Conclusion

The coordinate refinement and camera orientation program was successfully applied to real live data for the first time in April. Additional tests of the program are now being conducted prior to declaring it operational in the Bureau.

It is worth noting that the solution of a triplet, involving two or three iterations, takes about four seconds on the 7030 STRETCH computer and costs about 30 cents. The computational cost of a strip of photographs including strip adjustment, therefore, is 35 to 40 cents per photograph.

The advancement of the analytical aerotriangulation art has taken place because of, and concurrent with, the development of increasingly sophisticated automatic data processing equipment. The rapid proliferation of computing facilities throughout the country has also increased their accessibility to large and small mapping organizations. However, the one who develops an analytic aerotriangulation system is severely punished by the cost and effort of computer programming. Consequently, the adoption or adaptation of obtainable analytic programs at small cost in the future seems to be inevitable. The Coast and Geodetic Survey hopes that its decision to make the results of its programming work available to the public will be of some significance in expanding the interest in analytic aerotriangulation techniques among other members of the profession.

References

1. Schmid, H., "An Analytical Treatment of the Problem of Triangulation by Stereophotogrammetry," BRL Report 961, 1955.
2. Harris, W. D.; Tewinkel, G. C.; and Whitten, C. A., "Analytic Aerotriangulation," Technical Bulletin No. 21, U. S. Coast and Geodetic Survey, July 1962; also published in PHOTOGRAMMETRIC ENGINEERING, v. 28, n. 1, page 44, March 1962.
3. Harris, W. D., "Report on Analytic Aerotriangulation," PHOTOGRAMMETRIC ENGINEERING, v. 28, n. 3, July 1962.
4. Harris, W. D., "Aerotriangulation Adjustment of Instrument Data by Computational Methods," Technical Bulletin No. 1, U. S. Coast and Geodetic Survey, January 1958.
5. Harris, W. D., "Vertical Adjustment of Instrument Aerotriangulation by Computational Methods," Technical Bulletin No. 10, U. S. Coast and Geodetic Survey, September 1959.
6. Keller, M., and Tewinkel, G. C., "Aerotriangulation Strip Adjustment," Technical Bulletin No. 23, U. S. Coast and Geodetic Survey, August 1964.