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Microdensitometer Transfer Function Correction

ABSTRACT: *Present methods of determining modulation transfer functions from edge gradients such as that given by Scott, Scott and Shack*[1] *provide only an approximate correction for the effect of the microdensitometer transfer function by dividing the Fourier transform of the exposure image by the microdensitometer transfer function,* $T_\mu(k)$*, rather than by dividing the transform of the microdensitometer output by* $T_\mu(k)$*. The magnitude of this error is estimated by calculations with a simplified, though fairly realistic, camera-atmosphere-film-microdensitometer system.*

INTRODUCTION

THE IMPORTANCE of edge gradient analysis has arisen in the last few years from the need for photo-interpreters and systems maintenance personnel to know the in-flight performance of aerial camera systems. A knowledge of the transfer function of the total system along with a knowledge of the transfer functions of such components as film, atmosphere, micro-densitometer, etc., theoretically enables one to deduce the transfer function of the camera system itself. This transfer function not only yields a great deal of information about the performance of the camera system but also gives important clues as to the sources of degradations in the performance of the system.

Theoretically the transfer functions can be deduced from any known object with sufficient contrast but practically the analysis is much too laborious for any but the simplest targets. Ideal targets are arrays of sine wave or square wave (bar) targets of all spatial frequencies of interest. However, images of such ideal targets are seldom found on reconnaissance photographs. A slightly more difficult image to analyze is that of a step function in brightness—an edge gradient. The image of such a target does occur fairly often in reconnaissance photographs such as in images of sidelighted roofs, changes in paving materials of airfields, parking lots, etc.

The analysis of edge gradients has been the subject of a number of investigations in the last few years[1–6] In most of the procedures proposed to date, the deduction of the camera-atmosphere transfer function has proceeded somewhat as follows. A suitable edge is located on the film and the variation of the transmission of the image on the film in a direction perpendicular to the edge is determined by means of a microdensitometer. The transmission of a series of controlled exposure steps on the same film is also measured to furnish the proper relation between exposure and transmission. The transmission as measured by the microdensitometer is then converted by the above relations to the corresponding exposure edge gradient. From this edge gradient the modulation transfer

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function is determined in one of a number of ways.

The trouble with this procedure is that the microdensitometer itself has a transfer function less than unity and eventually a cut-off frequency, so that the microdensitometer output is not transmission and the wrong function is converted to exposure units. The final transfer function deduced is less than the actual transfer function. An attempt is made in some procedures to compensate for degradation caused by the microdensitometer by dividing the final transfer function by the microdensitometer transfer function. This is obviously not the true correction since the wrong transmission function has still been converted to exposure units. That is, the correction for the microdensitometer has been made on the wrong (exposure) side of the *H* and *D* curve.

Outline of Procedure

Several possible correct methods exist for correcting the effect of the microdensitometer but they are all equivalent to deriving the exposure edge from the transmission edge after the microdensitometer observation of the transmission edge has been corrected for the degradation caused by the microdensitometer. All these methods, however, involve much more effort and programming than the presently used incorrect method of determining the transfer function. For this reason it was thought desirable to get some reasonable estimate of the magnitude of the error in the presently used procedure. In order to estimate this error as simply as possible, the following method was used:

(a). Assume a simplified but realistic camera-atmosphere transfer function and similar simplified transfer functions for other

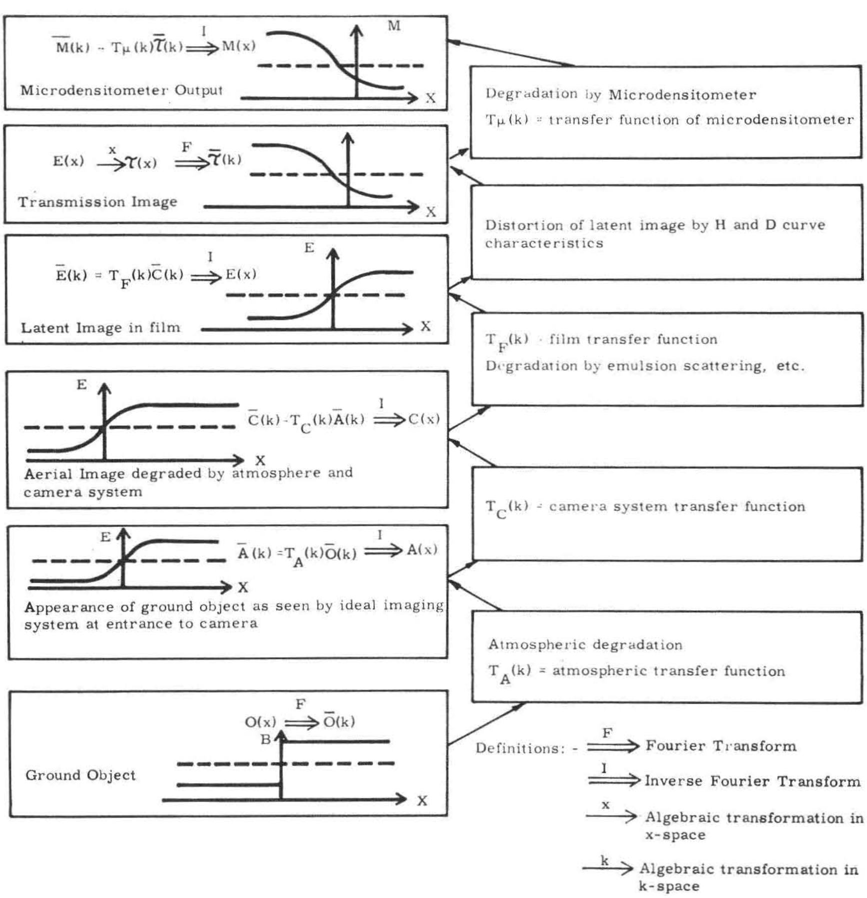

FIG. 1. Schematic diagram of steps in the formation of the microdensitometer output.

TABLE 1

DERIVATION OF MICRODENSITOMETER OUTPUT WITH DEFINITIONS AND SIMPLIFYING ASSUMPTIONS

Assumed Transfer Functions	*Assumed H & D Relationship*
$T_A(k)T_c(k) = e^{-Ck^2}$	$\gamma = \text{const.} = 2$
$T_F(k) = e^{-Fk^2}$	or $D = D_{\min} + 2 \log_{10}\left(\frac{E}{E_{\min}}\right)$
$T_\mu(k) = e^{-\mu k^2}$	or $T = T_{\max}\left(\frac{E_{\min}}{E}\right)^2$

Assumed equations of the object and successive images

$$A = \frac{E_{\max} + E_{\min}}{2}$$

$$B = \frac{E_{\max} - E_{\min}}{2}$$

$$O(x) = A + B \operatorname{Sgn} x \quad \overset{F}{\Longrightarrow} \bar{O}(k) = A\delta(k) + \frac{B}{i\pi k}$$

$$\downarrow k$$

$$C(x) = A + B \operatorname{erf}\left(\frac{\pi x}{\sqrt{C}}\right) \quad \overset{I}{\Longleftarrow} \bar{C}(k) = T_A(k)T_c(k)\bar{O}(k) = e^{-Ck^2}\left[A\delta(k) + \frac{B}{i\pi k}\right]$$

$$\downarrow k$$

$$E(x) = L(x) = A + B \operatorname{erf}\left(\frac{\pi x}{\sqrt{C+F}}\right) \quad \overset{I}{\Longleftarrow} \bar{L}(k) = T_F(k)\bar{C}(k) = e^{-(C+F)k^2}\left[A\delta(k) + \frac{B}{i\pi k}\right]$$

$$x \downarrow$$

$$\tau(x) = T_{\text{ma}}\left(\frac{E_{\min}}{E}\right)^2 = \frac{4T_{\max}}{\left[\left(\sqrt{\frac{T_{\max}}{T_{\min}}} + 1\right) + \left(\sqrt{\frac{T_{\max}}{T_{\min}}} - 1\right) \operatorname{erf}\left(\frac{\pi x}{\sqrt{C+F}}\right)\right]^2} \quad \overset{F}{\Longrightarrow} \bar{\tau}(k)$$

$$\downarrow k$$

$$M(x) \overset{I}{\Longleftarrow} \bar{M}(k) = e^{-\mu k^2}\bar{\tau}(k)$$

components degrading the final microdensitometer output.

(b). Assume a simplified but reasonable H, and D, relationship.

(c). From these relationships mathematically synthesize the microdensitometer output for an edge object.

(d). Use this synthesized microdensitometer output with the assumed transmission-exposure relationship to compute the camera-atmosphere transfer function by the presently used method.

(e). Compare this computed transfer function with the original assumed one to determine the error.

Figure 1 shows schematically the usual process by which the microdensitometer output is generated from the object. The atmospheric image is that formed by an ideal imaging system viewing the step through the disturbing atmosphere. The camera system then further degrades the original object at the image plane and the latent image is further degraded by film scattering and grain. This image is then distorted by the H and D relationships and the resulting transmission image is finally degraded by the microdensitometer. The images, corresponding Fourier transforms, transfer functions and the H and D relationship are shown schematically in the figure.

Table 1 shows the actual relations assumed and the steps taken in the synthesis. Note that all transfer functions are assumed to be gaussian with reasonable parameters and the D vs. *log* E curve is assumed linear with $\gamma = 2$.

DETAILS OF SYNTHESIS

The analytical synthesis of the microdensitometer output from the transmission output was performed as follows: The odd and even parts of the transmission function were approximated by the analytically transformable functions shown in Table 2. The approximations with appropriate values of the curve-fitting parameters turned out to be quite good. The error was less than 0.005 per cent of the total transmission for the even part of the transmission and less than 0.03 per cent for the odd part. Also shown in Table 2 is the frequency representation of the transmission, the conversion to frequency

TABLE 2

APPROXIMATION OF $\tau(x)$ BY ANALYTICALLY TRANSFORMABLE FUNCTIONS. SYNTHESIS OF $M(x)$ FROM THESE FUNCTIONS

$$\tau(x) = \frac{2T_{\max}}{a^2}\left[\left(1+\frac{b}{a}\operatorname{erf}(hx)\right)^{-2} + \left(1-\frac{b}{a}\operatorname{erf}(hx)\right)^{-2}\right]_e \qquad \text{where} \quad a = \sqrt{\frac{T_{\max}}{T_{\min}}}+1$$

$$+\frac{2T_{\max}}{a^2}\left[\left(1+\frac{b}{a}\operatorname{erf}(hx)\right)^{-2} - \left(1-\frac{b}{a}\operatorname{erf}(hx)\right)^{-2}\right]_0 \qquad b = \sqrt{\frac{T_{\max}}{T_{\min}}}-1$$

$$= \tau_e(x) + \tau_0(x) \qquad h = \frac{\pi}{\sqrt{C+F}}$$

$$\tau_e(x) = \frac{(T_{\max}+T_{\min})}{2} - \left[\frac{(T_{\max}-T_{\min})}{2} - \frac{4T_{\max}}{a^2}\right]e^{-p^2x^2} - A_e x e^{-q^2x^2}\sin(2\pi Sx)$$

Curve-fitting parameters

$A_e, A_0, p, q, h_0, S, a_0, b_0$

$$\tau_0(x) = -\frac{(T_{\max}-T_{\min})}{2}\operatorname{erf}(h_0x) - A_0 x e^{-b_0^2x^2}(1-e^{-a_0^2x^2})$$

$$\bar{\tau}_e(k) = \frac{(T_{\max}+T_{\min})}{2}\delta(k) - \left[\frac{(T_{\max}-T_{\min})}{2} - \frac{-4T_{\max}}{a^2}\right]\left(\frac{\sqrt{\pi}}{p}\right)(e)^{-\pi^2k^2/p2}$$

$$-A_e\frac{\pi^{3/2}k}{2q^3}\left[\left(1+\frac{S}{k}\right)(e)^{-\pi^2(k+S)^2/q^2} - \left(1-\frac{S}{k}\right)(e)^{-\pi^2(k-S)^2/q^2}\right]$$

$$\bar{\tau}_0(k) = -\frac{(T_{\max}-T_{\min})}{2}\,\frac{(e)^{-\pi^2k^2/h_0^2}}{i\pi k} + \frac{iA_0\pi^{3/2}k}{2b_0^3}\left[(e)^{-\pi^2k^2/b^2} - \frac{(e)^{-\pi^2k^2/(a_0^2+b_0^2)}}{\left[1+\left(\frac{a_0}{b_0}\right)^2\right]^{3/2}}\right]$$

$$\overline{M}_e(k) = (e)^{-\mu k^2}\bar{\tau}_e(k)$$

$$\overline{M}_0(k) = (e)^{-\mu k^2}\bar{\tau}_0(k)$$

$$M_e(x) = \frac{(T_{\max}+T_{\min})}{2} - \left[\frac{(T_{\max}-T_{\min})}{2} - \frac{4T_{\max}}{a^2}\right]\frac{\exp\left\{\dfrac{-p^2x^2}{1+\dfrac{\mu p^2}{\pi^2}}\right\}}{\sqrt{1+\dfrac{\mu p^2}{\pi^2}}}$$

$$-A_e\frac{\dfrac{S\mu}{\pi}\exp\left\{\dfrac{-S^2\mu-q^2x^2}{1+\dfrac{\mu q^2}{\pi^2}}\right\}}{\left(1+\dfrac{\mu q^2}{\pi^2}\right)^{3/2}}\left[\cos\left\{\frac{2\pi Sx}{1+\dfrac{\mu q^2}{\pi^2}}\right\} + \frac{\pi x\sin}{S\mu}\left\{\frac{2\pi Sx}{1+\dfrac{\mu q^2}{\pi^2}}\right\}\right]$$

$$M_0(x) = \frac{-(T_{\max}-T_{\min})}{2}\operatorname{erf}\left\{\frac{h_0x}{\sqrt{1+\dfrac{\mu h_0^2}{\pi^2}}}\right\} - A_0x\left[\frac{\exp\left\{\dfrac{-b_0^2x^2}{1+\dfrac{\mu b_0^2}{\pi^2}}\right\}}{\left(1+\dfrac{\mu b_0^2}{\pi^2}\right)^{3/2}} - \frac{\exp\left\{\dfrac{-[a_0^2+b_0^2]x^2}{1+\mu\dfrac{(a_0^2+b_0^2)}{\pi^2}}\right\}}{\left(1+\dfrac{\mu(a_0^2+b_0^2)}{\pi^2}\right)^{3/2}}\right]$$

representation of the microdensitometer output, and the microdensitometer output transformed back to x-space.

The exact values of the parameters of the microdensistometer output are shown in Table 3. This figure also shows the value of the different transfer functions at 200 cycles per millimeter from which the parameters were derived. Only one parameter apiece was used for the camera-atmosphere and film transfer functions but seven different values were used for the microdensitometer transfer function.

The exposure and exposure-transmission relation with the resulting transmission image and microdensitometer output with $\mu = 12.767$ microns²/cycle² are shown in Figure 2.

RESULTS AND DISCUSSION

Table 4 compares schematically a correct process for computing the camera-atmo-

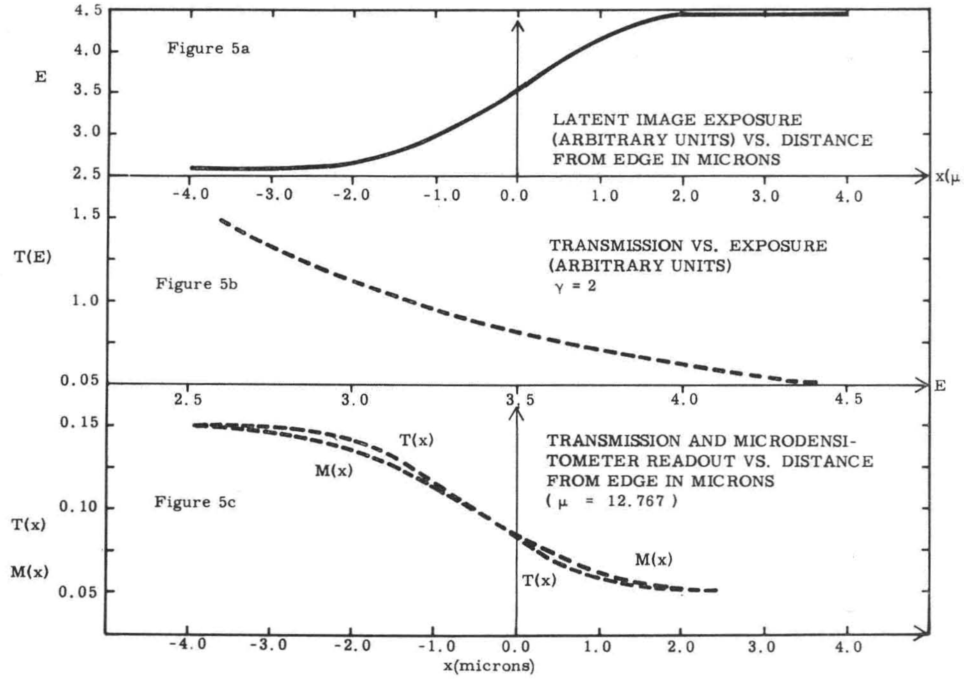

FIG. 2. E, T(E), T(x), M(x) versus Distance.

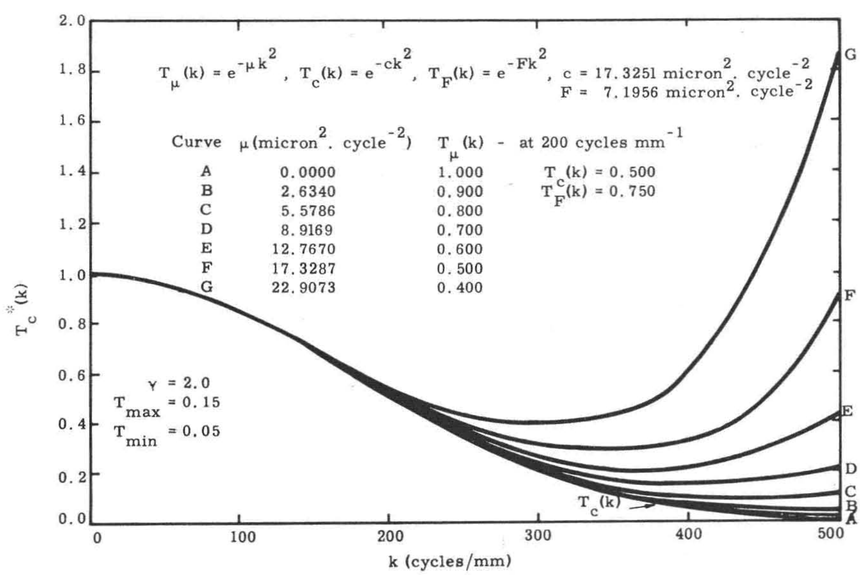

FIG. 3. Effect of microdensitometer transfer function on error in present method of estimating camera-atmosphere transfer functions.

TABLE 3

EXPRESSIONS AND PARAMETERS FOR MICRODENSITOMETER OUTPUT

$$\frac{(T_{\max}+T_{\min})}{2} = 0.1000000,\quad \frac{p^2}{1+\left(\frac{p^2}{\pi^2}\right)\mu} = \frac{0.462}{1+0.04681\mu},\quad \frac{S^2\mu+q^2x^2}{1+\left(\frac{q^2}{\pi^2}\right)\mu} = \frac{0.14793\mu+0.336x^2}{1+0.034044\mu}$$

$$\left[\frac{(T_{\max}-T_{\min}}{2} - \frac{4T_{\max})}{a^2}\right] = 0.0196152,\ A_e\left(\frac{S}{\pi}\right) = 9.3044\cdot 10^{-6},\ \frac{\pi}{S} = 8.16813,\ \frac{2\pi S}{1+\left(\frac{q^2}{\pi^2}\right)\mu}$$

$$= \frac{2.41661}{1+0.034044\mu}$$

$$M_e(x) = 0.1000000 - \frac{0.0196152}{\sqrt{1+0.04681\mu}}\exp\left(\frac{-0.462x^2}{1+0.04681\mu}\right)$$
$$- 9.3044\cdot 10^{-6}\mu\,\frac{\exp\left(-\left[\frac{0.14793\mu+0.336x^2}{1+0.034044\mu}\right]\right)}{(1+0.034044\mu)^{3/2}}$$
$$\cdot\left[\cos\left(\frac{2.41661x}{1+0.034044\mu}\right) + 8.16813\left(\frac{x}{\mu}\right)\sin\left(\frac{2.41661x}{1+0.034044\mu}\right)\right]$$

$$\frac{(T_{\max}-T_{\min})}{2} = 0.05000000,\ A_0 = 4.271\cdot 10^{-3}\text{ microns}^{-1},\ h_0 = 0.5470\text{ microns}^{-1}\ \frac{b_0^2}{1+\left(\frac{b_0^2}{\pi^2}\right)\mu}$$

$$= \frac{0.342}{1+0.034652\mu}\text{ cycles}^2/\text{micron}^2,\ \frac{(a_0^2+b_0^2)}{1+\frac{(a_0^2+b_0^2)\mu}{\pi^2}}$$

$$= \frac{0.622}{1+0.063022\mu}\text{ cycles}^2/\text{micron}^2,\ 1+\left(\frac{h_0^2}{\pi^2}\right)\mu = 1+0.030316\mu,$$

$$M_0(x) = -\ 0.05000000\ \mathrm{erf}\left(\frac{0.5470x}{\sqrt{1+0.030316\mu}}\right)$$
$$- 4.271\cdot 10^{-3}x\left[\frac{\exp\left(\frac{-0.342x^2}{1+0.034652\mu}\right)}{(1+0.034652\mu)^{3/2}} - \frac{\exp\left(\frac{-0.622x^2}{1+0.063022\mu}\right)}{(1+0.063022\mu)^{3/2}}\right]$$

At 200 cy/mm., $T_c(k)=0.500$ (=0.2 cy/μ)	$C=17.325$ micron²/cycle²
	$T_cT_F=0.375$
$T_F(k)=0.750$	$F=7.1956$
$T_\mu(k)=0.300$	$\mu=30.09933$ micron²/cycle²
$=0.400$	$=22.9073$
$=0.500$	$=17.32868$
$=0.600$	$=12.767$
$=0.700$	$=8.9169$
$=0.800$	$=5.5786$
$=0.900$	$=2.6340$

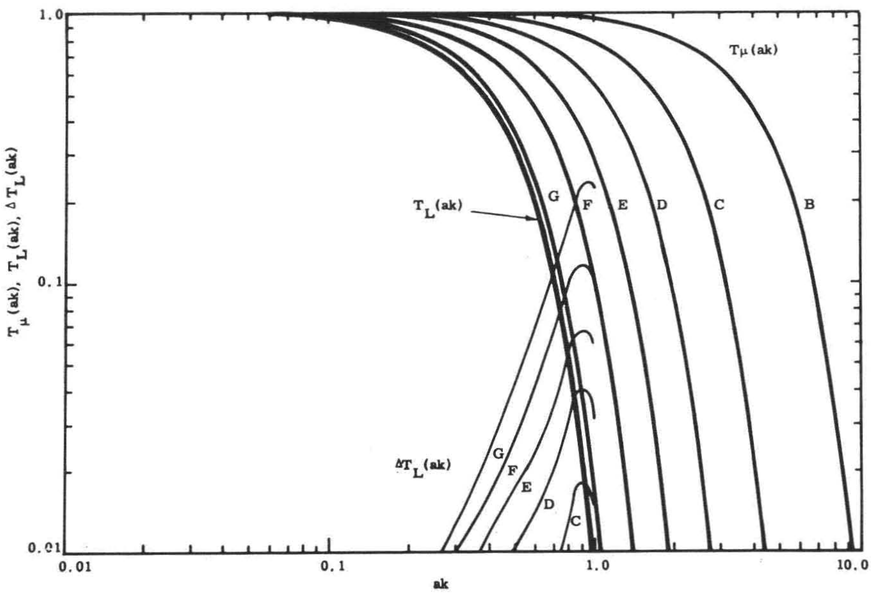

FIG. 4. Error in modulation of latent image due to improper correction for microdensitometer relative to modulation of latent image.

sphere transfer function with the one presently used. The microdensitometer output as computed from the relations of Table 3 was then used as input for the present program. The results are plotted in Figure 3. As can be seen the result of the program matches the assumed camera-atmosphere transfer function only when $\mu=0$, (i.e., the microdensitometer transfer function is always unity) as is to be expected. All other microdensitometer transfer functions result in an overly optimistic estimate of the camera-atmosphere transfer function—in some cases extremely inaccurate at the higher spatial frequencies.

Table 5 and Figure 4 show the relationships involved in a normalized presentation of the error in the computed modulation of the latent image (film plus camera-atmosphere modulation) due to use of the present improper method. The relationship of microdensitometer transfer function, $T\mu(ak)$, to latent image modulation, $T_l(ak)$, is also shown. Thus, from these graphs an approximation of the error can be made when the transfer functions are approximately gaussian, the modulations at some frequency are approximately known and the slope of the H & D curve is about 2.

It should be noted that when $\gamma=1$ on the linear portion of the H & D curve the correc-

TABLE 4

COMPARISON OF PROCESSES FOR ESTIMATING CAMERA TRANSFER FUNCTION

A correct process

$$M(x) \overset{F}{\Longrightarrow} \overline{M}(k) \overset{k}{\rightarrow} \tau(k) = \frac{\overline{M}(k)}{T_\mu(k)} \overset{I}{\Longrightarrow} \tau(x) \xrightarrow{x(H.\&D.)} E(x) \xrightarrow{\frac{d}{dx}} L(x) = \frac{dE(x)}{dx} \overset{F}{\Longrightarrow} \overline{L}(k)$$

$$\downarrow k$$

$$T_c(k) = \frac{\overline{L}(k)}{L(k=0)T_F(k)T_A(k)}$$

The presently used incorrect process

$$M(x) \xrightarrow{x(H.\&D.)} E^*(x) \overset{F+D}{\Longrightarrow} \frac{\overline{L}^*(k)}{\overline{L}^*(k=0)} \overset{k}{\rightarrow} T^*_c(k) = \frac{\overline{L}^*(k)}{\overline{L}^*(k=0)T_A(k)T_F(k)T_\mu(k)}$$

TABLE 5

RELATIONS INVOLVED IN CONSTRUCTION OF CURVES OF FIGURE 4

$$T_L(ak) = e^{-(2/M)(ak)^2} = e^{-Lk^2}$$
$$T_\mu(ak) = e^{-(2/M)(\mu/L)(ak)^2} = e^{-\mu k^2}$$
$$\Delta T_L(ak) = T_L^*(ak) - T_L(ak)$$

$$L = C + F,\ a = \sqrt{\frac{ML}{2}},\ M = \text{Log}_{10}\, e = 0.434294$$

By definition $T(ak)=0.01$ at cut-off, so limiting frequency of Microdensitometer is $(ak)_L = L/\mu$

Curve	$(ak)_L$
B	9.3110
C	4.3956
D	2.7503
E	1.9205
F	1.4144
G	1.0705

tion is, in general, still necessary. In this case the exposure vs. transmission curve is still not linear but is hyperbolic. However, if the range of the transmission is quite small, no correction is necessary for any smooth H & D curve since any small portion of the curve would be approximately linear. In this case, the accuracy of the calculation would be more limited by the noise level of the system than by the non-linearity of the exposure vs. transmission curve.

CONCLUSION

These results indicate that under some conditions a correct method of computing the desired transfer function is mandatory, under other conditions, sufficient accuracy can be obtained from use of the graph of Figure 4, and that when the microdensitometer transfer function cut-off is much higher than that of the latent image (about 9 or 10 times) no correction may be necessary.

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