Ronald G. Davis*
D. Brown Associates, Inc.
Eau Gallie, Florida†

Analytical Adjustment of Large Blocks

The normal equation matrix is "collapsed" and a Block Successive Over Relaxation Method is applied to yield a practical and rigorous solution for very large photogrammetric nets.

(Abstract on next page)

Introduction

Since the advent of large-scale high-speed digital electronic computers in the early 1950's, considerable effort has been directed toward applying numerical techniques to solve the fundamental problems of photogrammetry. An early goal of these efforts was the development of a practical, rigorous and nonrestrictive technique which would permit the simultaneous adjustment of the entire store of observational material arising from a general photogrammetric net. In 1958 Duane Brown developed a least squares theory which was sufficiently general and comprehensive to effect such a solution. Application of this theory has been highly successful in a variety of applications wherein the normal equations arising from the adjustment were of modest order. Attempts, however, to apply this, and other prominent theories, to the task of performing aero-triangulation and control extension by simultaneously adjusting large photogrammetric blocks have proven impractical or impossible. In all approaches reported to date, direct solution of the normal equations has been attempted by one or another of the many variants of Gaussian elimination. This has set a practical limit (on the order of 25 photographs) to the size of the photogrammetric net which can be handled, for computational difficulties with Gaussian elimination, due to roundoff and truncation, increase severely with an increasing number of unknowns. In addition, the number of computations required for the formation and solution of the normal equations increase as the square and cube respectively, of the number of unknowns.

Because of this, interest toward implementation of a rigorous analytical adjustment of large photogrammetric blocks has largely died out in favor of suboptimal solutions which are compromises based solely on computational considerations. These solutions have largely consisted of piecewise adjustments ranging from extension by analytical pairs, triplets or sub-blocks, with subsequent adjustment of the model to absolute control, to the adjustment of strips on blocks of modest size (typically 25 photos or less).

Nonetheless, the theoretical merits of simultaneous adjustment are conceded by almost all, and the military and commercial needs for such an adjustment have continued to mount. Consequently, D. Brown Associates, Inc., under the sponsorship of the Rome Air Development Center (RADC), began a study early in 1963 which sought to evaluate an approach designed to overcome the limitations which have precluded the successful application of Brown's solution (1958a, 1958b, 1959) to the simultaneous adjustment of large photogrammetric blocks. This paper briefly describes the rationale of this approach and outlines some of the results achieved. A full and detailed treatment of this study is to be found in Brown, Davis and Johnson (1964).

Rationale of Approach to the Problem

It was evident that if large blocks of photography were to be successfully adjusted as organic units, an effective alternative to the

* Presented at the Annual Convention of the American Society of Photogrammetry, Washington, D. C., March 1965. The work reported in this paper was performed under sponsorship of the Air Force Systems Command Research and Technology Division, Rome Air Development Center, Griffiss Air Force Base, New York, under contract No. AF30 (602)-3007.

† Mr. Davis is now located at the Washington branch office, 7100 Baltimore Ave., College Park, Md.

reduction of the normal equations had to be developed.

In an attempt to alleviate the undesirable consequences arising from the round-off error introduced by direct methods of normal equation solution, attention was directed toward recently developed matrix iterative analysis techniques to accomplish this solution. The advantage of such methods is that the original system of equations, or some simple transformation thereof, remains unaltered throughout all stages of calculation, and thus a great stabilizing factor is contributed to the process. Consideration was also given to the fact that, for large blocks of aerial photography possessing a fairly consistent pattern of forward and side overlap and a consistent pattern of control or pass points, such as the 4-by-5 block illustrated in Figure 1, the coefficient matrix of the general normal equations is both highly patterned and consists predominantly of zero elements, as shown in Figure 2*a*. This suggested the possibility of:

1. Utilizing an appropriate indexing algorithm to collapse the coefficient matrix of this system of equations into a far more compact matrix consisting of few or no zero elements;
2. Computing this 'collapsed system' of normal equations directly, and thus bypassing the unnecessary computation of zero elements;
3. Formulating a computing algorithm to exploit the collapsed system of normal equations to conceivably effect a practical solution by means of the aforementioned iterative procedures which are vast improvements over the classical methods;
4. Employing a process, which was named "intertwining," to rearrange the coefficient matrix of the general normal equations in such a manner as to achieve a highly diagonal form which could conceivably be conducive to a more rapid convergence of the iterative process.

The logic of generating only the nonzero constituents of the normal equations is a relatively simple and straight forward matter. Similarly, the coefficient matrix of the general normal equations can be "collapsed" to the compact form indicated in Figure 2b and an algorithm can be established by means of which the elements of the collapsed matrix can be related to their counterparts in the original matrix. This permits the collapsed system of equations to be formed directly, and as a result, the total computational time in setting up the system increases only *linearly* with the number of photographs.

ABSTRACT: *An investigation developed a highly practical approach to the problem of the rigorous simultaneous analytical adjustment of very large aerial photogrammetric nets. Difficult problems are associated with the implementation of the uncompromisingly rigorous analytical adjustment of large nets. A general approach to the solution of these problems is utilized. The photogrammetric theory essential to the approach includes an extended and refined version of an earlier theory, as developed by Duane C. Brown. Several specific approaches, within the framework of the general approach, are compared to determine the particular variant of numerical and computational techniques that leads to the most effective results. The method is completely general and nonrestrictive and will accommodate any conceivable configuration of ground control data and combination of auxiliary sensors.*

In Figure 2a, it is noted that considerable seperation exists between submatrices associated with elements of orientation and submatrices associated with control points. This raised questions as to the role of the internal arrangement of the normal equations and suggested the possibility of the existence of some optimal internal arrangement which might offer significant advantages. Considerable effort was expended to devise 'intertwined' normal equation arrangements which were highly diagonal in character and which also placed the coefficients of the unknowns corresponding to a given control point as close in the matrix as possible to the coefficient of those photos on which the point appears. Figure 3 illustrates one such intertwining arrangement as applied to the 3 by 5 block of photographs.

An analysis of the rapidly developing field of iterative techniques for the solution of simultaneous linear equations revealed three methods which appeared to merit consider-

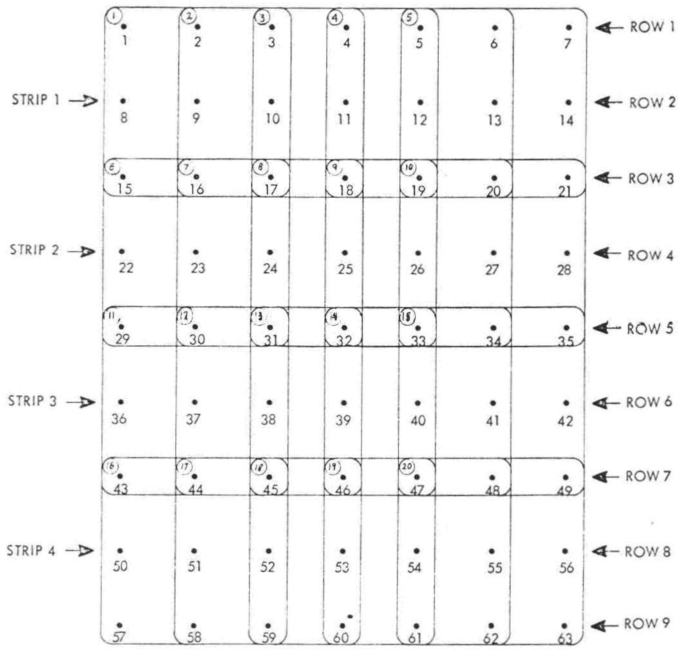


FIG. 1. A uniform 4-by-5 block. The numbering of the control points and photographs is according to the rows.

ation. While it was a relatively simple matter to adapt these solutions to operate only on the nonzero elements of the "collapsed" equations the pivotal question was whether or not a prohibitive number of iterations would be required for adequate convergence. Unfortunately this question could not be answered in advance on the basis of purely theoretical considerations. The answer was to be obtained only by actual trial through numerical simulation of various realistic situations.

NUMERICAL INVESTIGATION OF THE APPROACH

Two variations (Point and Block) of each of three iterative techniques (Gauss—Sidel, Gauss—Sidel with Luisternick Acceleration and Successive Over Relaxation), and four different normal equation arrangements (Figure 4–7) were selected for testing the various combinations of these iterative techniques, and arrangement of equations were evaluated by actual reduction of simulated strips of photography. Both economics and logic dictated such a course of action for, if an acceptable solution could not be found for a strip of photography, the possibility of developing an acceptable solution for larger blocks would be even more remote. A process of elimination was carried out in order to determine which specific combination of normal equation arrangement and iterative method would yield the highest rate of convergence. This process began with the simulation and solution of the basic two photograph problem and proceeded, in successive stages, to six photo strips and twenty-five photo strips until a single iterative method and normal equation arrangement were clearly demonstrated superior to all others. A near minimal control configuration (the four corner points in the initial photo) was utilized in all of these exploratory simulations.

Through this process it was established that the basic Gauss-Sidel technique converged too slowly to be considered practical and that the Gauss-Sidel with Luisternick acceleration was numerically unstable. The Block Successive Over Relaxation method, on the other hand, yielded results which were about equivalent to noniterative techniques. The "intertwined" forms of the normal equations failed to provide the hoped for acceleration to convergence of the iterative solutions and were discarded in favor of one of the more prosaic equation arrangements which turned out to be superior in this regard.

DETAILS OF THE ADOPTED TECHNIQUE

Inasmuch as the iterative approach was not expected to be computationally superior on small photogrammetric nets, the fact that a satisfactory solution had been achieved at all was considered to be encouraging enough to warrant additional study of the technique. Before proceeding with this however, it was necessary to reprogram the computer reduction, for the programs utilized to support the exploratory stages of the study, having experienced frequent and extensive modification as the various approaches were tested, were both inefficient and restrictive in character. This revised set of programs, were coded for the CDC 1604-B computer which is a part of the experimental facility at RADC. They fully exploit the concept of the direct formation of the collapsed form of the normal equations arising from a general photogrammetric block wherein the photos and ground points are numbered in a columwise manner, the technique of solution by the Block Successive Over Relaxation iterative technique and the statistical rigor, flexibility and comprehensiveness characteristic of Brown's theory. The development of the normal equations is largely the same as originally presented by Brown in 1958 and 1959, however, a number of refinements have been added.

By treating every unknown parameter in the adjustment as a correlated observation, subject to error of varying (but specified) degree, convenient and flexible provision is made for implementing almost any conceivable type of information or variation in the basic measuring without requiring alteration of the general adjustment program itself. In particular, the solution can exploit the metric output of external sensors in an altogether rigorous manner. No restrictions are placed on the type of distribution of control information, on the exposure station parameters or on the plate measurements. By allowing the plate coordinates for a given point to be correlated, a variety of possible plate measuring techniques (e.g., goniometric, polar coordinates) in addition to those which

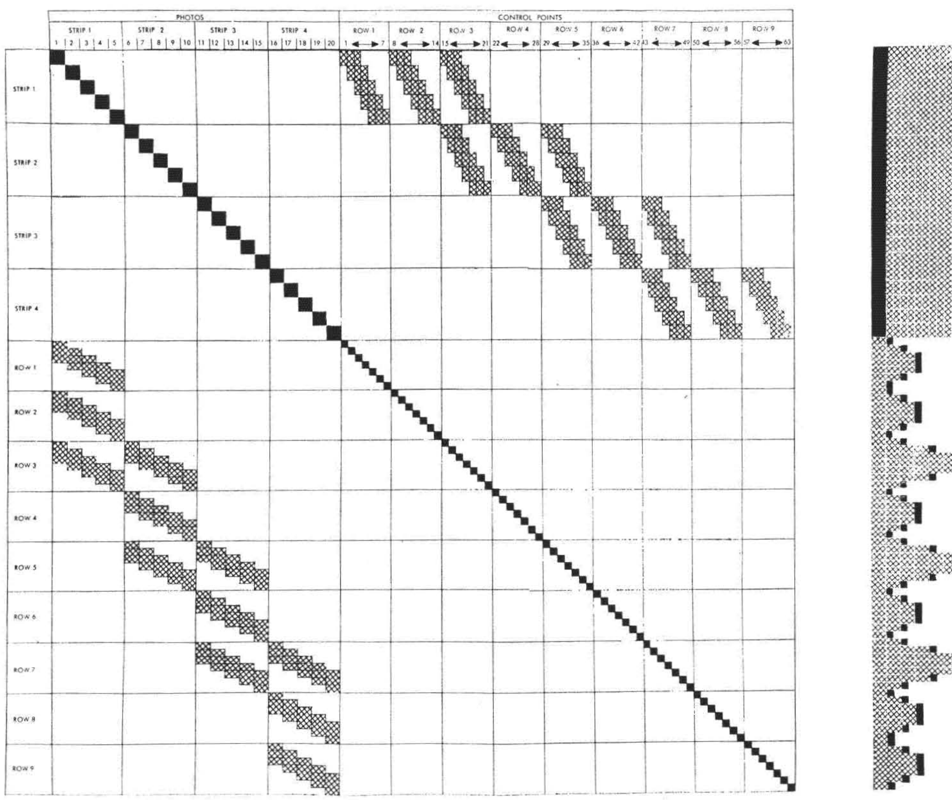
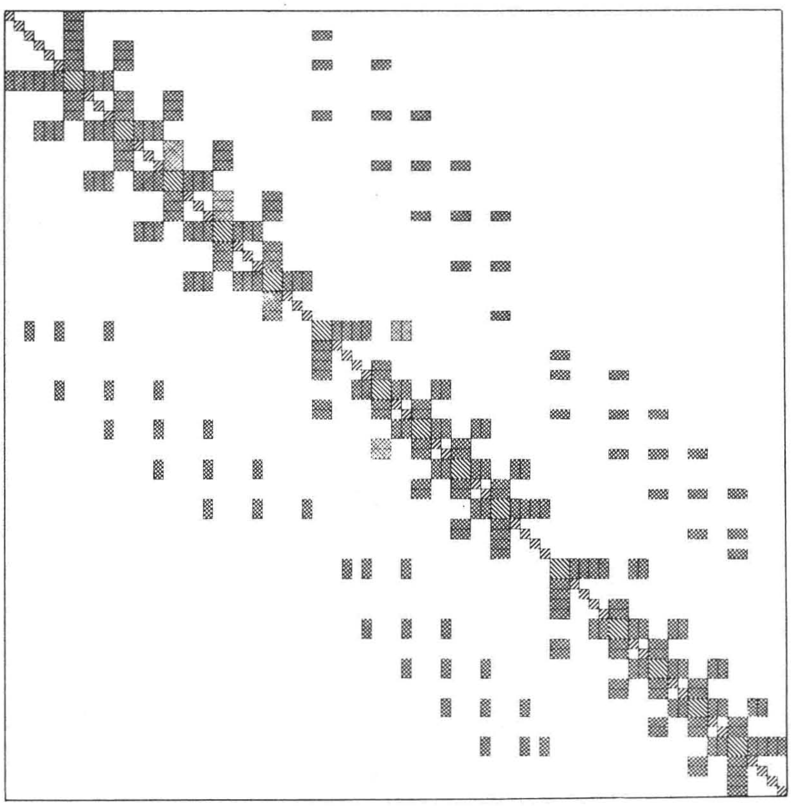


Figure 2a. Figure 2b.

FIG. 2. (a) Form of coefficient matrix generated by a uniform 4-by-5 block with points and photographs numbered as in Figure 1. The order of the full matrix is 308 by 308. (b) Collapsed form of the same matrix.



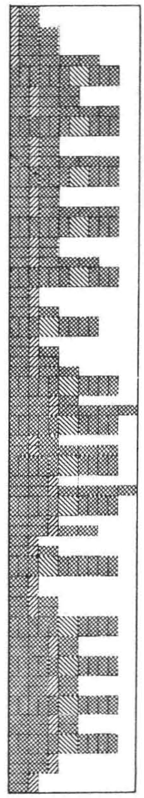


FIG. 3. (a) One possible intertwined form of the general normal equations for a 3-by-5 block employed in numerical simulations. (b) Collapsed form of the same matrix.

directly produce Cartesian coordinates are admitted. This also admits the possibility of employing cameras which do not have flat fields (e.g., panoramic cameras, meteor cameras, Baker Nunn Satellite Tracking Cameras, CZR cameras), for here the plate coordinates to be carried in the adjustment would be those derived from the appropriate transformation (usually from cylindrical to plane coordinates) of the original film measurements.

In addition to the simulation data generator program and the programs for generating and solving the normal equations, a number of programs for the preparation, correction and editing of actual data were coded and logically linked together to form a completely operational system.

SIMULATION STUDIES

In order to effectively evaluate the solution, a carefully planned set of simulation cases, employing 25 and 41 photograph strips and 3-by-5 and 4-by-8 photograph blocks, were processed through the rigorous simultaneous adjustment. Variations in control distribution and accuracy, external exposure station constraints, accuracy of initial parameter approximations, flying height, focal length, plate format and plate measuring accuracy were among the factors considered in this evaluation. While detailed results of these simulations are beyond the scope of this paper, some of the more significant findings indicate that:

1. The amount of distribution of absolute control has a pronounced effect on the rate of convergence of the Block Successive Over Relaxation iterative solution, up to a certain critical level. A strip having only the minimal absolute control required for determinacy (four points) was found to converge appreciably more slowly than one having a moderate sprinkling of control throughout the strip. With a 41-photo strip generating a system of normal equations of order 633, for instance, adequate con-

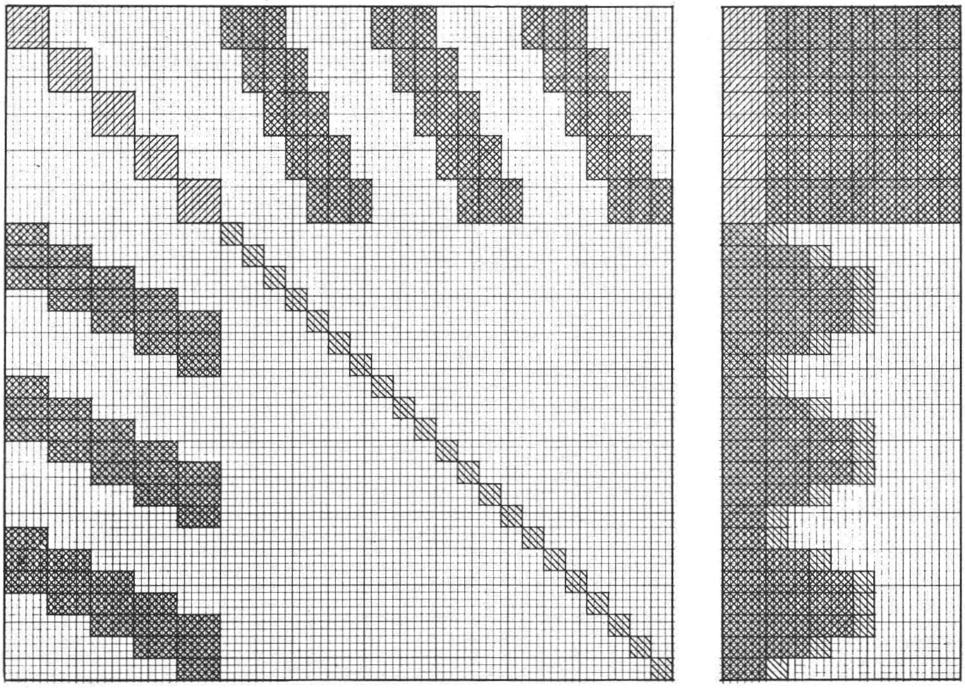


FIG. 4. (a) The A form of the coefficient matrix of the general normal equations arising from adjustment of a 1-by-5 strip of photography. (b) Collapsed form of the same matrix.

vergence was obtained within 150 iterations (seven minutes on an IBM 7094) when a pair of fresh absolute control points was introduced on about every fifth photo. On the other hand, on the order of 600 iterations were required when the same strip was adjusted with absolute control limited to the beginning of the strip. Indications are that the number of iterations required for satisfactory convergence is roughly equal to the order of the normal equations for the case of strips with minimal absolute control; for strips having a moderate level of well-distributed absolute control, the number of iterations may be as few as one fifth to one tenth of the order of the normal equations. There is even indication that once a certain level of absolute control is attained, the number of iterations for satisfactory convergence may depend only very weakly on the order (total number) of the normal equations. From this, the possibility emerges that the simultaneous adjustment of a long strip of aerial photography may entail no more than the general order of computational effort as is required for the reconstruction of the strip by means of analytic canti-lever extension operating on pairs of photos.

2. The introduction of external constraints imposed by auxiliary sensors serves to accelerate the iterative process in a manner similar to ground control.
3. The rate of convergence of the iterative solution is several times faster for a compact block than for a strip of comparable number of photographs and level of control, thus the iterative approach is even more attractive for large blocks than for long strips. For blocks of moderately large dimensions the critical level of control for extremely rapid convergence appears to be the equivalent of three non-colinear points in each strip comprising the block.
4. For strips or blocks possessing very poor initial approximations for orientation parameters and pass point coordinates, a significant increase in convergence is realized by relinearizing the normal equations after every few (on

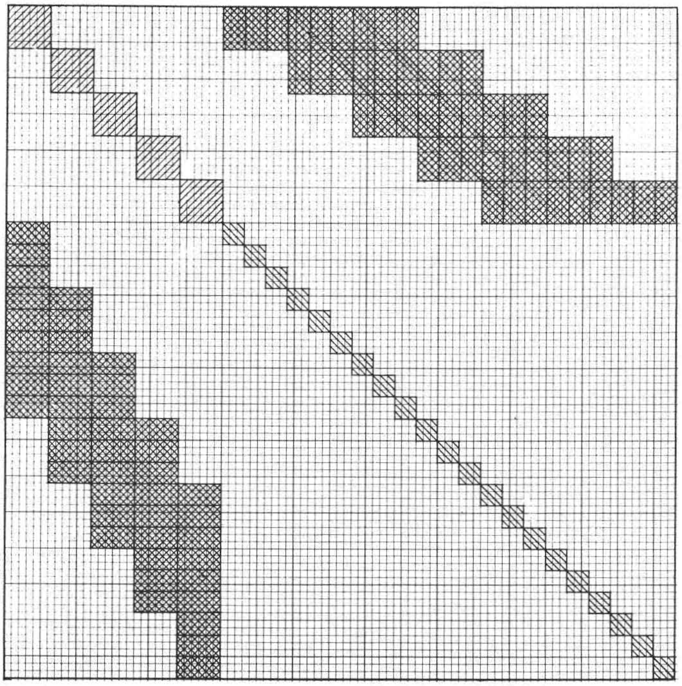


Figure 5a.

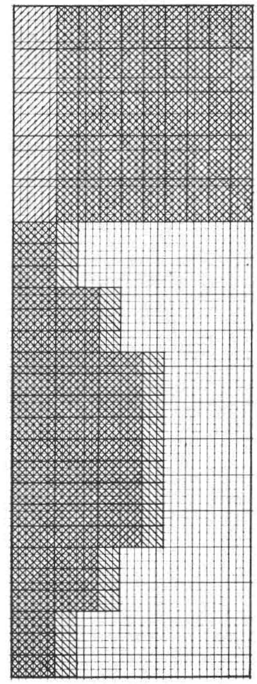


Figure 5b.

FIG. 5. (a) The B form of the coefficient matrix of the general normal equations arising from the adjustment of a 1-by-5 strip of photography. (b) Collapsed form of the same matrix.

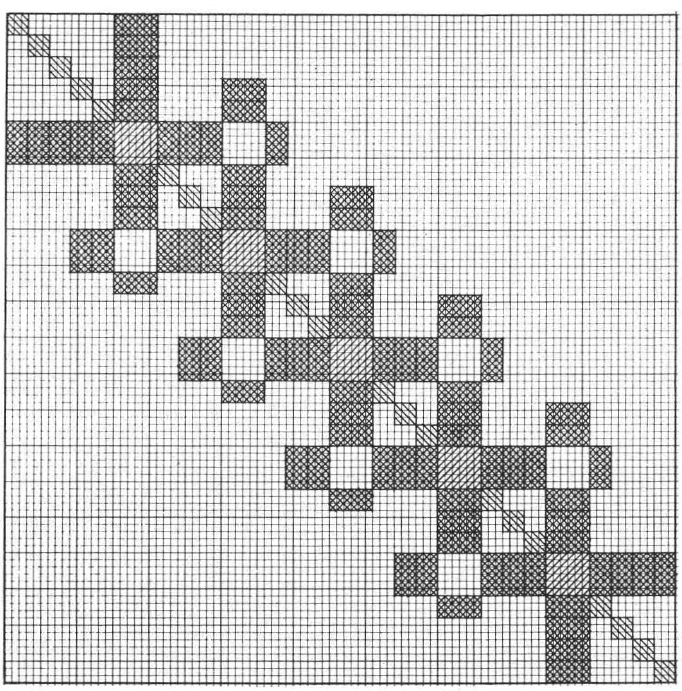


Figure 6a.

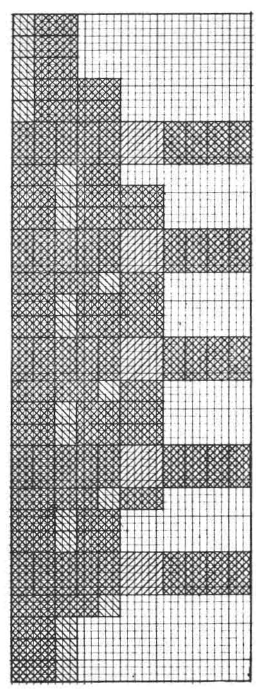


Figure 6b.

FIG. 6. (a) The D (intertwined) form of the coefficient matrix of the general normal equations arising from the adjustment of a 1-by-5 strip of photography. (b) collapsed form of the same matrix.

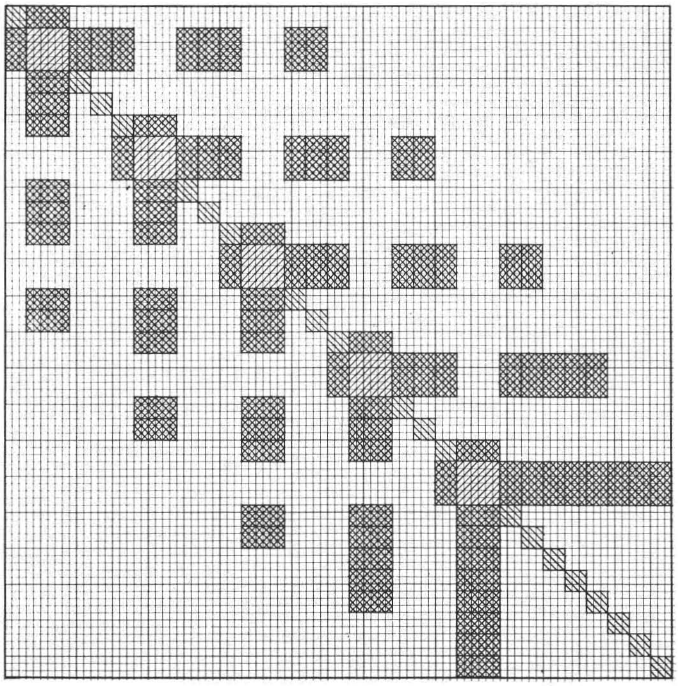


Figure 7a.

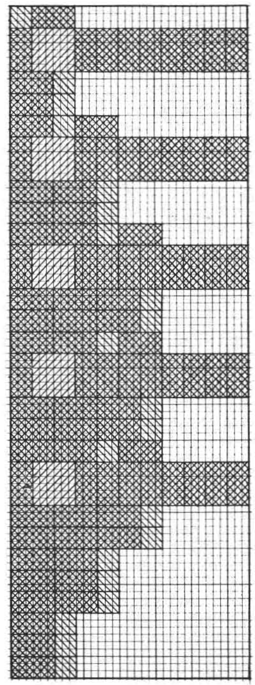


Figure 7b.

FIG. 7. (a) The *C* (intertwined) form of the coefficient matrix of the general normal equations arising from the adjustment of a 1-by-5 strip of photography. (b) Collapsed form of the same matrix.

the order of 25) iterations of the iterative solution.

The overall potential of the solution may best be appreciated by examining one specific simulation: Raw data consisted of a postulated 4-by-8 photograph block, possessing 60% foreward overlap and 20% side overlap, acquired from 10,000 feet with a 6-inch focal length camera with a 9-inch plate format. A uniform nine-point pattern of ground points was adopted, of which six were absolute control points. The ground points were assumed to be 5,000 feet apart in both the X and Y coordinates. Normally distributed random errors with a mean of zero and a standard deviation of 10 microns were applied to all plate measurements. Similar errors with a sigma of 2.5 degrees were added to the "true" values of the orientation angles to obtain initial approximations for these parameters, and errors with a sigma of 1,000 feet were applied to the "true" exposure station and pass point coordinates to obtain initial values for these parameters. As a result of this process, the initial values for orientation angles were in error by as much as 5 degrees, and exposure station locations were in error by as much as 3,100 feet. Pass point coordinate approximations were as much as 2,500 feet. By normal standards such approximations are very course.

The normal equations to be generated and solved for this 4-by-8 strip were of order 462. An initial solution and two relinearizations (each of which was iterated 25 times) were required to achieve satisfactory convergence (the final plate residual mean error of 9.9 microns is consistent with the 10 microns value applied to the plate measurements). Total computational time on the CDC 1604-B computer was 13.5 minutes (on the IBM 7094, this value would be decreased by a of about three). This is at least an order of magnitude better than the best non-iterative techniques for solving a block of this size. The results of this adjustment, which are consistent with what is to be expected from the assumed geometrical configuration and plate measuring accuracies, are presented in Tables 1 and 2 along with the errors in the initial values of each of the respective parameters.

CONCLUSIONS

Although the current programs are restricted to systems involving 500 or less unknowns (this is the largest coefficient matrix

TABLE 1

ERRORS IN EXPOSURE STATION PARAMETER INITIAL APPROXIMATIONS

Station No.	*Tip (Deg.)*	*Tilt (Deg.)*	*Swing (Deg.)*	*XC (Feet)*	*YC (Feet)*	*ZC (Feet)*
1	0.729	−0.782	0.521	−1,085.585	−373.083	828.221
2	4.031	−1.639	2.561	1,226.168	262.065	558.018
3	−2.227	−2.906	−1.868	855.769	−2.846	−2,272.782
4	−3.333	−0.268	−1.264	−301.056	−1,916.048	−562.965
5	3.153	−1.722	−1.244	1,007.207	345.550	−1,938.194
6	1.145	−0.270	−0.164	463.821	−772.839	−2,098.873
7	1.540	−6.205	0.934	−421.937	620.424	−1,215.323
8	−2.362	4.559	−1.575	−10.225	500.820	57.649
9	−1.570	4.260	2.513	468.993	292.452	790.042
10	−4.029	−2.619	0.414	−597.182	975.561	1,147.925
11	4.870	−4.236	−0.113	441.063	−695.601	−872.473
12	0.377	1.571	−0.039	−50.782	1,518.592	216.739
13	3.452	1.099	−0.391	−87.654	96.087	987.012
14	0.339	1.254	4.823	387.735	−1,785.486	−93.142
15	−0.655	4.009	−3.982	−257.864	120.173	−1,956.353
16	−1.998	1.294	2.886	−871.260	−851.192	199.369
17	1.783	0.331	−1.514	−158.423	501.083	199.319
18	−3.020	−1.654	−4.528	−1,324.327	−514.815	−1,670.048
19	2.016	0.870	−2.241	1,484.890	−431.914	−331.528
20	2.266	−2.027	2.298	−1,413.076	−113.311	23.640
21	2.424	−0.315	−1.312	−3,079.324	−2,282.232	561.096
22	−1.850	−2.779	−1.397	788.902	968.641	−639.835
23	−1.706	4.343	−1.377	462.476	−642.020	377.894
24	3.237	−0.690	−2.263	727.220	1,208.338	−1,431.626
25	2.257	−1.016	2.954	−533.238	−1,273.705	620.988
26	2.674	−1.786	−1.182	−1,538.741	727.787	1,204.974
27	4.898	3.387	−2.171	549.890	−16.392	−485.052
28	−2.550	2.362	2.183	−1,666.123	−1,686.995	132.602
29	−1.806	2.383	−1.125	1,545.494	2,153.811	1,232.270
30	4.630	1.352	−4.777	1,406.110	−981.360	−1,208.268
31	3.232	2.028	0.233	814.584	−517.474	−370.126
32	−4.407	0.636	0.175	185.823	−313.990	873.486

(Table 1 is continued at the top of the next page)

which, together with the iterative program, can be stored in a 32K computer memory), this by no means represents the limits for the theoretical capability of the technique nor is it a limit on the order of the normal equations which could be solved. The analysis that has been performed reveals that through the use of advanced programming, buffering procedures, and auxiliary storage devices (such as magnetic tapes, discs or drums), a comprehensive reduction of blocks or strips involving as many as 10,000 unknowns can be accomplished on a computer with a 32K main memory with extremely little loss in efficiency.

As the result of this, a method is now available which overcomes the major obstacles which have prevented the simultaneous adjustment of large n-photo blocks: the n^2 barrier to the formation of the normal equations, the n^3 barrier to their solution and the computational roundoff problem which has seriously compromised other approaches. Consequently the time is at hand when the photogrammetrists need no longer be intimidated by the enormous system of equations arising from the uncompromisingly rigorous adjustment of very large photogrammetric nets.

REFERENCES

Brown, D. C., "A Solution to the General Problem of Multiple Station Analytical Stereotriangula-

Errors Remaining after Adjustment

Station No.	*Tip (Deg.)*	*Tilt (Deg.)*	*Swing (Deg.)*	*XC (Feet)*	*YC (Feet)*	*ZC (Feet)*
1	−.025	.027	.007	7.584	−8.142	−1.415
2	.025	.017	−.006	3.439	7.077	−0.061
3	−.016	−.017	.009	−2.244	−5.221	−1.630
4	.012	−.028	−.003	−4.151	3.437	−1.982
5	.029	.024	.009	3.469	5.223	−1.700
6	.015	−.007	.003	−2.462	3.780	−1.192
7	−.026	−.006	.002	−1.770	−5.890	−0.606
8	.020	−.018	−.015	−4.135	4.645	−1.770
9	.011	−.012	.001	−3.794	1.369	−3.478
10	−.001	.009	.004	1.252	1.308	0.083
11	−.002	−.013	.003	−2.494	−1.325	0.756
12	.016	.038	−.006	8.420	1.635	−0.496
13	−.022	−.025	−.005	−4.972	−4.186	−0.538
14	−.008	−.008	.003	−1.942	−1.771	1.037
15	−.024	.014	−.002	3.793	−5.580	0.624
16	−.012	−.015	.008	−1.716	−4.276	0.563
17	−.009	−.008	−.002	−1.686	−2.739	0.459
18	−.016	.006	.000	0.494	−2.356	0.069
19	−.001	.007	−.004	1.502	−0.227	−1.285
20	−.027	.008	.005	2.704	−6.066	0.554
21	−.017	.010	−.009	2.129	−4.812	0.292
22	.001	.019	−.011	4.181	0.710	−0.008
23	−.007	−.002	.002	−1.210	−2.568	−0.486
24	−.001	.005	−.005	1.056	0.472	0.665
25	.014	−.030	−.007	−5.383	2.542	1.651
26	−.016	.004	−.007	−0.327	−3.589	0.467
27	−.012	−.012	−.006	−2.522	−3.234	−0.221
28	.029	−.009	−.001	−1.523	6.215	1.336
29	.007	−.030	−.005	−7.274	−0.171	3.500
30	−.005	−.052	.000	−10.928	−1.926	2.040
31	−.012	−.025	−.004	−4.664	−2.529	2.880
32	−.027	−.018	.004	−4.088	−5.063	1.118

tion," RCA Data Reduction Technical Report No. 43 (February 1958).

Brown, D. C., "Photogrammetric Flare Triangulation—A New Geodetic Tool," RCA Data Reduction Technical Report No. 46 (December 1958).

Brown, D. C., "Results of Geodetic Photogrammetry I," RCA Data Reduction Technical Report No. 65 (September 1960).

Brown, D. C., Davis, R. G., Johnson, F. C., "The Practical and Rigorous Adjustment of Large Photogrammetric Nets, Rome Air Development Center Report No. RADC-TDR-64-353 (May 1964).

(*Table 2 appears on the following page*)

Table 2

	Error In Initial Approximations				Error Remaining After Adjustment			
	Point No.	ΔX (ft.)	ΔY (ft.)	ΔZ (ft.)	Point No.	ΔX (ft.)	ΔY (ft.)	ΔZ (ft.)
	1	801.5	−891.8	0.0	1	−0.38	−1.11	0.00
	2	−140.9	−712.2	0.0	2	0.77	−1.06	0.00
	3	820.9	511.7	780.0	3	1.96	−0.20	2.26
	4	1,023.7	−1,587.4	0.0	4	−1.02	1.27	0.00
Column 1	5	1,424.4	−550.0	−1,089.5	5	−1.78	−0.80	−2.13
	6	−1,568.2	−438.1	0.0	6	1.06	−0.12	0.00
	7	−381.3	−868.6	924.8	7	0.82	0.14	−0.62
	8	862.0	1,495.7	0.0	8	0.72	1.30	0.00
	9	−421.8	−2,441.8	0.0	9	2.07	1.74	0.00
	10	797.5	−1,025.7	148.0	10	−0.03	−1.17	0.26
	11	25.1	1,383.4	1,403.8	11	0.82	−1.30	1.63
	12	905.5	−1,633.1	−198.3	12	0.86	−0.25	1.49
	13	−316.1	1,782.2	735.9	13	−0.54	0.13	−0.79
Column 2	14	−281.1	−399.3	−357.3	14	−0.08	−0.03	−0.48
	15	1,380.3	132.8	−596.3	15	0.92	−0.48	0.71
	16	1,030.3	−833.3	−1,744.8	16	0.70	0.13	−0.16
	17	−367.4	234.6	−536.7	17	−0.46	0.90	−1.24
	18	1,100.4	−833.6	827.1	18	0.24	0.28	−1.12
	19	732.7	653.3	649.4	19	0.20	−1.43	0.19
	20	768.5	1,786.5	−1,175.1	20	−0.40	−0.73	2.98
	21	0.0	0.0	0.0	21	0.00	0.00	0.00
	22	−408.8	−758.9	1,555.8	22	−0.20	0.52	1.29
Column 3	23	0.0	0.0	0.0	23	0.00	0.00	0.00
	24	−1,179.5	−1,129.9	622.2	24	0.13	−0.83	0.34
	25	0.0	0.0	0.0	25	0.00	0.00	0.00
	26	114.6	−1,211.4	275.7	26	0.00	0.43	1.37
	27	1,019.2	434.2	2,075.8	27	−0.66	−0.57	0.85
	28	716.1	302.3	−111.9	28	−1.07	1.04	−3.34
	29	1,630.7	222.7	−1,101.6	29	−1.19	0.18	−1.44
	30	1,039.2	−1,624.6	−1,151.2	30	0.05	0.47	−0.53
	31	1,697.3	579.8	933.8	31	−0.44	0.31	−0.60
Column 4	32	−1,049.5	−253.4	754.6	32	0.19	0.17	−0.32
	33	1,595.7	620.1	−1,153.2	33	0.44	−0.20	−0.36
	34	823.3	−1,029.0	1,437.1	34	0.37	−0.56	−0.98
	35	−1,398.0	−1,100.0	1,111.6	35	−0.64	−1.13	−0.71
	36	479.9	150.6	−356.4	36	0.37	−2.32	−2.16
	37	−526.3	1,199.8	571.8	37	−0.92	0.84	−1.35
	38	−35.5	−949.1	808.8	38	0.13	0.12	−1.92
	39	−17.5	−710.7	−1,109.6	39	−0.47	0.04	0.20
	40	−52.4	202.8	−2,437.4	40	−0.34	−0.14	0.39
Column 5	41	−489.4	624.1	1,275.5	41	0.28	0.22	−0.19
	42	−485.4	1,028.0	−757.1	42	0.17	−0.87	−0.12
	43	−1,364.2	−765.8	674.4	43	−0.08	−0.50	0.23
	44	−811.4	−396.8	473.9	44	0.15	−0.21	−1.00
	45	1,074.4	−1,218.4	131.6	45	0.65	−0.21	−0.08
	46	−1,011.3	1,268.0	−490.6	46	0.03	−0.53	1.29
	47	1,497.8	−590.2	307.8	47	0.46	−0.75	0.02
	48	127.4	437.0	−832.1	48	0.10	0.18	0.42
	49	26.6	25.7	882.7	49	0.15	0.84	0.45
Column 6	50	−934.8	606.6	123.2	50	0.12	−0.72	0.62
	51	−298.7	62.2	−681.2	51	0.01	−0.44	0.54
	52	380.5	1,459.8	389.4	52	−0.02	−0.05	0.32
	53	−737.2	1,837.9	−1,140.9	53	0.40	−0.90	0.97
	54	951.9	295.2	−886.8	54	0.61	0.25	0.62
	55	1,063.0	−1,545.0	−934.0	55	0.14	0.29	0.61
	56	−299.6	−1,298.5	−437.5	56	0.38	−0.29	−0.51
	57	−179.6	−207.1	1,252.7	57	0.41	0.66	0.94
	58	1,776.1	1,894.9	1,177.5	58	−0.07	0.19	1.85
Column 7	59	−691.6	1,115.9	735.8	59	−0.48	0.08	−0.84
	60	−2,038.1	−1,652.4	−618.4	60	0.38	−0.04	−1.26
	61	−1,534.6	1,389.6	318.0	61	−0.13	0.07	0.43
	62	−109.0	1,392.0	−218.7	62	−0.64	0.50	−0.54
	63	115.1	−457.2	2,143.7	63	−0.21	1.12	0.79
	64	−1,248.1	−646.4	−2,484.5	64	−0.88	−1.10	1.04
	65	−573.5	139.6	388.3	65	−0.16	−0.69	1.64
	66	0.0	0.0	0.0	66	0.00	0.00	0.00
	67	180.9	−86.1	−1,203.8	67	−0.19	0.28	1.08
Column 8	68	0.0	0.0	0.0	68	0.00	0.00	0.00
	69	1,972.7	−1,193.2	1,125.7	69	0.10	−0.32	0.39
	70	0.0	0.0	0.0	70	0.00	0.00	0.00
	71	1,896.7	1,095.2	−965.8	71	0.06	−0.16	−0.47
	72	−350.9	−1,855.8	−220.5	72	0.65	−0.51	−0.43
	73	−751.3	−1,014.2	647.0	73	−0.68	−1.05	3.98
	74	−9.0	905.3	−1,080.7	74	−1.18	−0.19	2.18
	75	−276.2	745.6	−1,377.2	75	−0.54	−1.15	1.36
	76	1,156.9	962.7	1,693.7	76	−1.38	−0.54	0.69
Column 9	77	−342.2	−432.0	−59.5	77	−0.28	−0.35	0.40
	78	−1,153.8	759.9	271.5	78	0.00	0.01	0.25
	79	1,342.8	1,985.9	−111.1	79	−0.83	−0.17	0.53
	80	−137.4	−672.2	963.4	80	−0.58	−0.07	1.40
	81	140.6	383.0	361.1	81	−0.66	2.62	4.24
	82	−2,137.3	1,348.3	0.0	82	1.48	1.68	0.00
	83	−115.0	−1,113.0	0.0	83	1.46	0.17	0.00
	84	−573.3	1,327.9	−1,000.6	84	−1.68	0.67	2.50
	85	−2,139.9	788.5	0.0	85	1.59	−1.54	0.00
Column 10	86	1,931.1	−954.9	−674.4	86	−1.50	0.26	3.27
	87	−1,492.2	741.6	0.0	87	1.60	−0.71	0.00
	88	242.3	621.4	−1,043.0	88	−1.22	−0.01	2.98
	89	268.3	688.5	0.0	89	0.60	−0.96	0.00
	90	437.6	−411.5	0.0	90	−0.56	−0.29	0.00