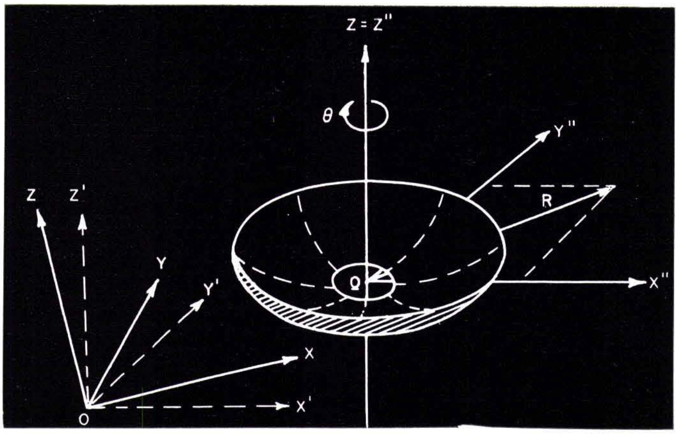

FRONTISPIECE. Relationship between the model and the cylindrical coordinate systems for the solar collector. See text page 323.

DR. SANJIB K. GHOSH
Ohio State University
Columbus, Ohio 43210

Solar Collector Calibration

A purely photogrammetric procedure using standard equipment throughout yields a sigma of 0.5 mm. for a 44-foot parabolic dish.

INTRODUCTION

THE MANUFACTURING OF solar collectors is a phase of the Air Force's program called "Advanced Solar Turbo-Electric Concept (ASTEC)". The ultimate objective is to provide a spacecraft with alternating electric current utility power derived from solar radiation only. In flight the collector is supposed to be aimed at the sun to capture solar energy throughout the active life of the space vehicle carrying it (e.g., a manned space station, a communication satellite, etc.). In operation the mount is supposed to track the sun automatically by means of a solar sensor.

The purpose of a solar collector or (parabolic) mirror is to bring all solar rays entering the aperture to focus at a point. To achieve this it is desirable to select a paraboloid (of revolution) as the ideal surface. A parabola, by definition, is the locus of a point which moves so that its distance from a fixed point (focus) is equal to its distance from a straight line (directrix). The axis of the parabola is the line perpendicular to the directrix and passing through the focus. Any wave front entering the parabola from the concave side of the parabola (the reflecting side) parallel to the axis will be brought to focus at the fixed point (focus of the parabola). The surface obtained by revolving the parabola around the axis is then the desired surface.

Due to imperfections in the manufacturing of such a solar collector, the actual mirror surface would fail to bring the wave front to a common focus. It is, therefore, of interest to the scientist to know the departures of a specific collector surface from the ideal one so that more valid decisions can be made concerning loss of energy, etc.

The investigations reported here were made on a particular solar collector, or mirror, which is approximately 44.5 feet in diameter, and is owned by the USAF and located at the Denver plant of the Sundstrand Corporation.

The objectives of these investigations were two-fold. First, the purpose was to measure and analyze the results using the above mentioned solar collector. The second and more important objective was to establish a general solution for such measurements (photogrammetrically) and to develop a technique in this respect that could be used in the future in similar problems.

General Approach

MIRROR SURFACE PREPARATION

To ensure the measurements with the desired accuracy it was necessary that a series of reference marks of good contrast be established directly on the surface of the mirror. The mirror consists of a very shiny and highly reflective surface.

Round pieces of $\frac{1}{2}$ inch diameter were cut off from a black plastic tape and these pieces were placed on the surface of the mirror to furnish the reference marks. The markers were placed along eight radial lines (gores), there being 37 markers along each gore. Further, to break the monotony and to avoid the resulting blunders, every fourth marker was covered by a larger square marker (1 inch square). The locations of the gores, the lines of placing the markers and the system of numbering of the gores are shown in Figure 1. Apart from these, three square markers were placed between each pair of gores to have an even distribution of points to help check the deviation of the actual surface from the ideal surface later on.

The total number of markers placed on the surface of the mirror to locate specific points for measurements was 320.

PHOTOGRAPHY

The photographs were taken with a Wild Phototheodolite camera (nominal focal length —165.49 mm; aperture—*f*/24).

The camera uses 10 cm by 15 cm photographic plates. For this project a topographic rapid ortho emulsion on a flat glass base obtained from Gevaert was used. Measurements were made on the glass negatives directly without going through the production of diapositives.

Two exposures were made, one at each of two camera stations with the camera axis tilted to $+7^g$ from the horizontal in each case. The mirror axis was tilted approximately $+7^g$ ($^-6°$) from the horizontal and the mirror was facing the camera base. Four targets placed at specific locations were also photographed along with the mirror. (See Figure 2.) These targets were meant to supply control for the photogrammetric measurements.

Abstract: *A purely photogrammetric procedure was successfully applied for the analysis of structural deviations of a 44-foot parabolic solar reflector with a standard error of only 0.5 millimeter. Conventional photogrammetric equipment was employed: a Wild Phototheodolite camera and a Wild A7 stereoscopic mapping instrument. The data were reduced with an IBM-7094 computer. Careful attention to the important details of the operation—photography, control, point identification, base-height ratio, plotter operation, a sound plan—helped assure rigorous and accurate results.*

The cameras were oriented in the normal case (i.e., camera axes parallel to each other and both at right angles to the base).

Because of the hugeness of the mirror, outdoor photography had to be resorted to. It was seen after long trials that the mirror surface gives the maximum contrast with respect to the black markers if the sky is clear (no clouds) and no direct rays from the sun on the mirror surface, i.e., the direction of sun's rays is about 90° with the axis of the mirror.

The base-distance ratio was kept at approximately $\frac{1}{3}$ to ensure an accuracy better than the usual terrestrial case, although the base was brought close enough to have the whole mirror in a single model.

ESTABLISHMENT OF GROUND CONTROL

Apart from the four targets (see Figure 2) that appeared in the picture, eight other control points were used. These consisted of one target along each gore, being the square marker nearest the rim of the mirror. All of these control points were observed from each of the camera stations with the theodolite

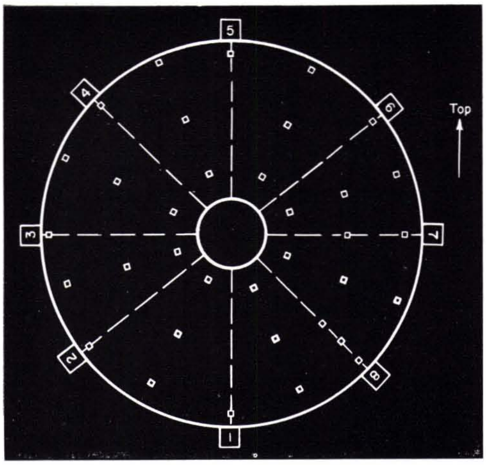

FIG. 1. The locations of the gores.

(Wild T2) portion of the phototheodolite. The theodolite angles were read on both faces and at two zeros. The base was measured with a steel tape. The instrument elevation at each camera station was measured with the same tape and, to supply good checks, four distances between the stations and the targets were measured. Later on the control coordinates (X, Y, Z (elevation)) were established by computations with a least-square adjustment.

WORK AT THE STEREO-PLOTTING INSTRUMENT

For this purpose the Wild Autograph A7 of the Department of Geodetic Science, The Ohio State University, was used.

The original negatives were inserted into the non-compensating plate holders and placed into the autograph cameras. The effect of objective distortion was ignored in this case (the radial distortions were less than 10 μ at the negative).

Interior and relative orientations were established. The relative orientation was done by the method of elimination of Y-parallax and was done empirically. The precision of the relative orientation was checked at the end analytically and was found to have a standard error (based on observations at 25 well distributed points) of residual Y-parallax to be less than ± 0.01 mm. Inasmuch as this was the least count of reading any coordinate on the Wild A7, no further improvement was attempted at this stage. The model was scaled with respect to all the available known distances between the control points.

After the relative orientation and scaling was performed in the model at the Wild A7, it was checked for the proper insertion of the focal length. Because it was a normal case of photography (i.e., camera axes were parallel to each other and were normal to the camera base), the introduction of a wrong focal length to the autograph cameras would result in a scale along the camera axes different from the scales along other directions normal to the camera axes. This would have caused in the model space (see Figure 2) a scale along line 2–4 different from the scale along line 1–3 between the targets. It was observed that the agreement between the two distances was within reasonable limits, thus indicating that the correct value of the focal length was introduced at the A7 cameras.

No attempt was made to establish absolute orientation (rotation of the model). This was done later by computation.

Measurements were made and recorded for the X, Y, Z model coordinates of each of the 232 (out of the total 320) available points.

The model scale determined with the help of the control points was 1:100. At each point in the model space the measuring mark was centrally placed on the marker and then three coordinates (X, Y, and Z) were read off the instrument counters. In order to increase the accuracy of reading the coordinates, and to avoid the effects of instrumental backlash, etc. each point was observed by two operators, and each of them made observations in two rounds (by moving the instrument carriage in opposite directions). The averages of such four readings of a coordinate at each point, rounded off to microns, were considered for the subsequent data processing.

DATA PROCESSING

All measured data (the average of four

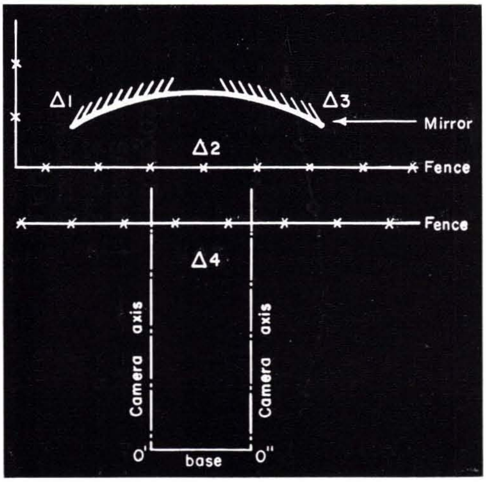

FIG. 2. Sketch plan showing locations of camera, mirror, and target.

sets of observation) were prepared in punch card form for use in an IBM 7094 computing system, located at The Ohio State University Research Center. The program was later prepared according to the SCATRAN source language which permits great flexibility in developing modified programs during the progress of the work.

The data processing was done in several stages: the situation may be described and easily illustrated by the Frontispiece.

In the X, Y, Z system at the A7 the Z axis is approximately parallel to the camera axis, positive Z being towards the camera (upwards), positive Y being towards the rear, and positive X being towards the right side.

The X', Y', Z' system is a simple rotation through space of the X, Y, Z system. The rotation is considered in three components: α (around the Y-axis), β (around the X-axis), and γ (around the Z-axis).

The translation of the origin O (of both X, Y, Z and X', Y', Z' systems) to O'' (of the X'', Y'', Z'' and the cylindrical systems) involves three components X_{av}, Y_{av} and Z_a (which is explained later).

In the cylindrical system the Z (axis of the paraboloid) coincides with the Z''-axis and the radial distance R is obtained from $(X''^2+Y''^2)^{1/2}$. The angle θ is computed with respect to the X'', Y'' coordinates and assuming one particular radial line as the zero line.

Phase 1. Rotation of X−Y−Z *System to Obtain* X′−Y′−Z′ *System*

This is done in two iterations. The accepted condition is that points on the mirror surface that are equidistant from the axis should have the same elevation Z. With a view to having a good first approximation, the 34th point (from the center) along each of the lines, 1, 3, 5, and 7 were considered in the first iteration. The rotation angles are:

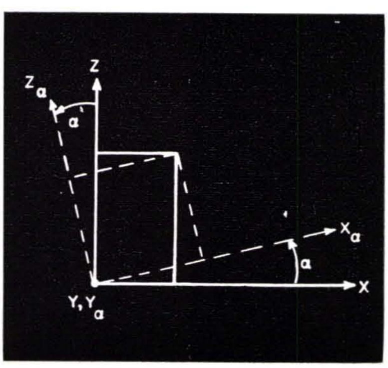

FIG. 3. Rotation α around Y-axis.

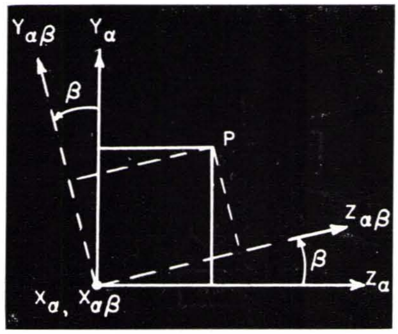

FIG. 4. Rotation β around X_α-axis.

$\alpha = \text{arc tan } (\Delta Z/\Delta X)$

$\beta = \text{arc tan } (\Delta Z/\Delta Y)$

γ = zero, considered for the sake of simplicity in this case.

The solution of the rotation angles was perfected in the second iteration where 64 points were considered, i.e., 16 sets of 4 points along diametrically opposite radial directions. The averages of α's and β's thus obtained were considered for rotation.

This second iteration is expected to give, within reasonable accuracy (1 second of arc), the Z' axis to be parallel to the Z (axis) of the paraboloid.

An explanation of the three rotations may be pertinent here:

Primary rotation (α). Where rotation α (around Y-axis) transforms the axes X and Z into the positions X_α and Z_α respectively. (See Figure 3.)

This is expressed by the rotation matrix

$$M_\alpha = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}.$$

Secondary rotation (β). Where rotation β is performed around the X_α-axis and the other axes Y_α and Z_α are transformed into the positions $Y_{\alpha\beta}$ and $Z_{\alpha\beta}$, respectively. (See Figure 4.)

This is expressed by the rotation matrix

$$M_\beta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & \sin\beta \\ 0 & -\sin\beta & \cos\beta \end{bmatrix}.$$

Tertiary rotation (γ). Where rotation γ is performed around the $Z_{\alpha\beta}$-axis and the other axes $X_{\alpha\beta}$ and $Y_{\alpha\beta}$ are transformed into the positions $X_{\alpha\beta\gamma}$ and $Y_{\alpha\beta\gamma}$, respectively. (See Figure 5.)

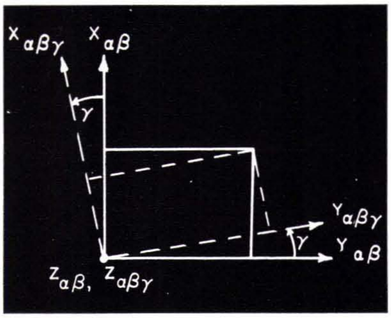

FIG. 5. Rotation γ around $Z_\alpha\beta$-axis.

This is expressed by the rotation matrix

$$M_\gamma = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The total effect of all the three rotations is then expressed by the orientation matrix $\boldsymbol{M}$

$$\boldsymbol{M} = \boldsymbol{M}_\alpha \cdot \boldsymbol{M}_\alpha \cdot \boldsymbol{M}_\gamma$$

and

$$\begin{bmatrix} X' \\ Y' \\ Z \end{bmatrix} = \boldsymbol{M} \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}.$$

Phase 2. Translation of the Origin (See Figure 6).

For the translation of the origin (i.e., shifting of the zero) all available symmetrically located points were considered. A total of 226, i.e., 113 pairs of symmetrical (with respect to the axis of the paraboloid) points, were used out of the observed 232 points. Averages of the X' and Y'-coordinates for these 226 points were taken, i.e.,

$$X_{av} = \frac{\Sigma X'_{i,j}}{226}$$

and

$$Y_{av} = \frac{\Sigma Y'_{i,j}}{226}.$$

Next, for the final translation the working equations are

$$X''_{i,j} = X'_{i,j} - X_{av}$$
$$Y''_{i,j} = Y'_{i,j} - Y_{av}$$
$$Z''_{i,j} = Z'_{i,j} - Z'_{\text{lowest point}} + 1 \text{ mm} = Z'_{i,j} - Z_a.^*$$

Phase 3. Transformation of Coordinates (See Figure 7).

The transformation is from the rectangular three dimensional X'', Y'', Z''-system to the cylindrical R, θ, Z-system. Here Z'' of the three dimensional system, and Z of the cylindrical system are identical. The transformation equations are:

$$R_{i,j} = [X''^2_{i,j} + Y''^2_{i,j}]^{1/2}$$
$$\theta_{i,j} = \text{arc tan } (Y''_{i,j}/X''_{i,j}) \cdots \text{in radians}$$
$$Z = Z''.$$

The angles were further reduced with respect to line 1 being zero. The Z-coordinates obtained at this stage are considered as the observation data for subsequent computations of the deviations from the desired surface.

Phase 4. Establishing the Best Fitting Paraboloid

This was done in two steps.

Step 1. A best fitting parabola for each row of points was obtained by using the general expression of a parabola:

$$a \cdot R^2 + b \cdot R + c \cdot Z - 1 = 0$$

* In $Z''_{i,j}$ the addition of 1 mm was made with a view of ensuring positive values for the eventual ease in transformation and data analyses. After this translation, the X''-axis is parallel to X'-axis, the Y''-axis is parallel to Y'-axis, and the Z'' is parallel to the Z'-axis. Furthermore, the Z''-axis coincides with the axis of the paraboloid.

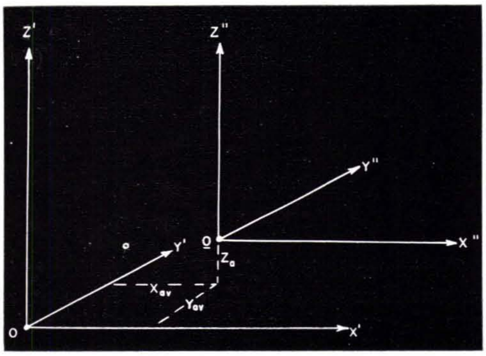

FIG. 6. Translation of the origin.

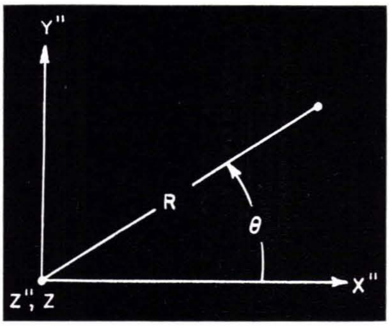

FIG. 7. Transformation from the three dimensional rectangular coordinate system to the cylindrical system.

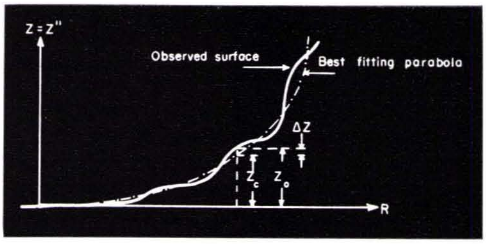

FIG. 8. The Z-deviations ΔZ.

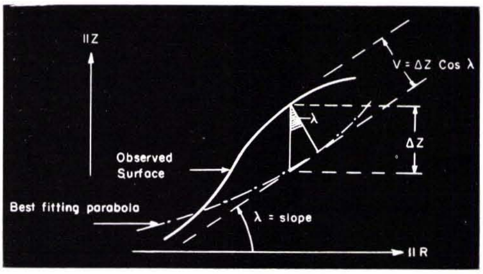

FIG. 9. The actual deviation v.

where a, b, and c are certain constants. In finding the best fitting parabola, the principles of least squares were followed by taking into consideration all available points in each row. Thus, eight different parabolas were obtained that correspond to the eight different rows of points.

Step 2. An average of these eight parabolas gives the final parabola by rotating which (around the axis) is obtained the best fitting paraboloid (which is assumed to be the ideal surface).

Phase 5. Computation of the Z-Deviations (ΔZ)

The cylindrical system of coordinates was of great help in this respect. The Z-deviations ΔZ) are obtained from

$$\Delta Z = Z_o - Z_c$$

where Z_o is the observed Z coordinate at a point and Z_c is the Z-coordinate at the same point (the same R) obtained from the best fitting parabola. (See Figure 8.)

Phase 6. Computation of Actual Deviations from the Best Fitting Paraboloid

The actual deviations in directions normal to the surface of the best-fitting paraboloid were next computed by multiplying the Z-deviations with the cosine of the slope of the best fitting parabola at the specific locations. The working equation is

$$v = \Delta Z \cdot \cos \lambda$$

where v is the deviation, ΔZ is the Z-deviation, and λ is the slope. (Also see Figure 9.)

Results of the Measurements

The final deviations v can be tabulated and represented graphically for ready reference separately along each row of points (gore). An example is presented in Figure 10.

In addition to this, an analysis of the data can be presented in terms of a map giving iso-deviation lines. The planimetric locations of the points in this map can be in terms of X'', Y''-coordinates (R, θ could be considered).

Conclusions and Recommendations

The pictures were taken with the mirror standing at a particular attitude. These investigations show the deformations of the mirror only for that attitude. It is possible that the shape of the mirror may change if its orientation be changed. If one is to have a good general idea of such possible changes in the shape, it should be studied with different orientations. It is suggested that the axis of the mirror be tilted to the following angles (with the horizon), 0°, 15°, 30°, 45° for a series of similar measurements.

The accuracy of the measurement system could be discussed very broadly. In the present case on the Wild A7, the model scale was 1:100; the least count of the reading of each coordinate is 0.01 mm; two observers made the observations and each in two rounds; and the average of the readings were considered as the raw data. From previous experience, the standard error of reading any coordinate at the Wild A7 is less than ± 0.02 mm. A consideration of the above gives us directly a standard error of measuring any coordinate in this case at the Wild A7 to be less than $\pm 5\ \mu$. This, considering the model scale, may be referred to the surface of the mirror as ± 0.5 mm.

The following recommendations are made

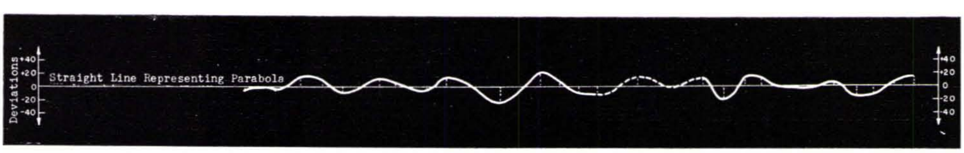

FIG. 10. Deviations along Gore 3 in mm.

in view of possible further development of the procedure:

1. Photographs should be taken (nearly) simultaneously, thus requiring two identical cameras with synchronized shutters.

2. Cameras with wider field angle but without lens distortion should be considered. This would enable taking the pictures from nearer locations ensuring larger picture scale and more precision in the observations.

Results of this investigation indicate that a feasible method of measurement of the surface of a solar collector (mirror) for the purpose of analysis and calibration has been developed based on purely photogrammetric procedures. This method, unlike other previously developed complicated and time-consuming methods, does not require special equipment usually unavailable in either governmental or commercial organizations.

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International Symposiums

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