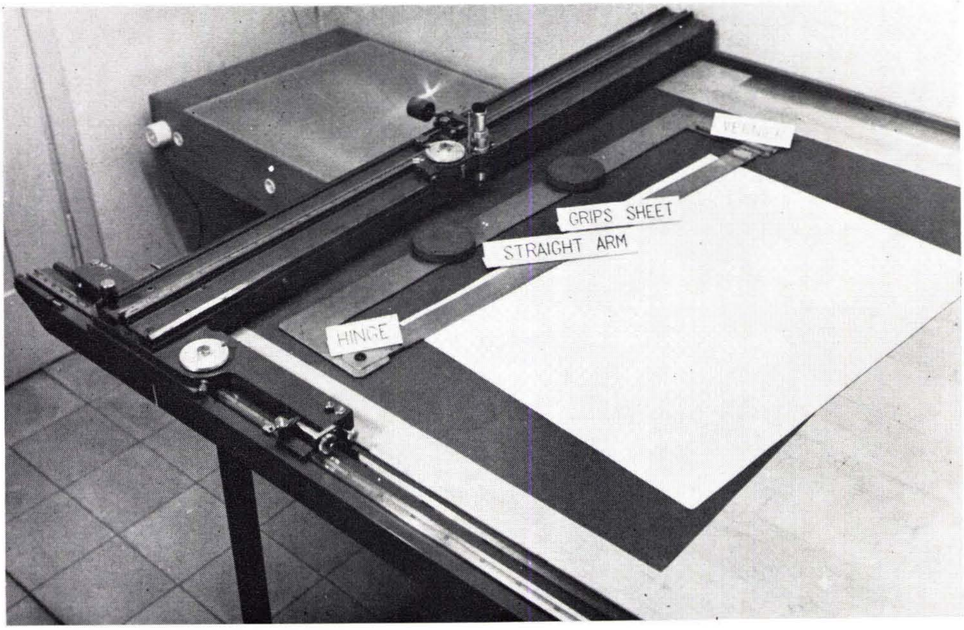

FIG. 1. A movable metal arm containing an emery paper undersurface, a vernier at one end, and hinged at the other end to a similar stationary bar to facilitate the orientation of a map sheet on the coordinatograph of an A7 plotter.

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Positioning the Drawing Sheet

A simple aid facilitates the adjustment of a map sheet on the drawing table of a stereo plotter.

(*Abstract on next page*)

INTRODUCTION

THE MAP SHEET IS POSITIONED on the drawing table of the stereoplotter for most mapping purposes by fitting two points of the plotted ground control to the corresponding points indicated by the drawing device. (A stand microscope is handy for this step.) The positions of the remaining ground control points of the model are compared with the plotted points and adjustment for distributing the planimetric discrepancies is done clumsily and arbitrarily. The reason for this is that existing stereoplotters possess at most three out of the four means of adjustment required to achieve the best possible fit. The inability to rotate the sheet through a definite small angle makes the positioning by adjustment extremely cumbersome. A device for rotating the sheet and a procedure for rational positioning are presented in the following.

JUSTIFICATION FOR THE PROPOSED PROCEDURE

The first three steps of the orientation of a model are carried out with care and precision, usually applying standardized adjustment methods which make use of superfluous data for distributing the discrepancies. It is only logical that the same degree of care and precision should be maintained throughout the absolute orientation, i.e. to position the map sheet with a convenient adjustment procedure. This will provide an objective criterion for completion of the positioning, instead of relying on the arbitrary sense of judgement of the operator. The least squares adjustment will ensure the best possible solution and usually will avoid mistakes and save time in the long run.

Application of the Proposed Procedure

The field of application is obviously for large scale mapping in general and cadastral mapping in particular, where a map of maximum planimetric precision can be attained directly, instead of subsequently plotting the linearly transformed machine coordinates. (This will depend, however, on the reliability of the enlarging gear of the stereoplotter.) Other applications will suggest themselves with experience.

The proposed procedure disregards the existence of automatic registration, electronic computers and automatic coordinatographs, but will undoubtedly prove useful in the numerous organizations that are unable to adopt these aids.

Description of the Proposed Device

As already mentioned, the problem of positioning the map sheet to more than two ground control points by an adjustment procedure is reduced to providing a simple means for rotating the sheet through a small angle. Figure 1 illustrates such a device consisting of two flat metal arms 80 cms long with emery paper attached on the underside. The bent arm rests on the drawing table (with weights added to increase stability) whereas the straight arm rests on the drawing sheet gripping it securely with the aid of the emery paper. The straight arm is hinged to the bent arm at one end, and at the other end has a vernier which slides over a scale graduated on the bent arm.

In this way the straight arm is able to rotate through a small angle (0.5^g to either side of the zero) relative to the bent arm and to impart the rotation to the drawing sheet, thus providing the fourth means of adjustment simply and reliably. The required rotation can be set to the nearest centigrade.

The Procedure

It is clear that the scale of the model will undergo an adjustment simultaneously with the positioning of the drawing sheet so that the "scaling" which is done before levelling the model is not required to the utmost accuracy but only sufficiently accurate to obtain the correct height differences for levelling the model. In order to save time and computations, start the procedure by positioning the sheet approximately. Then set the floating mark on the (four or more) control points in the model and read the discrepancies Δx, Δy between the points indicated by the microscope of the drawing device and the corresponding plotted points. One should read to the nearest 0.05 mm. At the same time read on the x, y-counters the x, y-coordinates (in the drawing table system and to the nearest cm) of the positions of the microscope.

Abstract: *A simple device facilitates the orientation and positioning of a map sheet on the coordinatograph of a stereo plotter to fit ground control points. A routine procedure enables one to complete the adjustment quickly and methodically, obtaining a least-squares fit and a quantitative measure of precision. The application should prove particularly advantageous for large-scale mapping.*

The computation will be simplified if the coordinates and discrepancies are referred to their respective centres of gravity, i.e.,

$$x_o = \frac{[x]}{n}, \qquad y_o = \frac{[y]}{n},$$

$$\Delta x_o = \frac{[\Delta x]}{n}, \qquad \Delta y_o = \frac{[\Delta y]}{n}$$

where

$$X = x - x_o \qquad Y = y - y_o,$$

$$\Delta X = \Delta x - \Delta x_o, \qquad \Delta Y = \Delta y - \Delta y_o.$$

Although the parameters of the linear transformation which the adjustment sets out

TABLE 1

NUMERICAL EXAMPLE OF ADJUSTMENT PROCEDURE

Model No. 0467/71/72 Scale of map 1:2000

Before Adjustment

Control Point	x *mm*	Δx *mm*	y *mm*	Δy *mm*
776	1000	+0.05	1000	−0.00
777	1290	+0.40	1010	−0.15
760	1010	0.00	740	0.00
787	990	+0.20	280	+0.50
788	1300	−0.30	330	+0.10
Σ	5590	+0.35	3360	+0.45
Σ/n	1118	+0.07	672	+0.09
	$X=x-\Sigma/n$	$\Delta X=\Delta x-\Sigma/n$	$Y=y-\Sigma/n$	$\Delta Y=\Delta y-\Sigma/n$
776	−118	−0.02	+328	−0.09
777	+172	+0.33	+338	−0.24
760	−108	−0.07	+ 68	−0.09
787	−128	+0.13	−392	+0.41
788	+182	−0.37	−342	+0.01
check $\Sigma=$	0	0.00	0	0.00

$$\mathrm{I}=[Y\cdot\Delta Y]+[X\cdot\Delta X]=+298 \qquad \Delta\lambda=\frac{\mathrm{I}}{\mathrm{III}}=+0.00050$$

$$\mathrm{II}=[X\cdot\Delta Y]-[Y\cdot\Delta X]=+248 \qquad bx=261.61$$

$$\mathrm{III}=[X^2]+[Y^2]=601760 \qquad \Delta bx=0.13$$

$$\Delta k=\rho\frac{\mathrm{II}}{\mathrm{III}}=2^c\cdot 6$$

After Adjustment

Control Point	Δx	$\Delta X=x-\Sigma/n$	Δy	$Y=\Delta y-\Sigma/n$
776	−0.40	−0.33	−0.35	−0.01
777	+0.20	+0.27	+0.40	−0.06
760	−0.25	−0.18	−0.45	−0.11
787	+0.25	+0.32	−0.15	+0.19
788	−0.15	−0.08	−0.35	−0.01
Σ	−0.35	0.00	−1.70	0.00
Σ/n	−0.07		−0.34	

$$\mathrm{I}=-39 \qquad \mathrm{II}=+109$$

$$\Delta\lambda=\frac{\mathrm{I}}{\mathrm{III}}=0.00006 \qquad \Delta k=\rho\frac{\mathrm{II}}{\mathrm{III}}=1^c$$

$$[\Delta X^2]+[\Delta Y^2]=0.3750$$

$$m_e=\sqrt{\frac{0.3750}{6}}=0.25\text{ mm} \qquad m_\lambda=\frac{0.05}{\sqrt{601760}}=0.00006$$

$$m_k=\rho m_\lambda=0^c\cdot 5$$

to solve are to be introduced partly by the autograph (scale), partly by the drawing device (x- and y-shifts), and partly by the drawing sheet (rotation), the formulas for computing the parameters are the same as in the standard least squares solution for the parameters of linear transformation.

Expressed as functions of X, Y, ΔX, ΔY, the formulas for scale change and rotation are:

$$\Delta\lambda = \frac{[Y\cdot\Delta Y] + [X\cdot\Delta X]}{[X^2] + [Y^2]}$$

$$\Delta\kappa = \frac{[X\cdot\Delta Y] - [Y\cdot\Delta X]}{[X^2] + [Y^2]}.$$

After introducing these two corrections, new values for Δx, Δy are measured and ΔX, ΔY computed as before. To determine whether the adjustment is completed, second corrections for $\Delta\lambda$ and $\Delta\kappa$ may be determined from the new values for ΔX, ΔY. The magnitudes of the second corrections will indicate whether the adjustment is completed, but it is unlikely that the solution will require more than one correction. Finally the microscope (drawing device) is adjusted so that the discrepancies observed in the microscope agree with the new values for ΔX and ΔY. The signs for the corrections $\Delta\lambda$, $\Delta\kappa$ are best determined experimentally so as to conform with the system of axes of the instrument considered etc. Care must be taken that the signs of Δx, Δy are always consistent.

A numerical example of the adjustment procedure is given in Table 1. The adjustment scarcely improved the range of ΔX (from 0.70 mm to 0.65 mm) but it did improve the range of ΔY considerably (from 0.65 mm to 0.30 mm). Theoretically the range of ΔX should have been reduced to 0.55 mm and ΔY to 0.26 mm. The reason this reduction was not achieved is due to residual errors in scale and rotation, inconsistencies in identification of the control points, and setting the floating mark on them in the two rounds of measurement.

Precision

The final discrepancies (ΔX, ΔY after adjustment) provide an estimate of the agreement between the plotted control points and the model. This is best summarised by the standard deviation

$$m_c = \sqrt{\frac{[\Delta X^2] + [\Delta Y^2]}{2n - 4}}$$

where n is the number of control points. To assess the required precision of rotation and scaling we require an estimate in the scale of the plan, of the precision m of identification, setting the floating mark on the control point, and reading the discrepancy in the microscope. Then

$$m_\lambda = \frac{m}{([X^2] + [X^2])^{1/2}} \quad \text{and} \quad \begin{array}{l} m_\kappa = \rho m_\lambda \\ m_\lambda = \\ m_\kappa = \delta m_\lambda \end{array}$$

In the numerical example above, m was assumed to be 0.05 mm.

Acknowledgment

The authors wish to express their thanks to the members of the Photogrammetric Section of the Israel Survey Department who kindly set an A7 Autograph at their disposal for testing the device and procedure.

Literature

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