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Infrared Scan Geometry

Several types of recordings are produced and each is associated with a specific geometric analysis.

(*Abstract is on page 775*)

INTRODUCTION

A LARGE PORTION OF INFRARED radiation can only be recorded with special sensing devices. At present, photographs produced by these sensor systems are used for photointerpretation only. An important application could be found in the field of mapping in special cases since such images possess certain advantages over the conventional type of photographs.

This paper summarizes the various types of recordings produced by infrared scanners and the relationships between system parameters, and examines certain problems concerning geometry.

TYPES OF RECORDING

Scanner devices may be divided into three groups, depending on the number of faces of the scanning prism or mirror and the number of detector elements employed.

1. The simplest arrangement is a one-faced prism and a single detector. Each revolution of the scanning prism produces only one scan line. The recording is discontinuous owing to the fact that no information is collected while the prism completes its rotation past the end of a scan line
2. Prisms with n number of faces and one detector produce n scan lines per rotation. The recording may or may not be discontinuous depending on the size of the maximum scan angle.
3. The most complicated system is a prism with n faces and a detector array of p elements, whereby p scan lines are recorded simultaneously and usually without any discontinuities.

In Figures 1 and 2 the image coordinates are plotted as a function of time for the first two types mentioned. Dashed lines signify no recording and both figures represent a panoramic presentation. In the event of rectilinear recording the y' scan lines take up the shape of a tangent curve.

SCANNING SYSTEM PARAMETERS

The simplified scanning geometry is shown in Figure 3 in which the principal parameters are:

θ_m = half the total angle of scan
α = instantaneous angular field of view
s = object scanner prism or mirror rotation rate
ω = angular velocity of object scanner prism or mirror in radians
v = ground velocity of vehicle
v' = velocity of film advancement

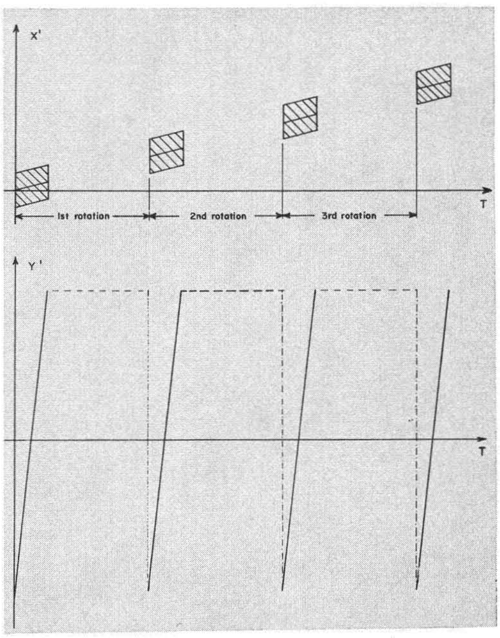

FIG. 1. Recording with a one-faced prism and one detector.

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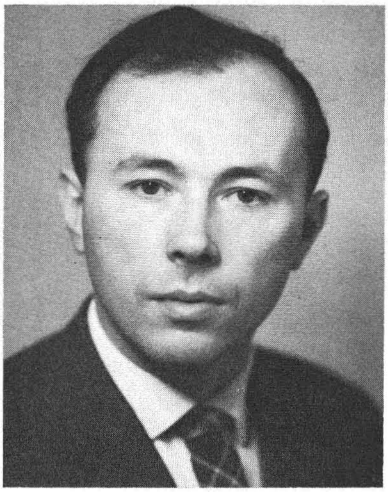

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h = flying height
n = number of faces of the scanning prism
p = number of elements in the detector array
d' = width of the strip image.

It is important to have the scanning system operate in such a manner as to avoid leaving gaps or creating overlaps in the coverage directly under the vehicle. Consequently the correct ground velocity of the vehicle is

$$v = \alpha hspn. \qquad (1)$$

By rearranging Formula 1 the scan rate requirement becomes

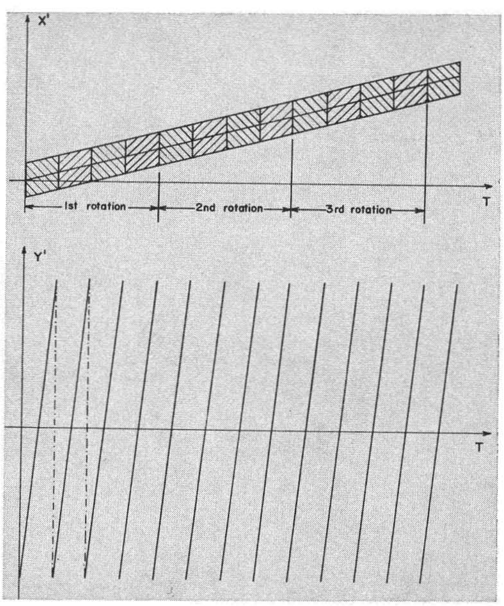

FIG. 2. Recording with an n-faced prism and one detector.

$$s = \frac{v}{h\alpha pn},$$

and the angular velocity of the scanner prism is

$$\omega = 2\pi s. \qquad (3)$$

The velocity of film advancement must be in accordance with the following formula:

$$v' = \frac{1}{SF} v, \qquad (4)$$

where SF is the scale factor.

It is also important to assure that the same scale factor is valid for both the longitudinal and lateral direction of the image whereby

$$SF = \frac{v}{v'} = \frac{2h\theta_m}{d'} \qquad (5a)$$

for panoramic presentation and

$$SF = \frac{v}{v'} = \frac{2h \tan \theta_m}{d'} \qquad (5b)$$

for rectilinear presentation.

Formulas 1 to 5 cover basic demands concerning a correct scale relationship between object and image. In addition several other requirements must be imposed on a line scan system to achieve geometric fidelity.

PHOTOGRAMMETRIC PRINCIPLES

Photogrammetry is based on the fundamental principles of projective geometry.

According to these principles, two geometrical figures are in a perspective position when a unique relationship exists between their elements, whereby each of the elements (points, lines, angles) in one of the figures has a corresponding element in the other. The object and image are in such a perspective position at the moment of exposure.

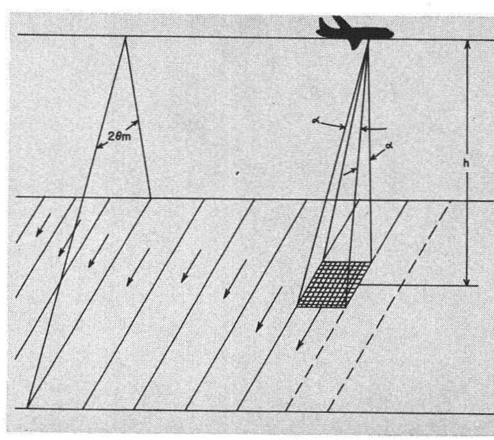

FIG. 3. Simplified scanning geometry.

Figure 4 shows the analytical relationship of a frame camera photograph.

Vector ξ_0, the space relationship A, between ground-system (i,j,k) and photo system (a,b,c) and the scale factor λ uniquely define the position and orientation of the photograph with reference to a fixed (i,j,k) system. Vector ξ_i, representing the position of a point P_i, can now be determined from an image vector p_i', which corresponds to image point p_i'.

The relative position between object and image in space can be expressed by the equation of collinearity of an object and image point and the exposure station.

$$\xi_i = \xi_0 + \lambda p_i', \tag{6}$$

or in matrix notation

$$\begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} = \lambda A \begin{bmatrix} x_i' \\ y_i' \\ -f \end{bmatrix} + \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} \tag{7}$$

where

X_i, Y_i, Z_i = coordinates of P_i in a fixed ground system,
X_i', y_i', f = image coordinates,
X_0, Y_0, Z_0 = coordinates of exposure station in the fixed ground system,
A = rotation matrix,
λ = scale factor.

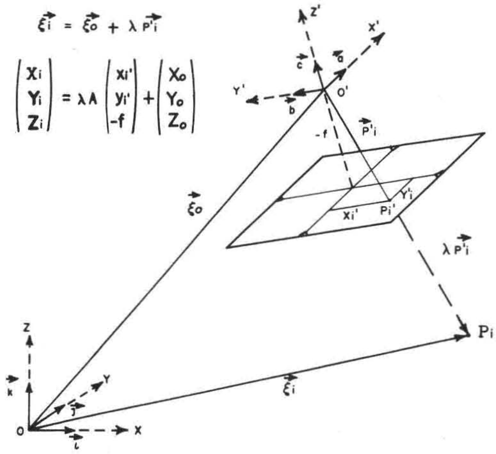

FIG. 4. Analytical Relationships of Frame Camera Photography.

of a narrow angular field of view are employed, only a very small portion of the object is projected onto the image plane at a particular instant. Images recorded with such a device from a moving vehicle will maintain the collinear relationship between a particular image and object point but no perspective will result since the recording is not instantaneous. Conventional photogrammetric procedures are no longer applicable here. Each image point becomes a function of time and must be considered individually.

ABSTRACT: *Effects of continuously changing orientation elements create large distortions in line-scan imagery. Geometric fidelity of such imagery is low and point identification can be difficult unless these distortions are prevented or eliminated. Gyro stabilization and automatic pilot, or a continuous recording of orientation elements, are the best means for obtaining imagery for mapping. Special instruments, such as an OMI-Bendix analytical plotter AP-2, are required to produce maps. Geometrical information may also be extracted point by point with the use of comparator measurements. Stereo effect may only be helpful for stabilized type "stereo" imagery. Most imagery available at present is obtained with single image scanners and have geometric fidelity which is far from desirable. It should be more generally recognized that "stereo" imagery is the only means to locate and identify objects properly. On the basis of the material presented, detailed techniques may readily be developed for the evaluation of "stereo" imagery once it becomes generally available.*

In the formulae $\xi_0, \lambda, A, X_0, Y_0$ and Z_0 have constant values applicable to the entire frame.

Numerous collinear rays connect object and image points at the moment of exposure if a photograph is taken with a frame camera. Space resection, relative and absolute orientation can then be performed by such bundles of collinear rays.

When dynamic camera systems or scanners

The analytical relations of line scan imagery are illustrated in Figure 5 and the equation of collinearity is

$$\xi_i = \xi_{0i} + \lambda_i p_i' = \xi_0 + V_i + \lambda_i p_i', \tag{8}$$

where V_i is the displacement vector of exposure station, and

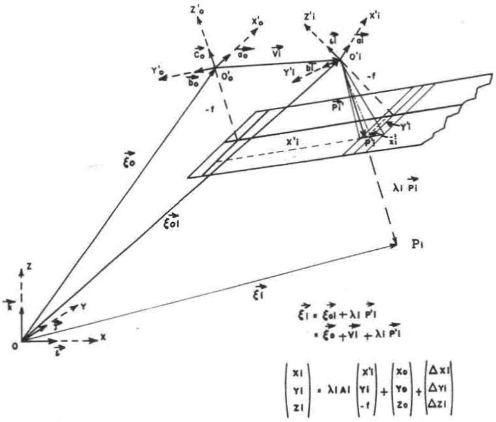

FIG. 5. Analytical Relationships of Line Scan Imagery.

$$\begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} = \lambda_i A_i \begin{bmatrix} x_i' \\ y_i' \\ -f \end{bmatrix} + \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + \begin{bmatrix} \Delta X_i \\ \Delta Y_i \\ \Delta Z_i \end{bmatrix} \quad (9)$$

where

X_0, Y_0, Z_0 = ground coordinates of the ideal exposure station at the beginning of the recording.

$\Delta X_i, \Delta Y_i, \Delta Z_i$ = ground coordinates of the scanner optical centre with reference to X_0, Y_0, Z_0.

All λ_i, ΔX_i, ΔY_i, ΔZ_i, A_i values are constantly changing and consequently gaps and double images may easily occur. This however makes it difficult to maintain projective relations, so that one image point uniquely corresponds to one object point. Gaps or overlaps which exceed the limits of resolution must be avoided, otherwise the imagery will not be suitable for mapping and will not be of general use for interpretation either, since objects cannot be properly recognized.

In other words, changes of the exterior orientation elements, X_0, Y_0, Z_0, φ, ω, κ introduce displacements in the image coordinates and distort the shape of objects. This problem is discussed in more detail in the next section.

EFFECT OF EXTERIOR ORIENTATION CHANGES

Changes of exterior elements from point to point have differential values within a scan line and also from one scan line to the other if the scan rate is sufficiently high. For example φ, ω and κ changes are not likely to exceed 20–30 seconds of arc per scan line at a scan rate of 100 scans/sec. Very low altitude flights are an exception since angular velocities can reach several degrees per second. At the same scan rate changes in angular orientation elements can then amount to 1 to 2 minutes of arc per scan line.

Point displacements caused by such orientation changes are generally tolerable in most cases. However, an accumulation of these changes over several scan lines creates serious distortions.

The magnitude of coordinate displacement of a particular point depends also on its location within the scan line and can be computed from the following formulae:

$$dX_c = hd\varphi + dX_0, \quad (10)$$

$$dY_c = -hd\omega + dY_0, \quad (11)$$

$$dX_\theta = h \tan\theta d\kappa + hd\varphi + dX_0, \quad (12)$$

$$dY_\theta = \tan\theta\, dh - h(1+\tan^2\theta)d\omega + dY_0, \quad (13)$$

where

dX_c, dY_c = coordinate changes in the centre line of a scan line,

dX_θ, dY_θ = coordinate changes at a scan angle θ,

$d\varphi$, $d\omega$, $d\kappa$, dX_0, dY_0, dh = changes in the orientation elements.

TABLE 1

VALUES OF $hd\phi = hd\omega$ IN FEET

h in feet	1,000	5,000	25,000
$hd\phi = hd\omega$	18	87	434

TABLE 2

VALUES OF $h \tan\theta\, d\kappa$ IN FEET

h / $\theta°$	10°	30°	45	60°	70°	75°
1,000 feet	3	10	17	30	48	65
5,000 feet	15	50	87	151	240	326
25,000 feet	77	252	436	756	1,199	1,628

TABLE 3

VALUES OF *tan θdh* IN FEET

$\theta°$	10°	30°	45°	60°	70°	75°
$\tan\theta\, dh$	9	29	50	87	138	187

TABLE 4

VALUES OF $h(1+\tan^2\theta)d\omega$) IN FEET

h \ $\theta°$	10°	30°	45°	60°	70°	75°
1,000 feet	18	24	35	70	150	261
5,000 feet	90	116	174	349	746	1,303
25,000 feet	448	580	870	1,743	3,728	6,511

Tables 1 to 4 list numerical values for the various terms in Formulas 10 to 13 based on assumption that $d\varphi = d\omega = d\kappa = 1°$ as compared to a truly vertical scan, perpendicular to the flight direction, with flying height variations of $dh = 50$ ft. The origin of the coordinate system is in the centre of the scan line and the positive X-axis points in flight direction.

The foregoing results lead to the following conclusions:

1. A change of 1° in the angular orientation elements can build up within one second of recording time. At a scanner rotation rate of 100 scans 1 sec. and a ground resolution of 1 foot in the X-ray direction, the imagery is capable of covering a strip of terrain of 100 feet in flight direction. Comparing this value with the Tables shows that distortions can add up to several times the size of an object.
2. The X-coordinate of points in line scan imagery are free of displacements due to height differences and changes in flying height if scanning is done in a near vertical plane.
3. The change in camera swing $d\kappa$, which does not play a major role in conventional photogrammetry, has considerable influence on the accuracy of line scan imagery.
4. From Tables I to IV it becomes evident that the $d\omega$ change has the largest influence on the accuracy of an image.
5. All displacements increase rapidly if the scan angle becomes larger than 60°. Displacements due to angular orientation changes will increase in proportion to the flying height.
6. It is impossible to eliminate all effects of changes in orientation elements with the aid of ground-control points, because magnitude and sign differ from scan line to scan line. Some other methods must be developed to correct for these distortions.

Evaluation of Infrared Scans

Two ways are possible to create projective relationships between object and image and to facilitate line scan imagery mapping. One is a fully automatic system employing gyro stabilized mounts and an automatic pilot with respect to position, rotation and elevation. The second suggested solution is to obtain a continuous recording of the exterior orientation elements. In both cases plotting can be executed on an analytical plotter or in the second case 'rectified' images can be produced on a specifically designed restitutor.

Both solutions have advantages and disadvantages. Experiences in conventional photogrammetry indicate that the recording of the orientation elements can be achieved to a higher degree of accuracy than to maintain them at a fixed value. On the other hand, images produced with an unstabilized scanner can have large distortions due to orientation element changes which make object identification difficult.

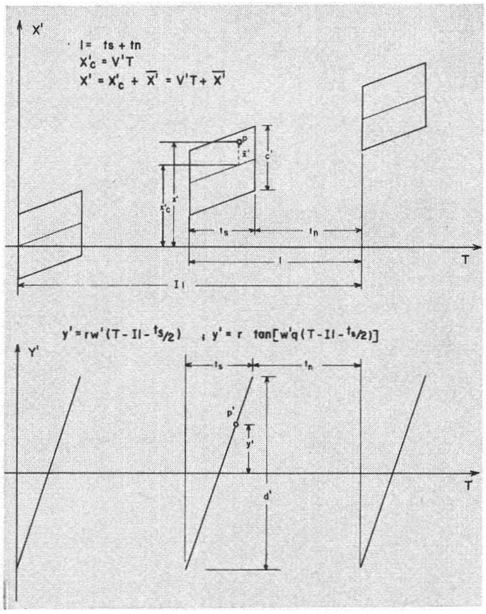

FIG. 6. Line Scan Image Parameters.

By examining the second scheme it becomes evident that orientation data must be properly coordinated with image points. In this case time can serve as a common parameter which in turn requires a knowledge of the moment of recording of each image point.

An examination of graphs of a discontinuous recording (Figure 6) shows that the x'-coordinates are piecewise linear function while y' has a periodic interval of $l = ts + t_n$ where

l = length of a period,
t_s = length of time required to record a scan line,
t_n = length of time during which no image recording takes place.

Theoretically speaking, all continuous recordings can be considered having very small discontinuities.

Image coordinates recorded at a time T may be expressed as based on Figure 6. The x' coordinate has a value of

$$x_c' = v'T, \qquad (14)$$

for all points situated along the centre line of a scan line. For other points

$$x' = x_c' + \bar{x}' = V'T + \bar{x}' \qquad (15)$$

where

$x'_c = x'$ coordinates of an image point on the centre line of a scan line

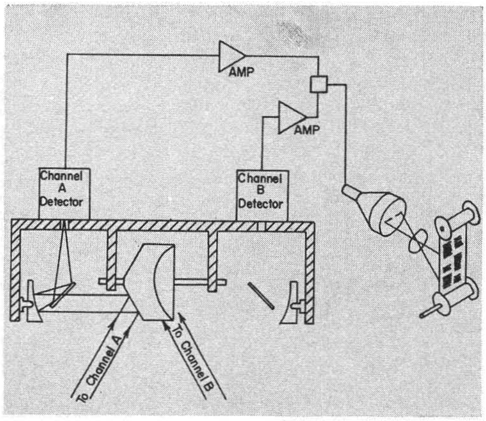

FIG. 7. Stereo Scanning System.

T = time of recording of the image point in question

$\bar{x}'$ = deviation of image point from centre line of scan line. (It is considered positive in the direction of flight).

The maximum value of $\bar{x}'$ is half the width of a scan line recording. As the resolution of a scanner recording is equal to the width of a scan line, the $\bar{x}'$ value is negligible and T can be obtained from Formula 14 as a close approximation. The prerequisite of the method is a known, uniform film speed.

The formulas for y' coordinates are

$$y' = r\omega'\left(T - Il - \frac{ts}{2}\right)$$

for panoramic presentation,

$$y' = r \tan\left[\omega p\left(T - Il - \frac{ts}{2}\right)\right]$$

for retilinear presentation.

The w' is the angular velocity of the recording light beam in radians. I is an integer which signifies the number of full periods completed during the time T. The r is the principal distance of the image scanner optics in case of a glow tube recording unit and the distance to the screen from the midpoint of the deflection plates for a cathode-ray tube recording unit. It is assumed here that the radius of curvature of the cathode-ray tube is also equal to the value of r, and that the photographic recording is made with a 1:1 object to image ratio.

"STEREO" IMAGERY

In order to have a complete mapping system, height differences must also be determinable. For this reason each object point must have two sets of collinear rays separated by a known base vector. This can be achieved either by two overlaping flight lines or by a stero scanner. (See Figure 7.) The principle of this device is similar to a Sonne continuous strip stereo camera in which two lines are scanned simultaneously at diverging angles, one ahead, the other behind the aircraft path.

CONCLUSIONS

The conclusions are stated in the Abstract on page 775.

ACKNOWLEDGEMENT

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