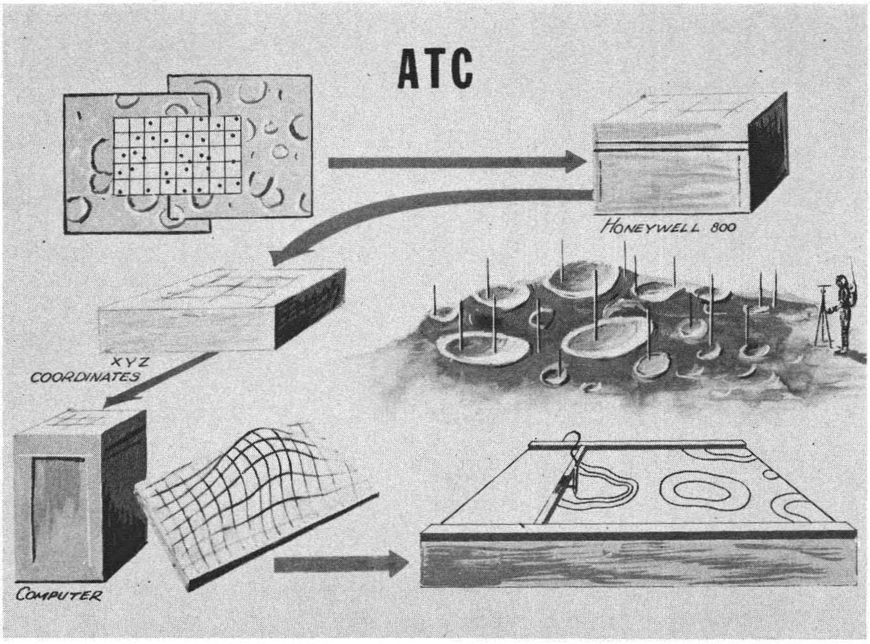

FRONTISPIECE. The Analytical Topographic Compilation System.

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Ranger Mapping by Analytics

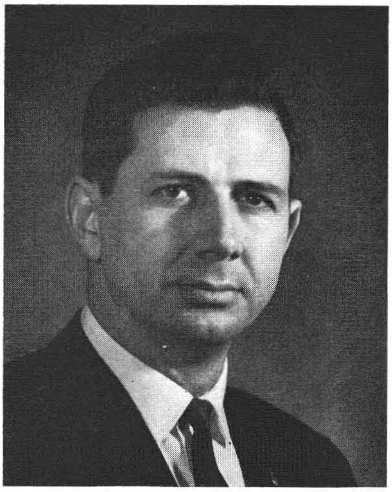
An automatically drawn contour map is produced by a series of computer programs based on stereoplotter or comparator measurements on lunar photos.

(*Abstract on page 794*)

INTRODUCTION

FOR AGES MAN HAS STUDIED and charted the heavens with a special interest in our closest celestial neighbor—the moon. Man first studied the moon with the naked eye, then with powerful telescopes, and now, thanks to the phenomenal success of the National Aeronautics and Space Administration's (NASA) Ranger missions, we have relatively close-up photography, with high resolution, of a portion of the lunar surface. Such rare imagery of our mystical neighbor, coupled with the need for landing-site maps, has led to a NASA-Army Map Service (AMS) agreement to attempt to produce topographic maps from this photography. It is more important however, to develop techniques and gain experience in reducing satellite photography in preparation for NASA's Lunar Orbiter missions. Although the imagery appears to be good, and some say that this is misleading, the geometry leaves much to be desired. In fact, the geometrical parameters

* Presented at the Annual Convention of the American Society of Photogrammetry in Washington, D. C., March 1966, under the title "Ranger Mapping by Analytical Topographic Compilation."



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exceed the physical ranges of even our most universal instruments, i.e.; low B-H ratio, extreme tilts, and the small 25 mm. focal length, on a format about 11 mm. per side. In addition to these unconventional parameters, the transmission from the spacecraft to earth, and subsequent reassembly, no doubt contribute additional problems.

The requirement to produce maps from Ranger photography has forced Army Map Service Cartographers to utilize computational methods of extracting contour information from this geometrically unusual, yet fascinating, photography. The fact that the Ranger stereo models cannot be absolutely oriented (leveled) in the plotter is the primary reason for resorting to computational methods. This paper describes a completely computerized contouring method, called "Analytical Topographic Compilation (ATC),"† that is being used to compile this contour information. This analytical effort is in direct support of a project of my colleague L. D. Bowles, "Cartographic Experimentation with Ranger Photography." The latter project is being conducted for NASA under Contract No. T-21657(G).

In essence, a dense network of x,y-stereocomparator, or x,y,z-stereoplotter coordinates are measured from the Ranger stereo models and recorded in digital form. These coordinates are then adjusted by analytical methods to the selenodetic control to obtain a dense network of X,Y,Z terrain data. A computer contouring technique is subsequently employed to interpolate contours.

As an introduction, Figure 1 shows the flow of the data that begins with the mensuration phase and ends with the contour plots drawn by a computer-driven X,Y-plotter.

Considering each mensuration phase separately, we first review the Schmid[1] analytical method which utilizes stereocomparator measurements. The operator measures control, and image points (as in any analytical photogrammetry application); but, in addition, he measures a dense network of points (x,y) over the entire model. Most of our measurements have been at intervals of 1 or 2 mm. on the model (1,000 or 2,000 meters on the moon) with additional points to further define craters, rills, or any other significant topographic feature.

In general, Dr. Schmid's analytical method can be expressed by the projective transformation equations:

$$(x - x_0) = C\,\frac{A_1(X - X_0) + B_1(Y - Y_0) + C_1(Z - Z_0)}{D(X - X_0) + E(Y - Y_0) + F(Z - Z_0)}$$

$$(y - y_0) = C\,\frac{A_2(X - X_0) + B_2(Y - Y_0) + C_2(Z - Z_0)}{D(X - X_0) + E(Y - Y_0) + F(Z - Z_0)}. \qquad (1)$$

The stereoplotter method is similar, except relative orientation of the terrain model is attained with the stereoplotter, which, in our case, is a Stereoplanigraph. Then, a dense network of x,y,z-coordinates are recorded, along with the instrument positions of the ground X,Y,Z control. Absolute orientation of the model in the plotter cannot be attained, largely due to extreme tilts in the photog-

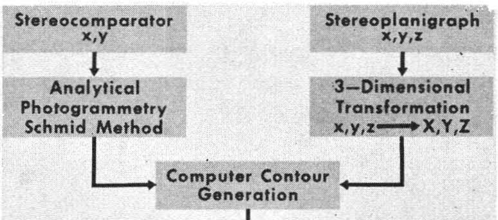

FIG. 1. Analytical topographic compilation.

† More precisely, the method produces hypsography only. In the case of the moon, which, apparently has no drainage or culture, the hypsography completes the topography. For purposes of this paper, therefore, the method can truly be called, "Analytical *Topographic* Compilation."

raphy. Therefore, a three-dimensional, linear transformation[2,3] is utilized first for the absolute orientation of the model, and, subsequently, to compute X, Y, Z terrain positions for all points measured. In general terms, this transformation can be expressed by Equation 2. Figure 2 illustrates the tilted spatial model, as measured by the plotter.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = K \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x-x_0 \\ y-y_0 \\ z-z_0 \end{bmatrix} + \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix}. \quad (2)$$

The unknown parameters in the transformation are three angles $(\omega, \varphi, \kappa)$, three translations (X_0, Y_0, Z_0), and a scale factor (K). Three scalars can be computed. The final output from both methods is Mercator northing and easting grid coordinates and elevation based on the lunar sphere.

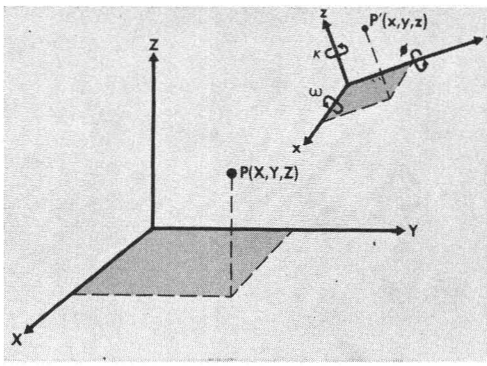

FIG. 2. Tilted model in space.

NUMERICAL SURFACE TECHNIQUES

To determine contours by computational methods from a three-dimensional stereo model, the usual procedure is to represent the surface by a network of measured points, then by an equation. In addition, an efficient contouring method should make use of the many techniques which are consciously or subconsciously employed in manual contouring. It is possible to obtain a statistically valid surface of a data configuration that is completely unrealistic if it is compared to what the cartographer might expect, due to his knowledge of topography.

With these considerations in mind, various ideas were explored with the idea of designing our own contouring program. First, however, we searched the literature, and inquired among various organizations to determine what, if any, digital contouring programs were already available. We determined that IBM[4], the Marquardt Corporation[5], the Control Data Corporation, Data Center (CDC)[6], and the X, Y-Plotter manufacturers, have these programs. The "Numerical Surface Techniques" described here are a consolidation of these references with some of the author's ideas, in order to describe a procedure for digital contouring of topographic data.

For both of our methods of mensuration, the adjustment is done at AMS on the Honeywell H-800 Computer. Our program transforms the instrument measurements to Mercator grid coordinates and elevations. It produces a printed copy for analysis purposes, and a magnetic tape suitable for plotting at the Control Data Center on their X, Y-plotter.*

The method described here assumes a random distribution of data points (X, Y, Z) well distributed over the model area. In general, the entire process can be represented by Figure 3.

ABSTRACT: *A digital contouring technique called "Analytical Topographic Compilation" is presented. In essence, a dense network of* x,y-*stereocomparator coordinates, or* x,y,z-*Stereoplanigraph coordinates, are recorded from the lunar surface of the Ranger stereo models in digital form. These coordinates are then adjusted by analytical methods to the selenodetic control to obtain a dense network of digital* X,Y,Z *terrain data. A digital computer contouring technique is then employed to generate contour maps which are drawn on an* X,Y-*plotter.*

CONTOUR PROGRAM INPUT DATA

Input to the contouring program consists of three coordinate values (X, Y, Z) for each data point. The X, Y and Z's correspond to easting and northing grid coordinates and the elevation of the point of interest. These coordinates are the transformed photogrammetric measurements. Normally, they are irregularly distributed (see Figure 3).

* AMS has recently acquired its own X, Y-plotter.

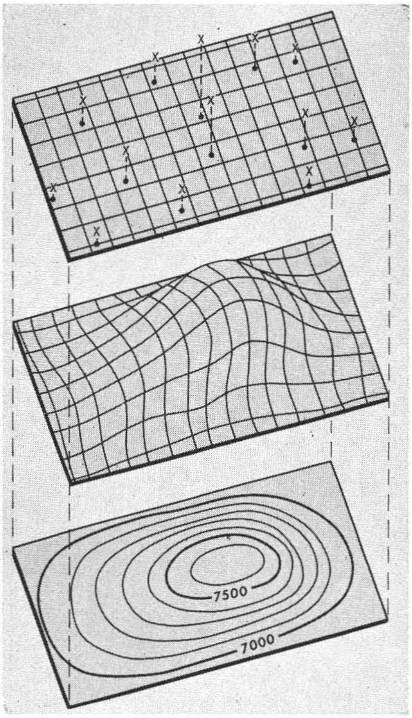

FIG. 3. Numerical surface representation.

MATHEMATICAL GRID CONSTRUCTION

A uniform mathematical grid must be established over the model area. According to IBM[4], the grid interval should not exceed one-third of the average distance between the network of data points for accurate interpolation of the surface.

MESH-POINT VALUES

Once the grid system has been established, the (Z) mesh-point values of the grid must be determined. This is accomplished by a continuous process of fitting planes to groups of observed data points. The size of these planes is defined by the grid (mesh) interval, which has been determined from the average interval between the measurements. Thus, the completeness of the contours is a function of the density of the measurements. The general form of the equation representing the plane is:

$$Z = K + HX + DY. \tag{3}$$

To fit these planes, from four to eight observed points should be selected, so the least squares solution will apply. Observed points closest to the area of interest are chosen for the plane fit. The number chosen can be controlled by defining a circle of given radius that these points must fall within (see Figure 4). Equation (3) is then fitted to these points by the method of least squares. Figure 5 illustrates the fitted plane to five data points. To fit these planes, first, the centroid of the selected data points is computed.

Next, the plane passing through this centroid is established by a weighted, least squares fit. The weights are determined such that data points closer to the centroid are given greater weight than those further away. The values of the plane, at the corners of a particular grid square, are the mesh-points involved (see Figures 5 and 6). Note from Figure 5 that the least squares criteria provide a best fit. Three of the points are above the plane and two are under it. The observation equations for the plane, $Z=K+HX+DY$ are:

$$\begin{bmatrix} 1 & X_1 & Y_1 \\ 1 & X_2 & Y_2 \\ 1 & X_3 & Y_3 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ 1 & X_i & Y_i \end{bmatrix} \begin{bmatrix} K \\ H \\ D \end{bmatrix} = \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ \cdot \\ \cdot \\ Z_i \end{bmatrix}, \quad 4 \le i \le 8$$

or

$$MA = C.$$

The weights are defined as $w = 1/D^2$ where D is the radial distance of point P from the centroid. Then, the weight matrix is

$$W = \begin{bmatrix} w_1 & & & & \\ & w_2 & & & \\ & & w_3 & & \\ & & & \ddots & \\ & & & & w_i \end{bmatrix}.$$

The normal equations can be given as:

$$[M^T W M](A) = [M^T W C].$$

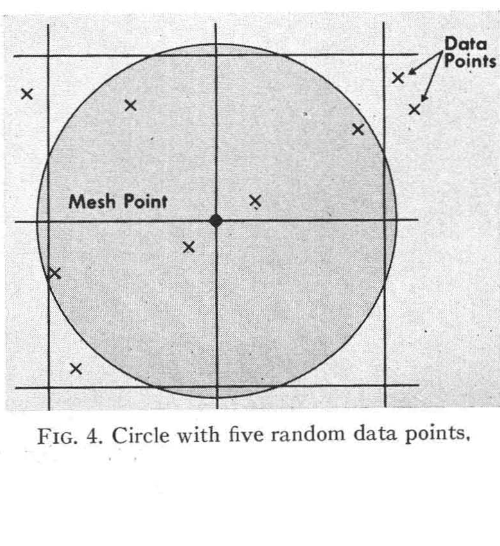

FIG. 4. Circle with five random data points.

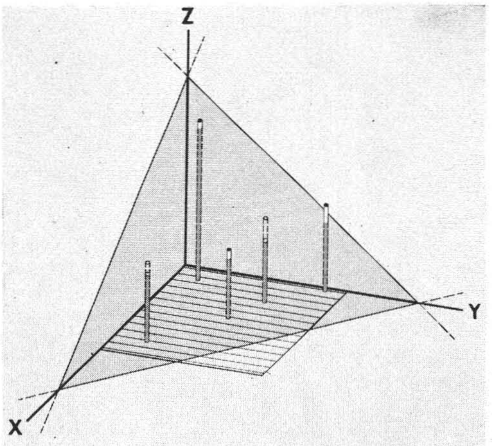

FIG. 5. Plane surface fitting.

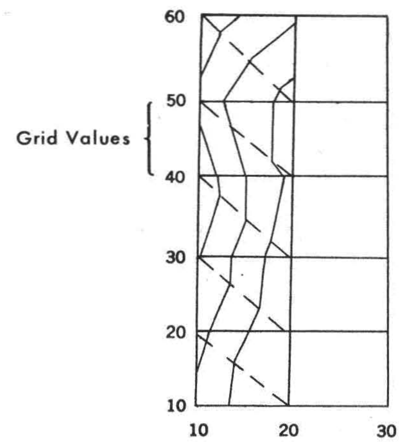

FIG. 7. Grid values.

The solution is

$$(A) = [M^T WM]^{-1}[M^T WC].$$

After all mesh-points, that surround the original data, have been determined by successive fitting of planes, these mesh-points are used to fill in the void areas where data do not exist. We finally arrive at mesh-point values for each grid point in the model. This is illustrated by Figure 6 where all mesh-points are dotted.

SMOOTHING

For topographic contouring, smoothing between mesh-point values should be performed with caution. The contours must portray the detail, and they must agree with the observed data. A suggested smoothing technique is to successively fit a second-degree equation to sets of three mesh-points in a row, in both the X and Y directions (see Figure 6). The Z-values corresponding to the required contours can be substituted into these equations, and the appropriate X, Y-coordinates, corresponding to the contour value of Z along the grid, can be determined. The general form of the equations for X and Y, respectively, are:

$$Z = AX^2 + BX + C,$$
$$Z' = A'Y^2 + B'Y + C'.$$

Once the curve is formed, the X, Y-coordinates for a given contour elevation Z are:

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

and similarly for Y.

Topographic detail can be retained by fitting a straight line across the diagonals of each grid square. This helps eliminate sharp turns in the contours, and should make smoother contours.

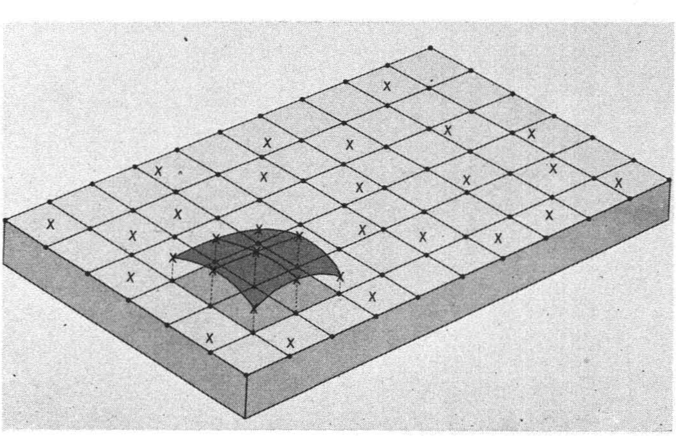

FIG. 6. Configuration for smoothing.

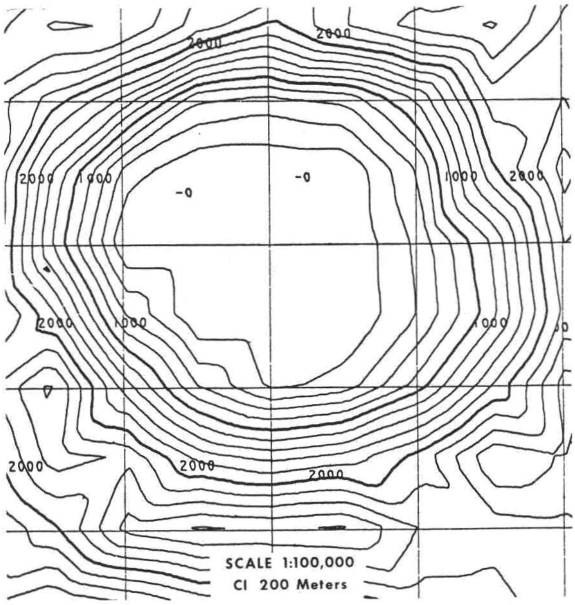

FIG. 8. Analytical topographic compilation raw data.

PLOTTING CONTOURS

By this time, a huge volume of data has been generated. The remaining item is to present the X, Y-coordinates along the contours to be drawn by the graph plotter. To minimize the storage problem, plotting by segments (one grid interval at a time) is suggested. In this procedure, it is easier for the plotter to join successive points that fall on the given contour (Figure 7). Operational tests to date have shown that the procedure provides a logical, and apparently reliable, representation of the given information.

ANALYTICAL TOPOGRAPHIC COMPILATION (ATC) EXAMPLE

Figure 8 is an exact reproduction of contours that were drawn by the ATC procedure.

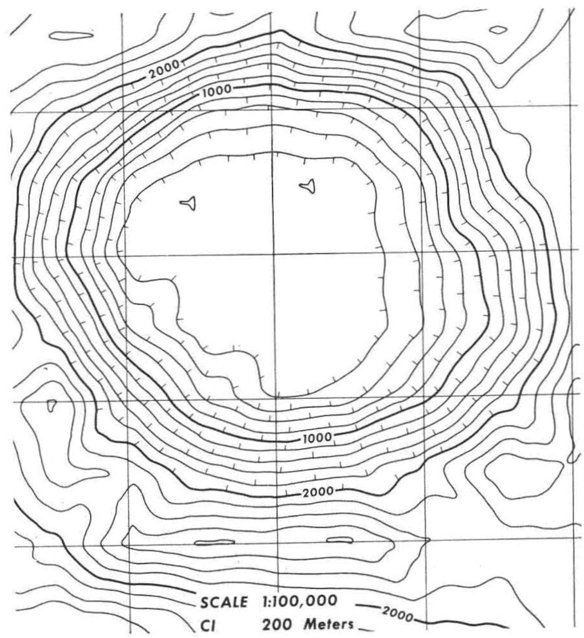

FIG. 9. Analytical topographic compilation refined.

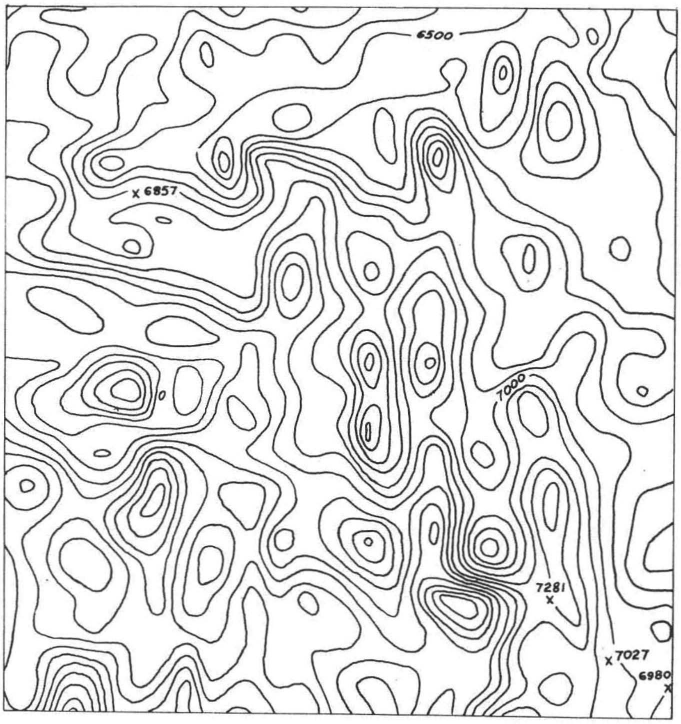

FIG. 10. Ranger *Raw* Compilation. Stereoplanigraph scale—1:1,000,000; *X, Y*-plotter scale—1:150,000; contour interval—100 meters; mesh network—900×900 meters. This compilation, not on the Selenodetic datum, was run for the purpose of testing the programs in the total ATC system.

Figure 9 is the same data that has been refined by the cartographer. The data for Figures 8 and 9 were scaled from a 1:1,000,000 Lunar Chart. The chart was chosen for the test run because of the crater topography. Two-thousand points were selected from a portion of the chart to be used as input to the contouring program. The resulting contours are comparable to the contours from which the data were taken.

Basically, this method of digital contouring is similar to what the plane table topographer has done for many years. But, thanks to computers, we have a means of processing this mass of data, once we get it, in about two hours per stereo model. Our first lunar model consisted of 4,552 observed points over an area of 72×93 km. The altitude is about 300 km. giving a photography scale of 1:12,000,000. The scale of the Stereoplanigraph model was 1:1,000,000. Plotting scale (CalComp Plotter) is 1:150,000 with 100-meter contour intervals. Although this plotting scale is unrealistic, when compared with the 1:12,000,000 photo-scale, we thought it desirable to have a large manuscript to analyze the contours more thoroughly. The manuscript is about 21×27 inches. Figures 10 and 11 are a small portion of this sheet. This compilation required the equipment and time increments shown in Table 1. We hope to reduce these times as we gain experience in computational methods. In summary, ATC provides an objectively contoured manuscript which later can be subjectively beautified by the cartographer. He can utilize his topographic license, by suppressing at one place and slightly exaggerating at another, according to his judgement and photo-interpretation. This combination of man and machine optimizes the use of these materials by producing a graphic expression of the imagery that is agreeable with the spot heights from which the contours were made.

PROJECT STATUS

A statement concerning the problems and progress of the ATC portion of the Ranger project seems appropriate. First, the programs for adjusting the data, interpolating and drawing the contours, in both the comparator and stereoplotter approaches are operational. But, much more remains to be accomplished.

To date, we have measured, adjusted and contoured, with both approaches, several experimental Ranger compilations, and also, a model from the Arizona test area to further

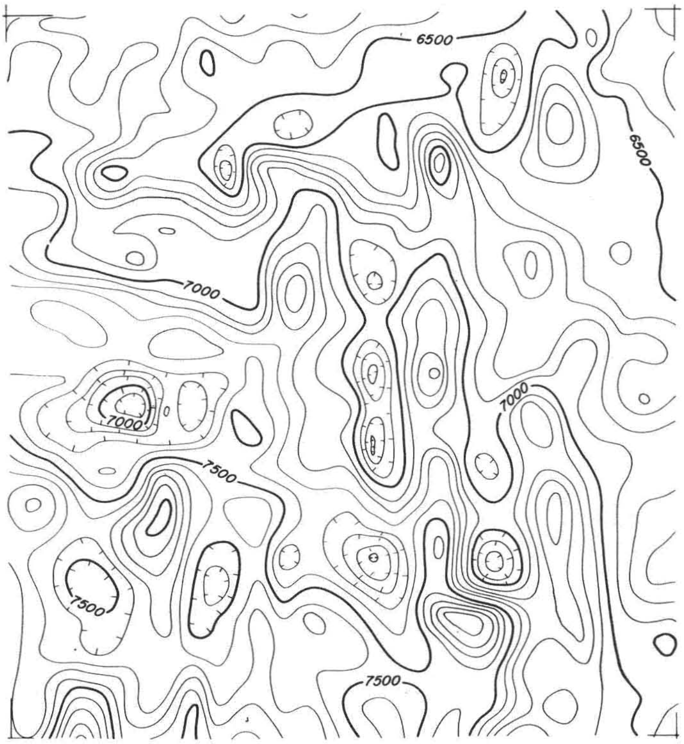

FIG. 11. Ranger *Refined* Compilation. Stereoplanigraph scale—1:1,000,000; X, Y-plotter scale—1:150,000; contour interval—100 meters; mesh network—900×900 meters. This compilation, not on the Selenodetic datum, was run for the purpose of testing the programs in the total ATC system.

compare the digital method with conventional contouring procedures. The Arizona stereoplanigraph model (1:20,000) was relatively oriented, and then; tilt, tip, and swing (ω, φ, κ), on the order of 10 grads, was introduced to simulate measurements from an unleveled model. The program, utilizing Equations 2 adjusted the model to its 16 control points with an RMSE in Z of 1.2 meters. The Comparator method computed a RMSE of 0.7 meters in Z. Forty-two hundred points were read from the planigraph model at 40 meter intervals on the ground and contours at 10-foot intervals were drawn on the x,y-plotter. A comparison of the digital contours to the conventionally compiled contours shows that, basically, the digital method is accurate, but to some extent, lacks topographic expression. Also, some of the sharp peaks were chopped off, due to the plane fitting process, when they should have had one more contour. We feel confident that this can be overcome by increasing the number of measurements that define the feature. Also, the cartographer can add the spot elevations for the peaks and special features and enhance the topographic expression prior to, or during, the final drafting process.

We are attempting to utilize these findings on the Ranger work, although additional problems have been identified. One is that, when using Ranger material, the computed Z-scale in the absolute orientation program is usually about one-third of the horizontal scale. This causes a flattening of the relief, which could be due to the camera-plotter geometry, or from the fact that adequate selenodetic control is not yet available in the Ranger area. The AMS Department of Geodesy is extending Selenodetic control into the Ranger VIII area. At present, the crater

TABLE 1
EQUIPMENT AND TIME USED IN COMPILATION

Operation	Equipment	Time (min.)
Adjustment	H-800 Computer	43
Interpolate Contours	CDC-3600 Computer	36
Plot Contours	Cal-Comp X, Y-Plotter	150

Schmidt is the only point with latitude, longitude and elevation given in the AMS Selenodetic control network.

Presently, we are utilizing latitude and longitude furnished by the Jet Propulsion Laboratory for the imagery beneath each photo reticle and interpolating corresponding elevations from existing charts and maps. It is evident that elevations determined from such sources are a long way from adequate, and this may be the cause for the flattening in the Z-scale. We have also assumed a level datum for crater rims as a source of vertical control which has increased the Z-scale up to about one-half the horizontal scale. A constraint on the Z-scale making it equivalent to the horizontal, has been applied, but the results often produce irregular profiles. This could be caused by several things such as large spreads in the random measuring errors, unknown distortions, inadequate control, or perhaps the lunar maria area near the craters Sabine, Ritter and Schmidt actually slope and undulate more than is apparent from the photography.

Another problem is the precision in measuring the z-coordinate. The standard error for repeating the z is about 120 meters in the stereoplanigraph, with a spread up to 400 meters. Fortunately, the contouring program is very sensitive to the measurements, but unfortunately, it frequently plots a small depression, or hill which is actually an error in measurement. The flat areas of low contrast are affected the most. Some of these depressions are inside the 6,500 contour on Figure 11. Increasing the number of measurements reduces the effect of this problem.

The comparator-analytical system has been utilized to compile contours of the same area. The relative depths of the craters are very realistic, and initial compilations show the slopes in the maria area to agree with the stereoplanigraph and AP2 plotter approaches. Continued analysis and re-runs are underway daily, with the Comparator-Analytical system to derive the optimum product from the photography. Further analysis will appear in the AMS Technical Reports on this project.

Conclusion

Progress to date has indicated that "Analytical Topographic Compilation" is technically feasible. It is a simplified version of an analytical plotter system. It could attract increased interest if the mensuration phase can be significantly speeded up. Most important, the system provides the cartographer a completely universal "tool," making possible what we could not accomplish on existing analog instruments. With conventional methods and these two computational methods in our library, we are prepared for the Lunar Orbiter project just ahead.

Acknowledgement

Special recognition goes to my colleague, Mr. H. R. Cook for his untiring effort in programming the X,Y,Z-transformation (GETRAN) for the stereoplotter approach, and to the AMS Department of Computer Services for programming support in modifying the Comparator-Analytical programs.

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