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AP/C Stability Tests

Thermal equilibrium was a major item of study.

ABSTRACT: *Personnel of the Institute of Geodesy, Topography and Photogrammetry, Politecnico, Milan, Italy, subjected an Analytical Plotter AP/C to numerous severe tests in an effort to determine the degree of thermal stability of the measuring system with respect to time. Major attention was payed to orthogonality, the measuring screws as they were affected by temperature change, and to the heating influence of the servo motors. Ambient operating conditions were determined. The standard coordinate error was* $\pm 1.6\ \mu$ *for one carriage, and* $\pm 2.0\ \mu$ *for the other. The standard error of pointing was* $\pm 0.84\ \mu$*. A calibrated grid was used for the tests; the tests showed as a by-product that the grid error was insignificant. One of the items determined was that either the power needs to be turned on three hours before operation, or left on over night.*

1 IN HIGH PRECISION PLOTTERS and, particularly, in stereocomparators, the instrumental stability during the use and, generally, in time, is of fundamental importance. It is generally realized that high precision is relatively easy to obtain when the instrument is new (especially for stereocomparators which have a relatively simple construction); however precision deteriorates with time if the instrument is not kinematically designed and constructed. As a consequence it follows that the effective precision of an apparatus is not the one that exists immediately after manufacture, but what the precision becomes after a rather long period of use.

For these reasons, before beginning a study on the applications of the Analytical Plotter, Model C, (which was supplied by Ottico Meccanica Italiana) it was desirable to perform a number of rigorous tests in order to establish the limits of the instrumental stability. At the same time, of course, various overall data were obtained in order to evaluate the intrinsic accuracy of the instrument.

The experiments were performed only on a grid so as to avoid the errors due to the identification of photographic images. As far as the instrumental *stability* is concerned, the errors of the grid were not relevant. However, as we also wanted to ascertain the *accuracy* of the apparatus, the errors of the grid had to be determined as well; actually they were of the same order as those of the AP/C, and their determination through measurements on the instruments would have been too difficult and uncertain even if theoretically possible. The calibration of the grid was made with a Genevoise comparator, which gives measurements of errors smaller than $\pm 0.5\ \mu$. Thus, knowing the grid errors, we were able to determine the errors due to pointing and those due to the instrument.

2 THE FIRST EXPERIMENT performed concerned the thermal stability of the instrument which was located in an air conditioned room. The screws are driven by electric servos which rapidly reach a temperature higher than 50°C, and which are directly connected to the screws although through an intermediate insulating material.

The initial measurements had shown an elongation of the screws during a day's work. This effect could be ascribed to two causes: either to the heating of the screws caused by the servos; or to the heating produced by friction with the lead nuts. Therefore, several tests were made under different operating conditions:

A. As soon as the instrument begins working, namely just after power is connected to the

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FIG. 1. Deviations of points on grid line relative to time as described by Type C. A dot refers to a direct reading, and a circle refers to the reverse reading.

servos, the abscissas of the points of a grid line were measured, starting first on the left side of it and then backward, in the direction of the x-axis. These measurements were repeated at regular intervals of about one hour. As each measurement requires about 8 minutes, the heating caused by friction is surely negligible.

B. Under the same conditions described in Type *A*, the carriage was moved back and forth in x without interruption for 15 to 30 minutes, during the interval between two successive measurements. The observations were started again as soon as the movements were stopped.

C. Always under the same conditions, between each measurement and the following one, were performed the normal relative orientation of a model with two grids.

Figure 1 shows, as an example, the results obtained with a measurement of Type C; on the x-axis are represented the position of the points measured on a grid line, and on the y-axis the deviations from the correct theoretical reading. As the measurements were always executed backward and forward, the deviations are indicated with two different symbols—a dot and a small circle.

Because the curves were almost linear (even though they contain the grid errors and the accidental instrumental errors) the linear regression coefficients were computed, that is, the slopes of the straight lines adjusted to the measurements according to the method of the least squares. These straight lines are drawn on the graph. The slope a represents the relative difference of length between the screw and the grid: as the measured zone l is 220 mm. long, the total elongation $-dl$ is equal to $-a$; therefore, if $a=10^{-6}$, it follows that $dl=-0.22\ \mu$. The a-values were then represented in a graph in function of time.

Figure 2 includes three diagrams, concerning the three tests: one each of Type A, Type B, and Type C. These graphs show that in the first three hours of work the servos cause heating of the screws with their consequent total elongation of about 10 μ. After that period, the instrument is considered to be stable because the residual elongations are only a few microns, which is the same order as the measuring errors. It should be pointed out that the slopes are computed with a mean square error of about $\pm 6.10^{-6}$ and therefore the dl have a mean square error of about $\pm 1.3\ \mu$.

Actually, for each series of measurements, the mean error of the a-coefficient was com-

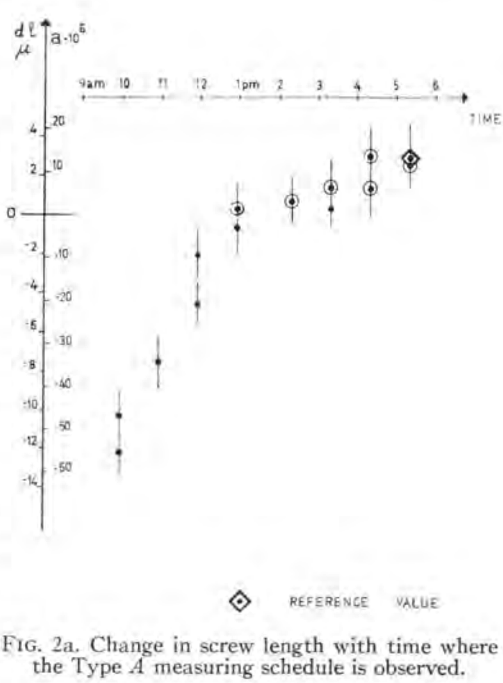

FIG. 2a. Change in screw length with time where the Type A measuring schedule is observed.

puted and, therefore, the corresponding dl. These mean errors have been represented in the graphs of Figure 2 with segments which, on the y-axis scale, are as large as the mean errors of a and dl respectively. The segments are drawn in the two directions, starting from each a point, so as to indicate the accidental character of the errors that include both the observation errors and the screw errors.

In order to evaluate which variations of a (or of dl) can be considered as accidental and which not, in each series a reference measurement was selected whose value was compared with all the other ones by means of the t-test according to the formula

$$t_i = (a_i - a_{ref})/Q_a(m^2_{oi} + m^2_{o(ref)})^{1/2},$$

where Q_a (whose value is constant) is the weight cofactor of a, whereas the m_o^2's are the mean errors of unit weight after taking away the contribution of the screw errors (see Section 4).

With a level of significance of 5 per cent, the evaluation was made as to whether each a_i significantly differed from a_{ref} or not. In all the graphs have been circled those a_i points whose deviation from a_{ref} is accidental.

Secondly, the comparison of Figure 2b with Figures 2a and 2c shows that friction has a modest but noticeable influence on the screws if the carriages move rapidly and for a long period. On the contrary, under ordinary operating conditions, the heating due to friction is absolutely negligible. In Figures 2a and 2c it

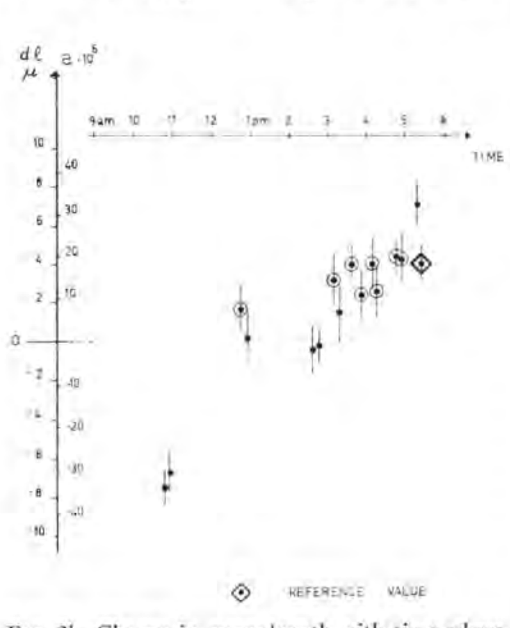

FIG. 2b. Change in screw length with time where the Type B measuring schedule is observed.

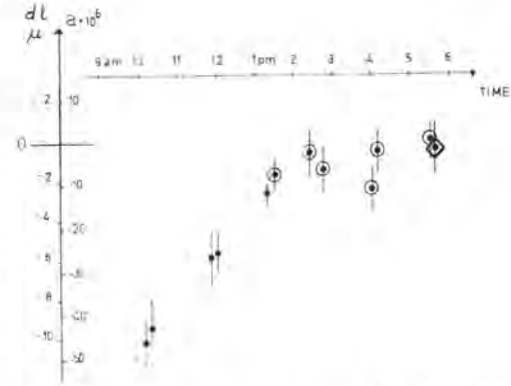

FIG. 2c. Change in screw length with time where the Type C measuring schedule is observed.

can be seen that the thermal equilibrium is reached after three hours: after that, the length variations of the screws are all accidental. Obviously, in Figure 2b, the thermal equilibrium needs a long time to be reached; however, this kind of test was purposely designed to work under definitely exceptional conditions.

In order to ascertain the possibility of reaching good stability in time, a test was protracted for more than three days without shutting the power off the apparatus: the results are fully satisfactory, because once the thermal equilibrium is reached, it remains unchanged. Only one measurement at the beginning of the second day showed a slightly anomalous value of dl but its deviation from the zone where the a-values must be considered as accidental is almost negligible. The results are shown in Figure 3.

Under the present conditions, in instances where operations of high precision are to be performed, it is necessary to follow one of the following procedures: (1) either turn the power on in the instrument early in the morning, namely about three hours before starting work; or (2) leave the instrument on for a whole week, which actually causes no inconvenience.

For normal plotting operations the errors are negligible because coordinates are measured with respect to the center of the plate and the total elongations of the screws cause variations in the measurement of point coordinates of about 6 μ at most, which is of the same order of accuracy as the stereoscopic pointing. As the heating of the screws is uniform, it is equivalent to a variation of the principal distance which is equal for the two cameras.

For the purpose of finding the most suitable

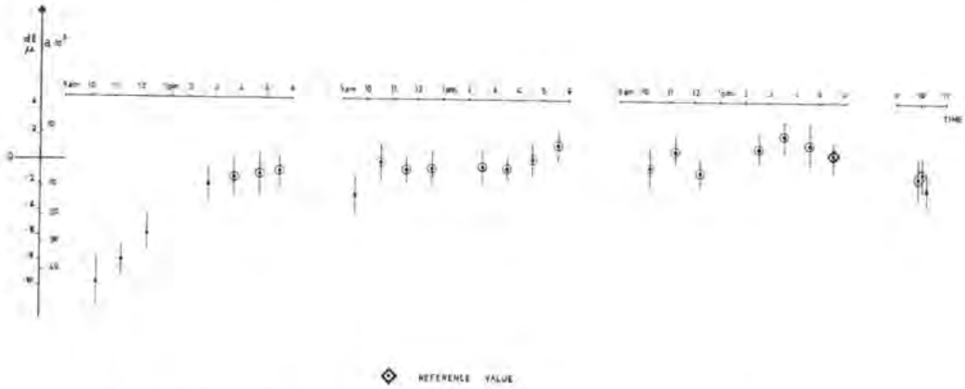

FIG. 3. Variation of screw length with time over a period of three days.

methods to eliminate this slight fault of the apparatus, the covers of the servos were removed in an attempt to cool them down with a small fan. An improvement was evident, as is shown in Figure 4, in which the total length variation of the screw is reduced to 7 μ and the thermal equilibrium is reached in about one hour.

3 THE INSTRUMENTAL STABILITY tests were always performed with the power on so that the measurements could be free from the influence of thermal variations. Moreover, the errors of the grid were not taken into account because in the first phase it was not necessary to know the absolute values of the measured coordinates—only their variation in time. For the same reason, the scale corrections of the screws were applied only in an approximately. In fact, it is obvious that a constant error in the length of the screws of each carriage can be eliminated by introducing a variation of the principal distance (F constant in the AP/C) into the computer, or by varying the projection distance (bz constant in the AP/C).

On the contrary, the length of the y-screw can be corrected and made equal to the x-screw by means of a suitable constant C_F, which is manually introduced into the computer for each of the two plate-holders. This is one of the remarkable advantages of the Analytical Plotter, which requires only ordinary screws, that is screws free from periodic and accidental errors, but whose lengths need not be the same.

As these corrections were applied only in an approximate way, it will be noted later that some differences of length still remain: however, they are of no importance relative to the study on the instrumental stability.

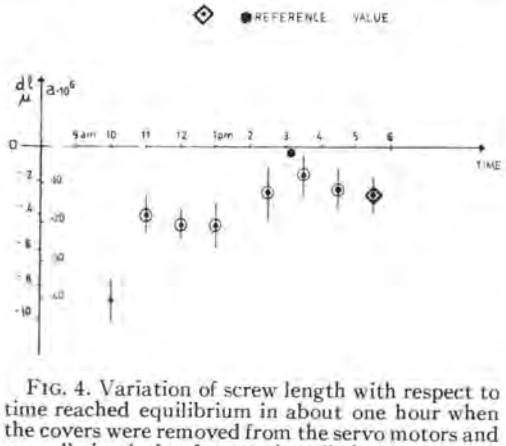

FIG. 4. Variation of screw length with respect to time reached equilibrium in about one hour when the covers were removed from the servo motors and a small circulating fan was installed.

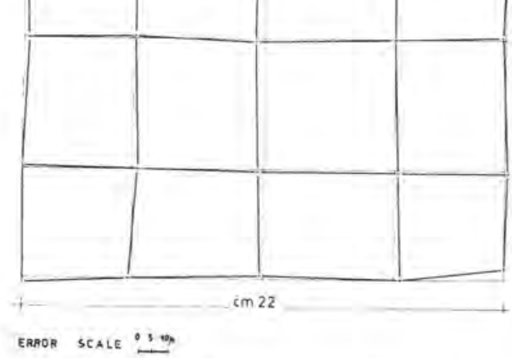

FIG. 5a. Residual discrepancies at 25 points on a grid for Plate Holder No. 1.

The tests were conducted in the following manner. On a grid were selected 25 points located as shown in Figure 5. Their coordinates were measured on both carriages several times, over a total period of almost one month. Twice at an interval of 20 days the grids were observed in four positions, rotated 90° one in respect of the other. These four positions were called *A*, *B*, *C*, *D*. All the other measurements, on the contrary, were observed only in the position *A*.

The possible methods for studying the instrumental stability are essentially two: (1) a more detailed and analytic one, consisting of considering the variations of all the observed coordinates (or of some of them) and in deducing their systematic and accidental parts; and (2), a more general and synthetic method, consisting of computing some constant elements for each test, such as the scale error in the *x*- and *y*-directions, a lack of orthogonality (which is the algebraic sum of a part due to the instrument and of another part due to the grid), and the mean square error of the measurement of a coordinate. This last one includes the pointing error of the grid etching, the local error of the measuring screws, and the position error of the grid points. This mean error is significant relative to instrumental stability only as it is constant in time or not.

The second procedure was preferred because it was less detailed than the first, it was quick to accomplish, and, above all, it allowed an immediate and significant comparison of four fundamental parameters. For the computations, use was made of Professor Hallert's* formulas which fit this purpose very well.

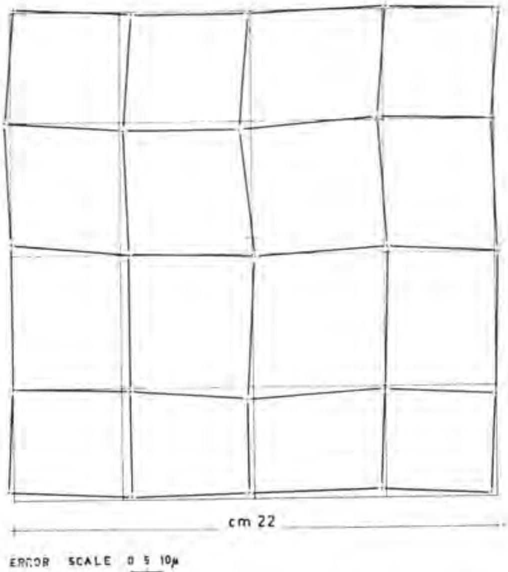

FIG. 5c. Additional computer corrections for scale errors through the use of C_F and b_z constants for Plate Holder No. 2 produced negligible overall improvement as compared to Figure 5b.

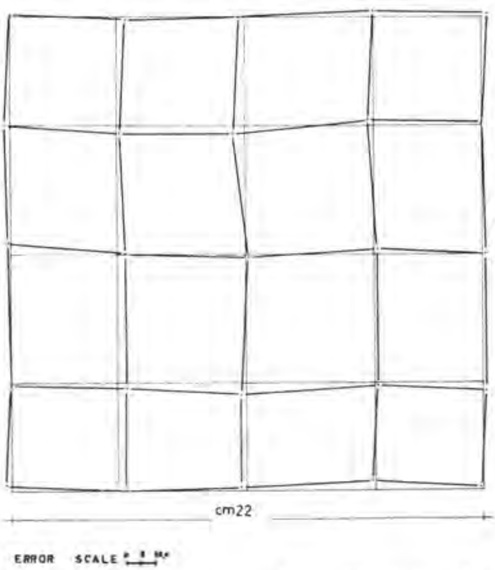

FIG. 5b. Residual discrepancies on the same grid as in Figure 5a used in Plate Holder No. 2.

The formulas that were applied are summarized later. If dx and dy are the errors of the measured coordinates of a point, dx_o and dy_o the instrumental coordinates of the grid center, dm_x and dm_y the scale errors in the two directions (algebraic sums of the screws and grid errors), α the angle between the *x*-axis of the grid and the *x*-axis of the instrument, β the algebraic sum of the errors β_s and β_g, β_s being the lack of orthogonality of the instrumental axes and β_g the lack of orthogonality of the grid, for each point one obtains the following error equations:

$$dx_o - xdm_x - (\alpha + \beta)y - dx = vx$$
$$dy_o - ydm_y + \alpha y - dy = vy$$

where α and β are small enough so that their squares may be disregarded. By writing the set of normal equations for the 25 observed points, values were obtained for 6 unknowns; among these, dx_o, dy_o and α have no interest in our case, because the grid was removed from the carriage after each measurement. The

* Prof. Bertil P. Hallert, Test Measurements in Comparators and Tolerances for Such Instruments, PHOTOGRAMMETRIC ENGINEERING, v. 29, n. 2, p. 301, March 1963.

TABLE 1. DATA OUTPUT FOR PLATE HOLDER NO. 1

Date 1966	Position	*X-screw* $dm_x \cdot 10^6$		dl_x (μ)	*Y-screw* $dm_y \cdot 10^6$		dl_y (μ)	β 10^{-6} (*radian*)		S_0 (μ)	
14-1	A	+31	+31.	+ 6.8	+33	+33.	+ 7.3	+11	+11.	±1.2	±1.2
19-1	A	+26 +27	+26.$_5$	+ 5.8	+54 +54	+54.	+11.9	+14 +13	+13.$_5$	1.7 1.5	1.6
20-1	A	+32 +34	+33.	+ 7.3	+48 +48	+48.	+10.6	+ 6 +13	+ 9.$_5$	1.8 1.9	1.8$_5$
3-2	A	+51 +43	+47.	+10.3	+58 +49	+53.$_5$	+11.8	+20 +24	+22.	1.9 1.8	1.8$_5$
7-2	A	+60 +59	+59.$_5$	+13.1	+82 +61	+71.$_5$	+15.7	+ 9 +11	+10.	1.6 1.9	1.7$_5$
9-2	A	+46 +43	+44.$_5$	+ 9.8	+55 +57	+56.	+12.3	+19 +18	+18.$_5$	1.8 1.9	1.8$_5$
11-2	A	+34 +30	+34.*	+ 7.5*	+53 +47	+50.*	+11.0*	+20 +20	+20.	1.6 1.6	1.6
11-2	A	+ 9 +18	+13.$_5$*	+ 3.0*	+24 +23	+23.$_5$*	+ 5.2*	+14 +24	+19.	1.9 1.5	1.7
28-1	B	+14 + 9	+11.$_5$	+ 2.5	+55 +30	+42.$_5$	+ 9.3	−22 −32	−27.	2.2 1.8	2.0
11-2	B	+ 7 + 9	+ 8.*	+ 1.8*	+46 +41	+44.$_5$*	+ 9.8*	−32 −29	−30.$_5$	1.9 1.9	1.9
20-1	C	+50 +47	+48.5	+10.7	+69 +62	+44.$_5$	+ 9.8	+ 1 + 6	+ 3.$_5$	2.4 2.1	2.2$_5$
11-2	C	+24 +20	+22.*	+ 4.8*	+48 +41	+44.$_5$*	+ 9.8*	+ 9 +16	+12.$_5$	2.1 1.8	1.9$_5$
28-1	D	+36 +35	+35.$_5$	+ 7.8	+41 +46	+43.$_5$	+ 9.6	−10 −15	−12.$_5$	1.6 1.7	1.6$_5$
11-2	D	+23 +18	+20.$_5$*	+ 4.5*	+43 +30	+36.$_5$*	+ 8.0*	−14 −15	−14.$_5$	2.3 2.3	2.3

* See explanation in text.

mean square error of unit weight of the measurement of a coordinate, is

$$s_o = (\Sigma v^2/44)^{1/2}$$

as $50-6=44$ is the number of degrees of freedom of the set of equations. This value includes, however, all the previously mentioned errors.

If the grid is observed in the four positions it is possible to separate the scale and orthogonality errors of the grid from those of the instrument. The first are of no interest for us; the second are given by the simple formulas:

$$\beta_s = -\tfrac{1}{4}(\beta_A + \beta_B + \beta_C + \beta_D)$$
$$\beta_o = -\tfrac{1}{4}(\beta_A - \beta_B + \beta_C - \beta_D)$$

where indices A, B, C, D indicate the four positions of the grid. The comparison between the values of β_s at the beginning and at the end of the experiments indicates how the orthogonality of the x and y instrumental axes behave with time.

The computations were performed on the IBM 7040 computer of the Politecnico of Milan. The results are shown in Tables 1 and 2, for the first and the second plate holders,

TABLE 2. DATA OUTPUT FOR PLATE HOLDER NO. 2

Date 1966	Position	*X-screw* $dm_x \cdot 10^6$		dl_x (μ)	*Y-screw* $dm_y \cdot 10^6$		dl_y (μ)	β 10^{-6} (*radian*)		S_0 (μ)	
14-1	A	−11	−11.	−2.5	+49	+49.	+10.8	−11.	−11.	±2.3	±2.3
21-1	A	− 4 − 5	$-4._5$	−1.0	+40 +38	+39.	+ 8.6	−10 − 9	$-9._5$	2.1 2.0	2.1_5
25-1	A	+ 1 − 3	− 1.	−0.2	+51 +41	+46.	+10.1	− 9 −14	$-11._5$	2.0 1.7	1.8_5
3-2	A	− 5 − 3	− 4.	−0.9	+42 +40	+41.	+ 9.0	− 9 − 9	− 9.	2.1 1.7	1.9
7-2	A	0 − 6	− 3.	−0.7	+39 +35	+37.	+ 8.1	−11 −10	$-10._5$	1.9 1.6	1.7_5
9-2	A	+11 + 6	$+8._5$	+1.9	+39 +25	+32.	+ 7.0	− 7 −13	−10.	2.3 2.3	2.3
11-2	A	− 3 − 5	− 4.*	−0.9*	+18 +15	$+16._5$*	+ 3.6*	−13 −10	$-11._5$	2.9 2.0	2.4_5
12-2	A	+ 5 +10	$+7._5$*	+1.6*	+39 +34	$+36._5$*	+8.0*	−22 −21	$-21._5$	2.0 2.0	2.0
28-1	B	+17 +15	+16.	+3.5	+37 +38	$+37._5$	+ 8.2	+22 +22	+22.	2.3 2.2	2.2_5
11-2	B	+ 7 − 1	+ 4.*	+0.9*	+13 +15	+14.*	+ 3.1*	+26 +18	+22.	2.8 2.4	2.6
21-1	C	− 7 − 5	− 6.	−1.3	+37 +31	+34.	+ 7.5	−14 − 7	$-10._5$	2.2 2.1	2.1_5
11-2	C	− 6 −14	−10.*	−2.2*	+18 +19	$+18._5$*	+ 4.1*	− 8 − 9	$-8._5$	2.6 2.3	2.4_5
28-1	D	+ 4 + 6	+ 5.	+1.1	+37 +37	+37.	+ 8.1	+18 +21	$+19._5$	2.2 1.9	2.0_5
12-2	D	+ 7 + 8	$+7._5$*	+1.6*	+47 +41	+44.*	+ 9.7*	+15 +13	+14.	2.2 2.5	2.3_5

* See explanation in text.

respectively. In the tables are included the dates of the experiments, the values of $-dm_x$ and $-dm_y$ in units of 10^{-6} and, next to them, the corresponding differences *dl* of length between the grid and the screws expressed in μ. Although the scale errors are indicated both for the forward and the back measurements, the variations *dl* were computed only with reference to the means of the two. Column 4 contains the values of $-\beta$ in units of 10^{-6} radians and the last column contains the mean square error of unit weight expressed in μ. The mean square errors of dm_x and dm_y amount, on the average, to $\pm 5.5 \cdot 10^{-6}$, and the mean square error of β to $\pm 7.9 \cdot 10^{-6}$ radians. The signs of the dm_x, dm_y and β values were changed because they were computed as corrections instead of errors.

One can see that, as the *C*-position is rotated 180° with respect of the *A*-position, their $-dm_x$ and dm_y values can be compared, whereas the β values cannot. Furthermore, the dm_x and dm_y values computed in the *B* and *D* positions should be equal but of oppo-

TABLE 3.—LACK OF ORTHOGONALITY, β, IN RADIANS

	$\beta_s \cdot 10^{-6}$ *Instrument Axes*	$\beta_g \cdot 10^{-6}$ *Grid*
Carriage 1, 1st Test	+6	−14
2nd Test	+3	−19
Carriage 2, 1st Test	+5	−16
2nd Test	+3	−15

site sign from the A and C values due to the grid scale errors.

In order to evaluate the stability of the x- and y-scales, the same method already described in Paragraph 2 was applied, that is, a test of significance on the difference of two means. The procedure is correct as each dm can be considered as a mean value computed with 44 degrees of freedom. The arithmetic means of dm_x and dm_y (for Carriages 1 and 2) were computed taking into account both positions A and C for the same reason. The values followed by an asterick (*) were disregarded.†

Then, for each value in columns 4 (x-screw) and 7 (y-screw), both for carriages 1 and 2, the following T-value was computed:

$$T = [dm_i - M(dm)]/[m_{dm(i)}{}^2 + m_{M(dm)}{}^2]^{1/2}$$

Because this T-value would have required an exceedingly large number of degrees of freedom, the comparison was made with the equivalent normal distribution. The value corresponding to a critical region of 5 per cent is therefore ± 1.96.

In Tables 1 and 2 the underlined dm values give rise to T-values larger than ± 1.96 and which, therefore, cannot be considered accidental.

One can see that Carriage 2 is perfectly stable, the variations of dm_x and dm_y from their mean being all accidental.

On the contrary, Carriage 1 shows some variations in the scale errors which cannot be considered accidental. However, the largest variations from $M(dm_x)$ and $M(dm_y)$, (both of which occurred on February 7th) amount to $18 \cdot 10^{-6}$ and $20 \cdot 10^{-6}$, respectively, that is, about 4 μ on the whole plate length.

The value of β is practically constant for both carriages as the variations between the largest and the smallest values are contained within twice the mean error.

† The measurements made on January 11 and 12 are not all reliable from the view point of dm_x and dm_y, because the air conditioner was out of order.

It can be concluded that the instrument exhibited a good stability with respect to time in spite of the fact that the instrument was not cleaned during the entire period of three months in which the experiments were performed.

The mean square error of unit weight varies between 1.5 and 2.4 μ for Carriage 1 and between 1.2 and 2.4 μ for Carriage 2; it is nearly always smaller than 2 μ. Considering that its value is the sum of three errors, this result appears to be very satisfactory; but we shall return to this point later.

If β_s and β_g are computed with the previously given formulas for the two carriages and for the initial and final tests, the values shown in Table 3 were obtained. One can see that the variations of β_s are very small and broadly within the mean error. The lacks of orthogonality between the instrumental axes are insignificant, being 0.8 and 0.6 seconds for the two carriages respectively—on the total length of the plate they cause errors smaller than 1 μ.

4 FROM THE MEASUREMENTS made to study the instrumental stability, one can also deduce some very important elements for the evaluation of the instrumental accuracy.

First of all, it was possible to compute the mean pointing error of the grid etching; for this purpose were used the residuals of the measurements made during the thermal tests with respect of the adjusted straight lines. In order to remove that part of the discrepancy at each point due to the local errors of the screws, the actual values of the discrepancies were not taken into account but instead their differences with respect to their mean. These means were computed for each of the 13 points and for all the tests of a series. The computation, repeated for ten series of tests, gave a mean square error of pointing amounting to 0.84 μ. This value reliably represents the operative conditions of the instrument, because the least count of the instrument was 2μ.

As already has been pointed out, in order to determine the mean error of the measurement of point coordinates, the grid that was used for all these tests was carefully calibrated. Therefore, it was possible to determine the differences between the measured and the true coordinates for each of the 25 points and for each carriage. With the usual procedure, the scale errors in the x- and y-directions were obtained and their effects were removed through computation. The results are shown in Figures 5a and 5b.

Another test was made on Carriage 2 in order to eliminate the scale errors by means of suitable values of the C_F and b_z constants. The discrepancies in the 25 points are still influenced by the lack of orthogonality β_s of the instrument (which was shown to be very small) and by the residual scale errors whose values were

$$dm_x = +4.10^{-6}, \qquad dm_y = +5.10^{-6}$$
$$(dz_l = 0.9\ \mu, \qquad dz_y = 1.1\ \mu).$$

It is therefore concluded that the computing devices provided by the AP/C for removing eventual scale errors serve their purpose perfectly. The results of this test can be seen in Figure 5c and, compared with those of the previous Figure 5b, the differences between the two grids are evidently negligible.

Finally, the mean error of a coordinate was $\pm 1.6\ \mu$ for Carriage 1, and $\pm 2.0\ \mu$ for Carriage 2.

ERRATA

Please note the following corrections to The Services and Equipment Guide which appeared in the January issue of this JOURNAL.

In the several (10, it seems) entries of Bendix Research Laboratories on pages 18 to 24 it should have been added that not only is Bendix a Sustaining Member of the American Society of Photogrammetry, but also they carried a two-page advertisement on pages 14 and 15.

Paul Rosenberg Associates should have been included in the Services section, pages 26ff, under the headings, "Consultants, Data Processing," "Consultants, Engineering," "Consultants, Photogrammetry," "Research & Development," as well as the "Alphabetical List of Companies." This is a firm of Consulting Physicists, established in 1945, located at 330 Fifth Avenue, New York, N. Y. 10803, Telephone (914) 738-2266.