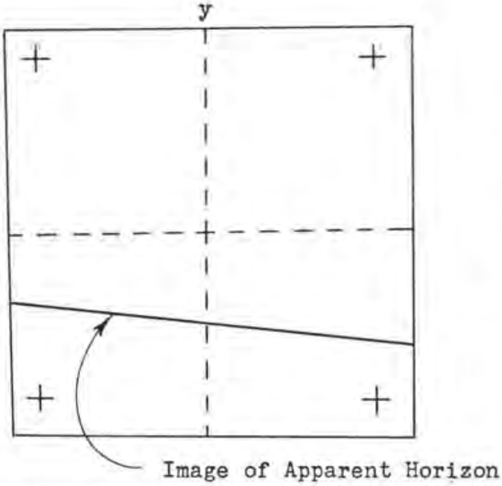

FIG. 1. The pitch and roll of the mapping camera are ultimately determined from measurements on the film of the horizon camera of the distances between the image of the apparent horizon and the fiducial axes.

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Horizon Camera Orientation

Matrix multiplication facilitates transformation of angles where the horizon and mapping cameras are separated.

(*Abstract on page 404*)

OBTAINING THE ANGLES of inclination from horizon-camera film records has been adequately covered in the literature (see References & Bibliography). All of the listed methods consider the usual case for the horizon camera in which the optical axis for each of two horizon cameras are directly aligned with the coordinate axes of the mapping camera. With this situation, the numerical values of pitch and roll may each be obtained directly from the respective horizon-camera records. Granted that this is the usual case, occasions do exist wherein the horizon camera is not so ideally located. Helava (1957) briefly touches on this possibility, but does not carry the analysis any further. The purpose of this paper is to present the mathematical technique for accommodating a precalibrated relationship between the optical axis of the horizon camera and the coordinate axes of the mapping camera.

The horizon camera must be oriented at a known azimuth, elevation, and roll with respect to the coordinate system of the mapping camera; the apparent horizon must appear in the field of view of the horizon camera. Data from the horizon camera is used to determine the orientation of the mapping camera; however only pitch and roll can be determined by the horizon camera method. Heading must be determined by some external means.

THE HORIZON-FILM coordinate system is defined as a right-handed, rectangular, Cartesian coordinate system with the xy-plane taken as the emulsion surface of the negative.

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The origin of the film-coordinate system is at the intersection of the fiducial axes and is assumed to coincide with the principal point. The y-axis is taken as the vertical line through the principal point, with the positive direction toward the top of the *negative*. The x-axis is taken perpendicular to the y-axis at the principal point, positive direction to the right when looking at the emulsion side of the negative.

The tangent-plane coordinate system is defined as a right-handed, rectangular, Cartesian coordinate system with the XY-plane passing through the origin parallel to the plane that is tangent to the earth's surface. The Y-axis is oriented at some given azimuth. The X-axis is oriented perpendicular to the Y-axis. The Z-axis is oriented perpendicular to the tangent plane at the origin with the positive direction upward. The origin may be considered as the nadir point of the mapping photograph. The local tangent plane is considered as the plane tangent to the surface of the earth at the time of occurence of the particular horizon photograph. The tangent plane is, therefore, determined by orientation to the local horizon.

ABSTRACT: *In the usual application of the horizon camera, the angles of inclination of the mapping camera can be determined directly because the optical axes of each of two horizon cameras are aligned directly with the coordinate axes of the mapping camera. However, in certain instances the horizon camera may be physically removed from the mapping camera. The relative orientation of the horizon camera with respect to the mapping camera must in this case be precalibrated. A matrix multiplication is then necessary to transform the angles of inclination from the horizon-camera coordinate system to that of the mapping camera. A bibliography lists techniques for determining the angles of inclination of the horizon camera.*

IT IS THE IMAGE of the *apparent* horizon, not that of the *true* horizon, which appears on the horizon film. The dip of the horizon is the angular difference between the true and the apparent horizons, and may be calculated for any altitude of exposure station. Considering the effect of atmospheric refraction, the value of the angle of dip in arc seconds takes the form:

$$D'' = 58.82H^{1/2}, \qquad (1.1)$$

where H is the altitude of the camera station in feet. For purposes of this paper, the formula is considered adequate.

The pitch and roll of the mapping camera are ultimately determined using measurements on the film of the distances between the image of the apparent horizon and of the fiducial axes, as indicated in Figure 1.

Let the orientation matrix representing the direction cosines between the xyz-axes of the horizon camera negative relative to the $x^c y^c z^c$-axes of the mapping camera diapositive, respectively, have the matrix form:

$$\boldsymbol{M} = \begin{vmatrix} A & B & C \\ A' & B' & C' \\ D & E & F \end{vmatrix} \qquad (1.2)$$

in which

$$\boldsymbol{M} = f(\alpha, \omega, \kappa) \qquad (1.3)$$

represents a function of the angles azimuth, elevation, and roll of the optical axis of the horizon camera with respect to the coordinate system of the mapping-camera diapositive.

Because the coordinate axes of the aerial diapositive essentially correspond to those of the tangent plane coordinate system, the orientation matrix takes the usual form for that of the negative in terrestrial photogrammetry (A.S.P., 1966):

$$\begin{aligned} A &= -\cos\alpha\cos\kappa - \sin\alpha\sin\omega\sin\kappa \\ B &= \sin\alpha\cos\kappa - \cos\alpha\sin\omega\sin\kappa \\ C &= \cos\omega\sin\kappa \\ A' &= \cos\alpha\sin\kappa - \sin\alpha\sin\omega\cos\kappa \\ B' &= -\sin\alpha\sin\kappa - \cos\alpha\sin\omega\cos\kappa \qquad (1.4) \\ C' &= \cos\omega\cos\kappa \\ D &= \sin\alpha\cos\omega \\ E &= \cos\alpha\cos\omega \\ F &= \sin\omega. \end{aligned}$$

Similarly, let the orientation matrix denoting the direction cosines between the xyz-axes of the horizon film relative to the XYZ-axes of the tangent plane, respectively, have the matrix form:

$$\overset{*}{M} = \begin{vmatrix} \overset{*}{A} & \overset{*}{B} & \overset{*}{C} \\ \overset{*}{A'} & \overset{*}{B'} & \overset{*}{C'} \\ \overset{*}{D} & \overset{*}{E} & \overset{*}{F} \end{vmatrix} \qquad (1.5)$$

in which

$$\overset{*}{M} = f(\overset{*}{\alpha}, \overset{*}{\omega}, \overset{*}{\kappa}) \qquad (1.6)$$

represents a function of the angles, heading, elevation and roll of the horizon-camera optical axis with respect to the tangent-plane coordinate system.

Because the optical axis of the horizon camera is essentially directed parallel to the horizontal plane, this orientation matrix also takes the usual form for that of the negative in terrestrial photogrammetry (A.S.P., 1966):

$$\begin{aligned}
\overset{*}{A} &= -\cos\overset{*}{\alpha}\cos\overset{*}{\kappa} - \sin\overset{*}{\alpha}\sin\overset{*}{\omega}\sin\overset{*}{\kappa} \\
\overset{*}{B} &= \sin\overset{*}{\alpha}\cos\overset{*}{\kappa} - \cos\overset{*}{\alpha}\sin\overset{*}{\omega}\sin\overset{*}{\kappa} \\
\overset{*}{C} &= \cos\overset{*}{\omega}\sin\overset{*}{\kappa} \\
\overset{*}{A'} &= \cos\overset{*}{\alpha}\sin\overset{*}{\kappa} - \sin\overset{*}{\alpha}\sin\overset{*}{\omega}\cos\overset{*}{\kappa} \\
\overset{*}{B'} &= -\sin\overset{*}{\alpha}\sin\overset{*}{\kappa} - \cos\overset{*}{\alpha}\sin\overset{*}{\omega}\cos\overset{*}{\kappa} \\
\overset{*}{C'} &= \cos\overset{*}{\omega}\cos\overset{*}{\kappa} \\
\overset{*}{D} &= \sin\overset{*}{\alpha}\cos\overset{*}{\omega} \\
\overset{*}{E} &= \cos\overset{*}{\alpha}\cos\overset{*}{\omega} \\
\overset{*}{F} &= \sin\overset{*}{\omega}
\end{aligned} \qquad (1.7)$$

In Equation 1.7 the quantity $\overset{*}{\alpha}$, the heading direction of the horizon-camera optical axis with respect to the tangent-plane coordinate system, cannot be determined directly. The value of $\overset{*}{\alpha}$ does not enter into the subsequent development because the determination of heading must be made exterior to the photogrammetric reduction. Therefore, its derivation is not presented.

THE QUANTITIES $\overset{*}{\omega}$ (elevation) and $\overset{*}{\kappa}$ (roll) of the horizon-camera optical axis with respect to the local horizon may be determined in a simplified manner directly from the horizon measurements on the film by the equations:

$$\begin{aligned}
\overset{*}{\omega} &= -\left[D'' + \tan^{-1}\left\{\tfrac{1}{2}(y_1 + y_2)/f\right\}\right] \\
\overset{*}{\kappa} &= \tan^{-1}\left[(y_1 - y_2)/(x_2 - x_1)\right]
\end{aligned} \qquad (1.8)$$

in which D'' is the dip angle in arc seconds. (Several other techniques are also available for determining the angles of inclination; see the attached Bibliography.)

The matrix defining the orientation of the mapping camera with respect to the tangent-plane coordinate system can be derived from the above Equations 1.2 and 1.5 by the relationship:

$$\bar{M} = M^T \overset{*}{M}, \qquad (1.9)$$

or

$$\begin{vmatrix} \bar{A} & \bar{B} & \bar{C} \\ \bar{A'} & \bar{B'} & \bar{C'} \\ \bar{D} & \bar{E} & \bar{F} \end{vmatrix} = \begin{vmatrix} A & A' & D \\ B & B' & E \\ C & C' & F \end{vmatrix} \cdot \begin{vmatrix} \overset{*}{A} & \overset{*}{B} & \overset{*}{C} \\ \overset{*}{A'} & \overset{*}{B'} & \overset{*}{C'} \\ \overset{*}{D} & \overset{*}{E} & \overset{*}{F} \end{vmatrix} \qquad (1.10)$$

in which

$$M = f(\bar{\omega}, \bar{\phi}, \bar{\kappa}) \qquad (1.11)$$

represents a function of the usual angles roll, pitch and kappa (A.S.P., 1966) of the mapping camera optical axis with respect to the tangent plane coordinate system.

The usual orientation matrix for aerial photogrammetry (A.S.P., 1966) is used in this case. The elements of the matrix are therefore:

$$\begin{aligned}
\bar{A} &= \cos\bar{\phi}\cos\bar{\kappa} \\
\bar{B} &= \cos\bar{\omega}\sin\kappa + \sin\bar{\omega}\sin\bar{\phi}\cos\bar{\kappa} \\
\bar{C} &= \sin\bar{\omega}\sin\bar{\kappa} - \cos\bar{\omega}\sin\bar{\phi}\cos\bar{\kappa} \\
\bar{A'} &= -\cos\bar{\phi}\sin\bar{\kappa} \\
\bar{B'} &= \cos\bar{\omega}\cos\bar{\kappa} - \sin\bar{\omega}\sin\bar{\phi}\sin\bar{\kappa} \\
\bar{C'} &= \sin\bar{\omega}\cos\bar{\kappa} + \cos\bar{\omega}\sin\phi\sin\bar{\kappa} \\
\bar{D} &= \sin\bar{\phi} \\
\bar{E} &= -\sin\bar{\omega}\cos\bar{\phi} \\
\bar{F} &= \cos\bar{\omega}\cos\bar{\phi}
\end{aligned} \qquad (1.12)$$

This orientation matrix denotes the direction cosines between the $x^c y^c z^c$-axes of the mapping camera relative to the XYZ-axes of the tangent plane, respectively. Based on this particular form of the angle definition and sequence to form the orientation matrix, the three rotational elements of orientation may be determined from the elements of the matrix in the manner:

$$\begin{aligned}
\sin\bar{\phi} &= \bar{D} & \cos\bar{\phi} &= [1 - \sin^2\bar{\phi}]^{1/2} \\
\sin\bar{\omega} &= -\bar{E}/\cos\bar{\phi}, & \cos\bar{\omega} &= \bar{F}/\cos\bar{\phi} \\
\sin\bar{\kappa} &= -\bar{A'}/\cos\bar{\phi}, & \cos\kappa &= \bar{A}/\cos\bar{\phi}
\end{aligned}$$

As indicated earlier in this article, for those cases where the optical axis of the horizon camera is aligned directly with the coordinate axes of the mapping-camera diapositive, the numerical values of pitch and roll may be obtained directly from the horizon-camera records.

If film records from several horizon cameras are available, the final values of roll and pitch can be taken as the average of the individual values obtained from each horizon camera.

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