FRONTISPIECE. Large copy measured with a glass scale.

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Least Squares Camera Calibration

Precision reproduction cameras are tested in situ.

(*Abstract on page 408*)

INTRODUCTION

THE DEVELOPMENT OF automated analytical aerial triangulation techniques and plotting has created a need for precise photogrammetric and cartographic reproductions. Since these reproductions are made on large flat field lens cameras, a method for analyzing, evaluating, and calibrating a precision cartographic camera is required. This paper develops the method for the *Least-Squares Calibration of Precision Cameras.*

CALIBRATION FOR REPRODUCTION CAMERAS

When a reproduction camera lens is to be used for a given range of enlargements, it must be designed to satisfy the conditions of focus and flatness of field for that range. This is different from an aerial camera lens which is fixed at one focal length setting for the life of the camera.

For example. assume that a lens is to be used for near ultra-violet, visual, and near infra-red photography. Then three different focal lengths must be satisfied for each range of the electromagnetic spectrum within which it will operate. Once these focal lengths are determined and the camera is made serviceable, it will remain fixed for the life of the camera. Only a new and better design will replace such equipment.

With the reproduction camera lens the conditions are quite different. The conditions

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for focus must satisfy a given range of conditions around a fixed range more nearly 1:1, a ratio of 1.000, or 100 per cent original size.

Let us assume that a lens has been designed for reproduction at the visible light range of the spectrum from 0.5 to 0.7 microns wavelength. Also, assume that the reproductions to be made with this lens will be made on photographic emulsions sensitive to light within the same range. Further, we must assume a range of reproduction enlargements, say near 1:1 and from 75 per cent to 125 per cent, with a flatness of field such that any line length on the copied material will be reproduced with an accuracy of 0.001 inch and a reproducibility or precision of 0.001 inch. Then we have established the criteria for the calibration of a precision reproduction camera.

ABSTRACT: *A unique solution is presented for: (1) the effective focal length of the camera lens in the system in which the lens is installed; and (2) the effective nodal point separation which also includes the standard deviation of the scales for positioning the principal planes on the camera system considered. This least-squares method was developed so that any precision reproduction for cartographic, photogrammetric, or miniaturization purposes could be predicted. In this manner, the method can be used for determining the utility and accuracy of the camera systems in addition to its value as a production aid.*

Assuming that we have had such a lens and camera system developed and that the lens has been calibrated by the National Bureau of Standards, we then have a calibrated focal length F_c with an error of $\pm\epsilon$. Now we are prepared to make an operational test on the camera system using the material copied on that system for performing the *Least-Squares Calibration*.

EQUIPMENT

Since we are limiting ourselves here to precision cameras (reproducibility to 0.001 inch), we must have had the parallelism of the principal planes of the camera system and the perpendicularity of the lens axis established by autocollimation or an equivalent technique. Then it is only necessary to measure the large copy material with a glass scale subdivided in 0.001 inch as shown in Figure 1. If miniaturized copy is to be measured, a device such as the Mann Stellar Comparator should be used. (Frontispiece) The measurements on the copied material compared to the same measurements on the original material is the actual enlargement M. With the value M and the recorded principal plane settings at the time of exposure, a system of n condition equations can be established for n exposures throughout the range of the camera system.

DETERMINING CONJUGATE DISTANCES

The standard formulas for determining the conjugate distances I and O apply here, where $O/I=M$, the magnification:

$$O=F(1+M) \quad \text{(lens to copy board distance)}$$

$$I=F(1+1/M) \quad \text{(lens to negative plane distance)} \qquad (1)$$

where F is focal length of the camera lens.

When the conjugate distances are both measured from the film plane, the pre-determined image distance places the lens and the object distance O, and the nodal point separation d must be added to the image distance for determining the easel position D, from the film plane. Hence,

$$D = I + O + d. \qquad (2)$$

Then Equation 2 can be rearranged using Equations 1 as follows:

$$\begin{aligned} D &= (O/M) + O + d \\ &= O(1 + 1/M) + d \\ &= F(1 + M)(1 + 1/M) + d \end{aligned}$$

and finally,

$$D = F(1 + M)^2/M + d = kF + d \qquad (3)$$

where:

- k is computed from the actual magnification M measured on the reproduced copy;
- D is the distance between the negative and easel planes as indicated on the camera system scales;
- F is the effective focal length of the lens as used in the camera system for obtaining k and D, as solved for in the least squares set using Equation 3;

FIG. 1. Miniaturized copy measured with a comparator.

d is determined as the effective nodal point separation of the lens as used in the system using Equation 3 in the least-squares set. This value also contains that standard deviation from any previously established or calibrated value *plus* any slight shift in the measuring scales which give the value for D.

Rearranging Equation 3 and adding the subscript for n equations, the normal equations are as follows:

$$\left(\sum_1^n k_n^2\right) F + \left(\sum_1^n k_n\right) d = \sum_1^n (k_n D_n)$$
$$\left(\sum_1^n k_n\right) F + nd = \sum_1^n (D_n) \tag{4}$$

Simplifying the notation, the normal equations are reduced to the conventional notation,

$$a_1F + b_1d = c_1$$
$$a_2F + b_2d = c_2, \tag{5}$$

and the matrix array of the normal Equations 5 is

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \cdot \begin{bmatrix} F \\ d \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \tag{6}$$

where $a_2 = b_1$ because the matrix is symmetrical and $b_2 = n$, the number of equations used in the solution because the coefficients of d are unity in each of the condition equations.

Analysis of Data

Equation 3 is very simple and requires only the total distance between negative and easel planes D and the measured magnification for this setting M to find the effective focal length F and spacing d in a least squares reduction. Once F and d are found for any set, they could be resubstituted into each condition Equation 3 along with the magnification M for finding the least squares value D'. Then $D' - D = \Delta D$ is the amount that each value used in the test deviates from the computed least squares value. A set of ΔD values for the whole range of magnifications of any camera system would show a trend such that if any value deviates greatly, it must be investigated. Once a smooth set is obtained (the deviations are accounted for), an interpolation or polynomial approximation method can be used for subtabulation of conjugate distance tables by electronic computations.

A check on the method can be performed at any time by rearranging Equation 3 as follows:

$$(D - d)/F = (1 + M)^2/M$$
$$= (1 + 2M + M^2)/M = C'$$

then,

$$M + 2 + (1/M) = C'$$

and,

$$M + (1/M) = C' - 2 = C \tag{7}$$

from which the quadric equation

$$M^2 - CM + 1 = 0 \tag{8}$$

results, where the quadratic formula for the solution of M is as follows:

$$M = \tfrac{1}{2}[C \pm (C^2 - 4)^{1/2}]. \tag{9}$$

Note that the distance between the negative and easel planes D is the distance measured on the same scales as used to find the least squares values of d and F.

As the camera system is used in production throughout its complete range, a greater volume of data becomes available and normal Equations 5 and 6 can be used for further refinement of F and d, until the changes in their values become insignificant. It should be anticipated that normal wear will have some effect on these results as refined. However, the maintenance of any given accuracy such as 0.001 inch can be accomplished, by changing the electronic computer printout of the conjugate distance values. It is simply necessary to record each setting and its resultant magnification for use as input data in this analysis.

Conclusion

This least squares analysis can be used to calibrate precision reproduction cameras using the copy that the camera system produces. The ability to predict the magnification of a given camera system with a specified degree of precision is an index of the reliability and reproducing capability of that system. When the index changes as a result of normal wear, the index can be used to indicate the need for preventative maintenance. It can also serve as a guide to what can be expected in any newly designed system.