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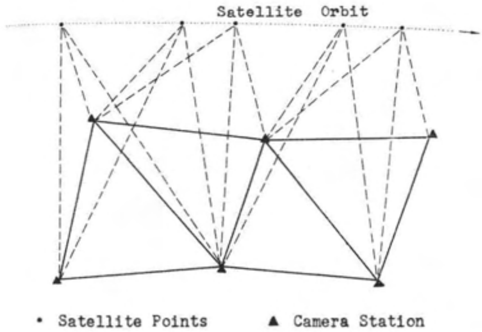

FIG. 1. A network of spatial triangles formed by a satellite and camera stations.

Satellite Triangulation Accuracy

(*Abstract on page 102*)

Error propagation will remain a major problem in an extensive network.

INTRODUCTION

SATELLITE TRIANGULATION extends geodetic controls by means of a three-dimensional network of triangles in space (Figure 1). Since August 1963, the United States Coast and Geodetic Survey (U.S.C. & G.S.) has engaged in the establishment of a continental network of geodetic reference points by the method of satellite triangulation. The fundamental principles underlying this new method of triangulation have been discussed by Dr. H. Schmid of the U.S.C. & G.S. in several recent papers.[3,4] Basically, a satellite is photographed against a star background simultaneously from two or more earth based cameras. Using the known directions to the stars, the orientation of the camera axis during the moment of exposure can be accurately determined. The direction to any recorded satellite position can then be deduced from the position of its image on the photographic plate. Once sufficient satellite positions have been observed, a three-dimensional triangulation adjustment may be performed to determine the geodetic positions of the unknown camera stations.

The ultimate accuracy of a satellite triangulation net depends on the accuracy with which the directions from the camera stations to the satellite can be determined. Two major sources of errors to these directions are: (1) the orientations of the camera axis; and (2) the plate coordinates of the satellite images. This study is concerned with the propagation of the errors from these sources in a three-dimensional triangulation adjustment.

A hypothetical triangulation net, consisting of five camera stations which formed three triangles, was used in this study. The true locations of the camera stations and those of the satellite positions to be observed were first computed. The true plate co-ordinates of the recorded satellite images and the true orientations of the cameras were also generated. Errors of known magnitude were subsequently introduced into the plate co-ordinates and the orientation elements. These fictitiously erroneous data were then used to determine the

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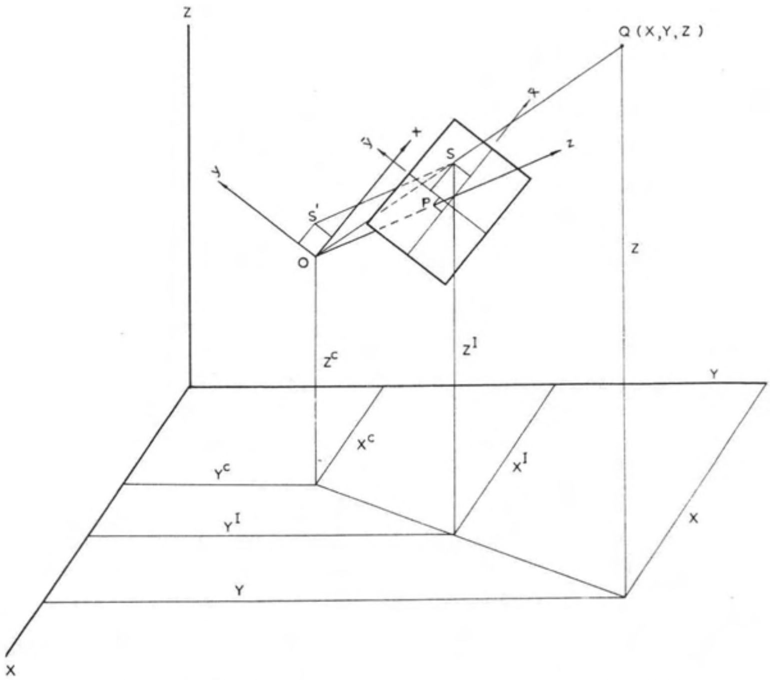

FIG. 2. Master, local and plate coordinate systems. Legend: X, Y, Z, master coordinates system (geocentric); x, y, z, local coordinates system with origin at the center of projection; $\bar{x}$, $\bar{y}$ image plate coordinates; Q a point in space; p principal point; c focal length.

locations of the camera stations and the satellite positions. These computed locations were finally compared with their corresponding true locations to give an indication of the accuracy of such a survey.

In the following paragraphs, the data reduction procedure employed in this study is first briefly described. This is followed by a description of the hypothetical triangulation net. Finally, the results from several independent adjustments using different control parameters are presented.

DATA REDUCTION PROCEDURE

The triangulation adjustment procedure used in this study was adopted from D. C. Brown's solution to the general problem of multiple station analytical stereotriangulation.[1,2] Since a satellite triangulation net can be scaled by using one or more measured base lines, a system of length condition equations is added to Brown's solution.

CO-LINEARITY EQUATION

The direction of each ray from a camera station to a satellite position can be described by two colinearity equations. These equations express the condition of colinearity among the satellite in space, the optical center of the camera and the image of the satellite on the photographic record. In Figure 2, let i represent the i-th photograph, and j denotes the j-th satellite point. The two equations generated by the j-th point being photographed in the i-th photo can be written as:

$$\frac{\bar{x}_{ij}-\bar{x}_{pi}}{c_i} = \frac{A_i(X_j-X_i^c)+B_i(Y_j-Y_i^c)+C_i(Z_j-Z_i^c)}{D_i(X_j-X_i^c)+E_i(Y_j-Y_i^c)+F_i(Z_j-Z_i^c)} \quad (1)$$

$$\frac{\bar{y}_{ij}-\bar{y}_{pi}}{c_i} = \frac{A_i^1(X_j-X_i^c)+B_i^1(Y_j-Y_i^c)+C_i^1(Z_i-Z_i^c)}{D_i(X_j-X_i^c)+E_i(Y_j-Y_i^c)+F_i(Z_j-Z_i^c)} \quad (2)$$

where $\bar{x}_{ij}$ and $\bar{y}_{ij}$ are the plate coordinates of the image of the j-th satellite point on the i-th photo; $\bar{x}_{pi}$ and $\bar{y}_{pi}$ are the plate coordinates of the principal point of the i-th photo; c_i is the focal length; X_j, Y_j and Z_j are the geocentric coordinates of the j-th satellite point; X_i^c, Y_i^c and Z_i^c are the geocentric coordinates of the camera station at which the i-th photo is taken; and A_i, B_i, C_i, D_i, E_i, F_i, A_i^1, B_i^1 and C_i^1 are the elements of the rotational matrix of the i-th photo.

In Equations 1 and 2, the rotational elements α_i, ω_i and κ_i of the principal axis of the i-th photo are implicit in the elements of the rotational matrix. Since α_i, ω_i and κ_i can be determined by a least square adjustment using an abundant amount of excess direction control points (stars), they are carried into the triangulation adjustment as completely known quantities.

By means of Taylor's Series expansion and by neglecting all second and higher order terms, Equations 1 and 2 can be linearized to the form

$$V_{ij} + b_{ij}^1\Delta X_i^c + b_{ij}^2\Delta Y_i^c + b_{ij}^3\Delta Z_i^c + b_{ij}^4\Delta X_j + b_{ij}^5\Delta Y_j + b_{ij}^6\Delta Z_j + \epsilon_{ij} = 0 \quad (3)$$

$$\tilde{V}_{ij} + \tilde{b}_{ij}^1\Delta X_i^c + \tilde{b}_{ij}^2\Delta Y_i^c + \tilde{b}_{ij}^3\Delta Z_i^c + \tilde{b}_{ij}^4\Delta X_j + \tilde{b}_{ij}^5\Delta Y_j + \tilde{b}_{ij}^6\Delta Z_j + \tilde{\epsilon}_{ij} = 0 \quad (4)$$

where V_{ij} and $\tilde{V}_{ij}$ are errors in the x and y plate coordinates of the satellite image; ΔX_i^c, ΔY_i^c, and ΔZ_i^c are the corrections to the approximate coordinates of the camera station at which the i-th photo is taken; and ΔX_j, ΔY_j and ΔZ_j are the corrections to the approximate geocentric coordinates of the j-th satellite point. Expressions for the coefficients b_{ij}'s can be easily obtained from Equations 1 and 2 by partial differentiation. If approximate values are assigned to X_i^c, Y_i^c, Z_i^c, X_j, Y_j and Z_j, the only unknown elements in Equations 3 and 4 are the corrections V_{ij}, $\tilde{V}_{ij}$, ΔX_i^c, ΔY_i^c, ΔZ_i^c, ΔX_j, ΔY_j and ΔZ_j.

LENGTH CONDITION EQUATION

The straight line distance L_{gh} between station g and h can be expressed as a function of the X^c, Y^c, Z^c coordinates of those two stations; i.e.

$$L_{gh}^2 = (X_g^c - X_h^c)^2 + (Y_g^c - Y_h^c)^2 + (Z_g^c - Z_h^c)^2 \quad (5)$$

The length condition can then be expressed as

$$L_{gh}^2 - (X_g^c - X_h^c)^2 - (Y_g^c - Y_h^c)^2 - (Z_g^c - Z_h^c)^2 = 0 \quad (6)$$

Expanding Equation 6 by Taylor's Series and neglecting all second and higher order terms yields the expression

$$dL_{gh} - \frac{(X_g^o - X_h^o)}{L_{gh}} dX_g^c - \frac{(Y_g^o - Y_h^o)}{L_{gh}} dY_g^c - \frac{(Z_g^o - Z_h^o)}{L_{gh}} dZ_g^c + \frac{(X_g^o - X_h^o)}{L_{gh}} dX_h^c + \frac{(Y_g^o - Y_h^o)}{L_{gh}} dY_h^c + \frac{(Z_g^o - Z_h^o)}{L_{gh}} dZ_h^c + \frac{\epsilon^o}{2L_{gh}} = 0 \quad (7)$$

where dL_{gh} is the error in the measured length; X_g^o, Y_g^o, Z_g^o, X_h^o, Y_h^o, and Z_h^o are the approximate coordinates for stations g and h; and dX_g^c, dY_g^c, dZ_g^c, dX_h^c, dY_h^c and dZ_h^c are their corresponding corrections.

Equations 3, 4 and 7 constitute all the condition equations needed in a three dimensional triangulation adjustment. Two colinearity equations of the form of Equations 3 and 4 are written for each pair of satellite plate coordinates, and one length equation of the form of Equation 7 is written for each measured base line. All of these condition equations are then combined in a simultaneous least square adjustment. It is essentially an iterative solution. The principal end products of the adjustments are the geocentric coordinates of all the unknown camera stations as well as those of the satellite points. It is beyond the scope of this paper to discuss the least square adjustment procedure. The in-

ABSTRACT: *Satellite triangulation extends geodetic control by means of a three-dimensional network of triangles in space. The final accuracy of the computed geodetic positions depends on the accuracy of the directions from the camera stations to the satellite. Two major sources of error affect these directions: the orientation of the camera axes, and the measured plate coordinates of the satellite images. The propagation of the errors from these sources in a three-dimensional triangulation adjustment was studied by means of a numerical model using fictitious data. The fictitious triangulations net consisted of five camera stations forming three triangles. The average length of the sides was about 1,500 kilometers. An error of ±0.2 second was introduced into the orientation elements of the camera axes, and an error of ±0.2 micron was introduced into the plate coordinates. The results from several adjustments showed that the absolute positions of the unknown stations could be determined to better than 1/200,000.*

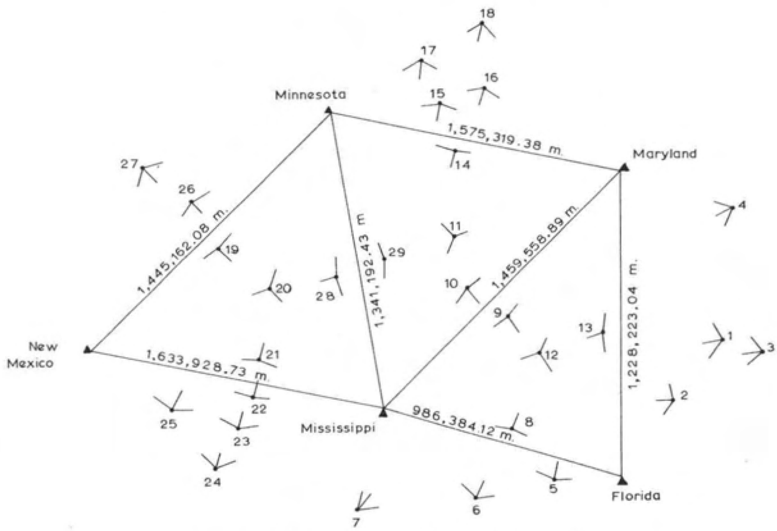

FIG. 3. A hypothetical triangulation net.

terested readers are referred to References 1, 2 and 5.

Three computer programs were prepared in connection with this investigation. Two of these, CØ-ØRD and EKAPPA, were used to generate fictitious data. The remaining one, SPATRI, was used to perform spatial triangulation. They were all written in FØRTRAN 63 computer language. Computations were performed on the Control Data 1604/1604A Computer at the Cornell Computing Center. Detailed descriptions of the functions and performances of these programs as well as their listings can be found in Reference 5.

A Hypothetical Triangulation Net

A hypothetical triangulation net consisting of five camera stations was constructed. The network closely resembles the eastern portion of the North American continental triangulation net being surveyed by the U.S.C. & G.S. The lengths of the sides of the triangle vary between 986 kilometers and 1,633 kilometers with an average length of about 1,500 kilometers. There were 29 satellite positions recorded. All but one of the 29 positions were observed simultaneously from three stations. The altitude range of the satellite was between 1,300 and 1,800 kilometers. The configuration is illustrated in Figure 3.

The following fictitious data were generated:

1. The true X, Y, Z geocentric coordinates of all the camera stations;
2. The true X, Y, Z geocentric coordinates of all the recorded satellite positions; and
3. The necessary data for each photographic record; that is,
   a. The three rotational elements (ω, α, and κ) of the camera axis, accurate to ± 0.00005 second of arc; and
   b. The plate coordinates of the satellite images, accurate to $\pm 0.00005\mu$. The principal point of all the photographs was assigned the coordinates $\bar{x}_p = 0$ and $\bar{y}_p = 0$, and the focal length of the camera was 305 millimeters.

An error of either $+$ or -0.2 second was introduced into all the rotational elements

TABLE 1. Fictitiously Correct Geocentric Coordinates of the Camera Stations

Station	*Fictitiously Correct Coordinates in Meters*		
	X	*Y*	*Z*
Florida	879,571.661	−5,508,534.488	3,082,095.112
Maryland	1,163,259.552	−4,788,556.895	4,035,869.337
Mississippi	−32,078.930	−5,368,717.225	3,431,806.374
New Mexico	−1,561,766.114	−4,899,379.414	3,762,117.577
Minnesota	−338,302.439	−4,546,414.413	4,446,125.047

TABLE 2. Fictitiously Correct Geocentric Coordinates of the Satellite Positions

Satellite Position	*Fictitiously Correct Coordinates in Meters*		
	X	*Y*	*Z*
1	1,682,812.955	−6,224,539.305	4,127,033.560
2	1,560,651.868	−6,748,729.403	4,336,160.107
3	1,928,551.825	−6,376,497.573	4,194,063.649
4	1,999,224.033	−5,802,055.836	4,765,811.844
5	742,914.288	−6,902,529.868	3,920,624.953
6	268,132.168	−6,829,018.950	3,703,584.482
7	−303,318.862	−6,893,459.411	3,791,455.725
8	571,870.188	−6,768,503.963	4,173,406.895
9	659,109.765	−6,554,710.207	4,576,048.298
10	344,274.247	−6,584,413.454	4,825,602.483
11	306,605.199	−6,225,081.002	4,961,080.392
12	843,540.765	−6,493,425.237	4,368,619.193
13	1,215,700.112	−6,530,569.342	4,584,562.448
14	332,539.404	−6,023,879.328	5,357,747.316
15	336,527.930	−5,799,759.384	5,598,084.383
16	734,683.203	−5,598,888.341	5,761,227.125
17	345,720.664	−5,450,374.925	5,868,539.108
18	718,653.491	−5,312,560.140	5,892,628.196
19	−1,065,774.046	−6,043,448.308	5,084,482.660
20	−840,398.938	−6,300,827.235	4,971,709.769
21	−1,023,063.404	−6,563,329.142	4,758,040.930
22	−1,114,607.528	−6,759,846.145	4,454,199.115
23	−1,206,198.840	−6,838,754.748	4,308,255.508
24	−1,255,255.532	−6,937,044.311	4,035,162.548
25	−1,621,564.742	−6,494,350.940	4,329,402.068
26	−1,510,280.461	−5,841,573.691	5,357,076.562
27	−1,655,048.027	−5,518,603.629	5,358,519.046
28	−355,329.821	−6,397,274.273	4,906,797.438
29	18,021.949	−6,300,480.746	5,121,647.041

ω, α and κ; and the plate coordinates of all the satellite images were displaced by either + or -0.2μ. The combination of these errors gives a net error of about $\pm 0.5''$ in the direction of the ray joining the camera station and the satellite position in space. The magnitudes of these errors correspond to the error limits presently encountered in practice.

The fictitiously correct X, Y, Z coordinates for the camera stations are listed in Table 1; and those for the satellite positions are given in Table 2.

RESULTS FROM SEVERAL ADJUSTMENTS

ADJUSTMENT CASE A

Only the Florida-Maryland-Mississippi triangle in the hypothetical triangulation net was used in adjustment Case A. Thirteen satellite positions were recorded from the three camera stations. The configuration is shown separately in Figure 4. Four independent least square adjustments of this triangle were made by using different sets of control parameters. The results are summarized in Table 3.

In adjustment A-1, the absolute position of the triangle in space and its scale were controlled by enforcing the coordinates of stations Maryland and Mississippi. The coordinates of Mississippi were left error free, while an error of either + or −6 m. was introduced into the coordinates of Maryland. The actual errors in the computed X, Y, Z geocentric coordinates of Florida are −5.181 m., +0.659 m., and 1.856 m. respectively. The total error in the absolute position of the station amounts to ±5.496 m., which corresponds to a relative accuracy with respect to the fixed station, Mississippi, of about 1/180,000. The standard deviation of the plate measurements after adjustment is $\pm 0.348\ \mu$, which is equivalent to ±0.3 seconds of arc.

The same system of control used in adjustment A-1 was also used in adjustment A-2. However, the coordinates of Maryland were each given a weight of 10^{-5} which corresponds to an accuracy of ±6 m. This offered more freedom to the solution by allowing limited adjustment on the coordinates of Maryland. The results show very little improvement over

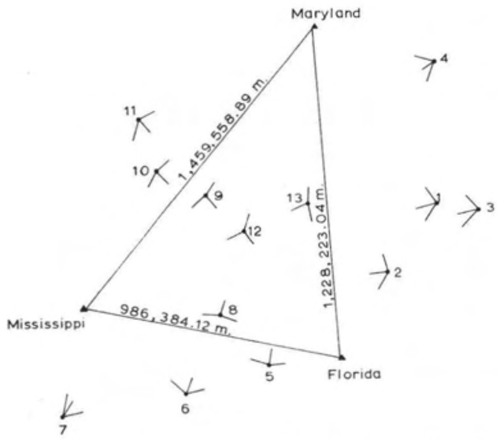

FIG. 4. Configuration used in adjustment Case A. Altitude range of satellite between 1,300 and 1,800 kilometers.

A-1. The actual errors in the computed X, Y, Z coordinates vary between ± 0.43 meter and ± 5.7 meters. The total error in the absolute position of Florida is ± 4.97 meters which gives a relative accuracy of 1/198,000. The total error in the position of Maryland is ± 9.28 meters, which corresponds to 1/157,000. The standard deviation in the plate coordinates after the adjustment is the same as in adjustment A-1.

In adjustment A-3, the spatial triangle was controlled by enforcing the coordinates of Mississippi and the length of the line from Mississippi to Maryland. The coordinates of Mississippi were again left error free; but an error was introduced into the length of the base line so that it has a relative accuracy of 1/500,000. The coordinates of Maryland and Florida were treated as completely unknown. The actual errors in the computed X, Y, Z coordinates vary between ± 0.178 meter and ± 6.72 meters. The errors in the absolute positions of Florida and Maryland are ± 2.534 meters and ± 6.78 meters respectively. This indicates a positioning accuracy better than 1/300,000. The standard deviation in the plate coordinates after the adjustment is $\pm 0.335\mu$, which is slightly better than those of the two previous adjustments.

In adjustment A-4, the triangle was again scaled by enforcing the coordinates of Mississippi and the length of the line from Mississippi to Maryland. A second control point was also enforced. The fictitiously correct coordinates of Maryland were each displaced by either + or -6 meters. They were subsequently each assigned a weight of 10^{-5} in the adjustment. The results show a significant increase in accuracy for the location of Florida. The actual error in the position of Florida is ± 1.128 meters, which indicates a relative accuracy of 1/750,000. However, the accuracy obtained for the location of Maryland remains about 1/200,000.

In addition to the coordinates of the unknown camera stations, those of the thirteen satellite points were also computed after each of the four adjustments. Only those computed from adjustment A-3 are listed in Tables 4a and 4b. The actual errors in the X, Y, Z coordinates of these thirteen satellite points vary between ± 0.07 meter and ± 6.1 meters. The actual errors in their absolute positions vary between ± 0.8 meter and ± 6.3 meters. The mean positioning accuracy is computed to be 3.6 meters with a standard deviation of ± 1.7 meters. This indicates a relative accuracy of about 1/400,000.

TABLE 3. Summary of Test Results from Case A

Adjustment	*Stn.*	*Weight* X	*Weight* Y	*Weight* Z	*Actual Errors* ΔX	*Actual Errors* ΔY	*Actual Errors* ΔZ	*Errors*[1] *in Absolute Position (m.)*	*Relative*[2] *Accuracy*	σ^3 (μ)	*Length Constraint*
A-1	Fla.	0	0	0	−5.131	0.659	1.856	±5.496	1/179,473	±0.348	None
	Md.	10^{20}	10^{20}	10^{20}	−6.0	−6.0	−6.0	±10.392	1/140,446		
	Miss.	10^{20}	10^{20}	10^{20}	0.0	0.0	0.0	0.0			
A-2	Fla.	0	0	0	−4.548	0.431	1.963	±4.972	1/198,388	±0.348	None
	Md.	10^{-5}	10^{-5}	10^{-5}	−5.428	−5.743	−4.874	±9.284	1/157,212		
	Miss.	10^{20}	10^{20}	10^{20}	—	—	—	—	—		
A-3	Fla.	0	0	0	0.903	−2.281	0.633	±2.534	1/389,260	±0.335	One
	Md.	0	0	0	0.178	−6.719	−0.885	±6.779	1/215,306		
	Miss.	10^{20}	10^{20}	10^{20}	—	—	—	—	—		
A-4	Fla.	0	0	0	−0.961	−0.056	0.588	±1.128	1/770,613	±0.343	One
	Md.	10^{-5}	10^{-5}	10^{-5}	−0.674	6.659	−2.444	±7.125	1/204,850		
	Miss.	10^{20}	10^{20}	10^{20}	—	—	—	—	—		

[1] Error in absolute position $=(\Delta X^2+\Delta Y^2+\Delta Z^2)^{1/2}$.
[2] Relative Accuracy = error in absolute position/straight line distance of station from the origin of the network.
σ = expected mean error of plate measurement after adjustment.

TABLE 4a. Computed Coordinates of the Satellite Positions

Satellite Point No.		*Computed Coordinates (m.)*	*Actual Error (m.)*	*Expected Mean Error ±m.*	*Total Error in Position ±m.*
1	*X* *Y* *Z*	1,682,810.829 −6,244,539.672 4,127,031.289	−2.126 +0.367 −2.271	1.759 2.036 1.510	±3.132
2	*X* *Y* *Z*	1,560,650.806 −6,748,726.187 4,336,158.259	−1.062 −3.216 −1.848	2.064 3.234 2.136	±4.040
3	*X* *Y* *Z*	1,928,548.117 −6,376,494.552 4,194,060.124	−3.708 −3.021 −3,525	2.402 2.551 1.785	±5.941
4	*X* *Y* *Z*	1,999,222.000 −5,802,056.657 4,765,809.025	−2.033 +0.821 −2.819	2.661 2.111 2.631	±3.571
5	*X* *Y* *Z*	742,915.556 −6,902,532.893 3,920,624.960	+1.268 +3.025 +0.007	1.237 3.191 1.583	±3.280
6	*X* *Y* *Z*	268,132.100 −6,829,020.633 3,703,583.783	−0.068 +1.683 −0.699	1.236 3.065 1.312	±1.824
7	*X* *Y* *Z*	−303,318.105 −6,893,459.260 3,791,455.606	−0.757 −0.151 −0.119	2.072 3.964 1.580	±0.781

TABLE 4b. Computed Coordinates of the Satellite Positions

Satellite Point No.		*Computed Coordinates (m.)*	*Actual Error (m.)*	*Expected Mean Error ±m.*	*Total Error in Position m.*
8	*X* *Y* *Z*	571,869.629 −6,768,506.400 4,173,409.229	−0.559 +2.437 +2.334	1.226 2.862 1.807	±3.420
9	*X* *Y* *Z*	659,108.592 −6,554,712.608 4,576,048.497	−1.173 +2.401 +0.199	1.258 2.514 2.210	±2.680
10	*X* *Y* *Z*	344,273.413 −6,584,411.663 4,825,596.560	−0.834 −1.791 −5.923	1.429 2.838 2.821	±6.244
11	*X* *Y* *Z*	306,606.083 −6,225,081.162 4,967,074.240	+0.884 +0.160 −6.152	1.391 2.171 2.782	±6.217
12	*X* *Y* *Z*	843,540.784 −6,493,427.890 4,368,619.286	+0.019 +2.653 +0.093	1.204 2.245 1.825	±2.655
13	*X* *Y* *Z*	1,215,697.645 −6,530,570.936 4,584,562.374	−2.467 +1.594 −0.074	1.504 2.616 2.222	±2.938

Mean positioning accuracy of a satellite point = 3.6 ± 1.7 m.
Relative positioning accuracy of a satellite point = 1/400,000.

TABLE 5. Summary of Test Results from Case B

Adjustment	Stn.	Weight			Actual Errors			Errors[1] in Absolute Position (m.)	Relative[2] Accuracy	σ^3 (μ)	Length Constraint
		X	Y	Z	ΔX	ΔY	ΔZ				
B-1	N. Mex.	0	0	0	12.653	−1.144	1.017	±12.745	1/215,056	±0.360	One
	Minn.	0	0	0	7.509	1.110	−2.973	±8.152	1/193,243		
	Miss.	0	0	0	5.542	4.497	1.598	±7.314	1/199,557		
	Fla.	0	0	0	0.345	4.510	−0.350	±4.537	1/270,713		
	Md.	10^{20}	10^{20}	10^{20}	—	—	—	—	—		
B-2	N. Mex.	10^{-5}	10^{-5}	10^{-5}	5.338	3.402	3.110	±7.053	1/388,618	±0.460	One
	Minn.	0	0	0	4.791	−1.125	1.161	±5.056	1/311,574		
	Miss.	10^{-5}	10^{-5}	10^{-5}	5.347	−1.295	−0.835	±5.565	1/262,275		
	Fla.	0	0	0	1.022	1.950	1.382	±2.599	1/472,575		
	Md.	10^{20}	10^{20}	10^{20}	—	—	—	—	—		
B-3	N. Mex.	0	0	0	11.165	0.124	−0.977	±11.209	1/244,529	±0.378	Two
	Minn.	0	0	0	8.457	1.771	−3.083	±9.174	1/171,717		
	Miss.	0	0	0	6.271	4.594	0.838	±7.819	1/186 668		
	Fla.	0	0	0	1.341	4.466	−0.612	±4.703	1/261,157		
	Md.	10^{20}	10^{20}	10^{20}	—	—	—	—	—		

Same footnotes as in Table 3.

ADJUSTMENT CASE B

The purpose of Case B is to obtain an insight into the seriousness of error propagation in satellite triangulation. Using different control parameters, three independent least square adjustments of the hypothetical triangulation net shown in Figure 3, were made. The results are summarized in Table 5.

In adjustment B-1, station Maryland was selected as the origin of the triangulation net. Its fictitiously correct X, Y, Z coordinates were subsequently held fixed in the adjustment. The length of the line from Florida to Maryland was used to scale the network. An error was introduced into the true length of the line so that it has a relative accuracy of 1/500,000. The coordinates of the remaining four stations, Mississippi, Florida, Minnesota and New Mexico, were treated as completely unknown. The actual errors in the computed coordinates of these stations range between ±0.350 meters and ±12.653 meters. The errors in the absolute positions of these stations vary between ±4.537 meters and ±12.745 meters. However, the relative positioning accuracy of the stations vary between a narrow range, from 1/193,000 to 1/270,000, with a mean better than 1/200,000. This indicates that the positions of all the stations are determined to about the same degree of relative accuracy. The standard deviation of the plate coordinates after adjustment is ±0.360 μ.

In adjustment B-2, two additional control points were used. The fictitiously correct coordinates of New Mexico and Mississippi were each given an error of either + or −6 meters, and they were each assigned a weight of 10^{-5} in the subsequent adjustment. The mean relative positioning accuracy is improved to better than 1/300,000. The standard deviation of the plate coordinates after adjustment is increased to ±0.46 μ. This is understandable: because the positions of Mississippi and New Mexico were only allowed to have limited movements, the plate coordinates were made to absorb more of the errors in the adjustment.

In adjustment B-3, instead of using additional absolute control points as in adjustment B-2, a second base line was used as constraint. The length of the line from New Mexico to Minnesota was assumed to be surveyed to 2 parts in 10^6. The results from this adjustment show that no increase in accuracy can be obtained by the addition of a second scale constraint. The mean relative positioning accuracy is still about 1/200,000, as in adjustment B-1. The standard deviation of the plate coordinates is slightly increased to ±0.378 μ.

SUMMARY

The test results from Case A show that a satellite triangulation survey, consisting of one triangle and controlled by a base line surveyed to 2 parts in 10^6, is capable of determining the absolute location of a camera station to better than 1/300,000. The upper limit in the accuracy of such a survey is, of course, restricted by the accuracy of the base line. If the base line is surveyed to 1/500,000, the final results cannot be expected to be better than 1/500,000.

The locations of 13 satellite points in space were also determined and studied in adjustment A-3. The mean discrepancy in the computed locations of the 13 points was found to be 3.6±1.7 meters. With a mean altitude of 1,500 kilometers, this corresponds to a relative accuracy of about 1/400,000.

The test results from Case B show that, in spite of the large number of redundant observations, the problem of error propagation still exists in satellite triangulation. The addition of two more unknown stations, thus forming two more triangles, lowers the positioning accuracy of the survey to 1/200,000. However, the mean error in the adjusted directions is still about 0.3 second of arc, which agrees with the results from Case A.

The accuracy of a triangulation survey can be improved by using additional absolute control points, as shown by the results from adjustment B-2. An absolute control point is equivalent to a constraint in both length and azimuth. Furthermore, the coordinates of such a station do not have to be rigidly enforced. They can be assigned proper weights and allowed to have limited amounts of adjustment. Two such stations, New Mexico and Mississippi, were used in adjustment B-2. The coordinates of these stations each had an error of ± 6 meters and were given a weight of 10^{-5} in the subsequent adjustment. The positioning accuracy of the triangulation net was improved to 1/300,000. Unfortunately, it is rather difficult, if not impossible at present, to determine by conventional means the coordinates of several geodetic stations, each about 1,500 kilometers apart, to ± 6 meters.

The employment of a second scale constraint in adjustment B-3 failed to bring about any significant improvement in accuracy. This shows that the scale of the net can be sufficiently stabilized with a single base line. What the triangulation network really needs is a constraint in azimuth. Although an azimuth constraint is not an absolute necessity, it will undoubtedly improve the accuracy of the net.

It should be remembered that in both cases A and B, the theoretically maximum values of the errors presently encountered in practice were used. They were applied to the rotational elements, ω, α and κ, and the plate coordinates, $\bar{x}_p$ and $\bar{y}_p$, for the satellite positions. Therefore, it is expected that in practice better results can be achieved. However, because of the propagation of errors in a more extensive network, the accuracy of a continental or world-wide satellite triangulation net probably will be much lower than 1/500,000.

In summary, the results obtained so far from this investigation show that error propagation will remain a major problem in an extensive satellite triangulation net. Redundant observations and scale constraints alone will not be sufficient to eliminate the problem of error propagation in a cantilever type of triangulation net.

It is quite probable that improvements on various phases of the data reduction procedures would further decrease the magnitude of the errors from various sources. The base lines may eventually be surveyed to better than 1 part in 10^6. A more accurate star catalogue may be available in the near future to improve the determination of the orientation of a photogrammetric camera. Furthermore, cameras of longer focal length may be used to reduce the effects of the errors in the plate coordinates. However, as it is now possible to measure accurately the distance between a satellite and a point on earth by electronic ranging devices,[6] the best solution to the problem of error propagation may well rest on the combined use of triangulation and trilateration. Because of its importance, a continental or world-wide reference system certainly deserves nothing less than the best efforts from geodesists and photogrammetrists.

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