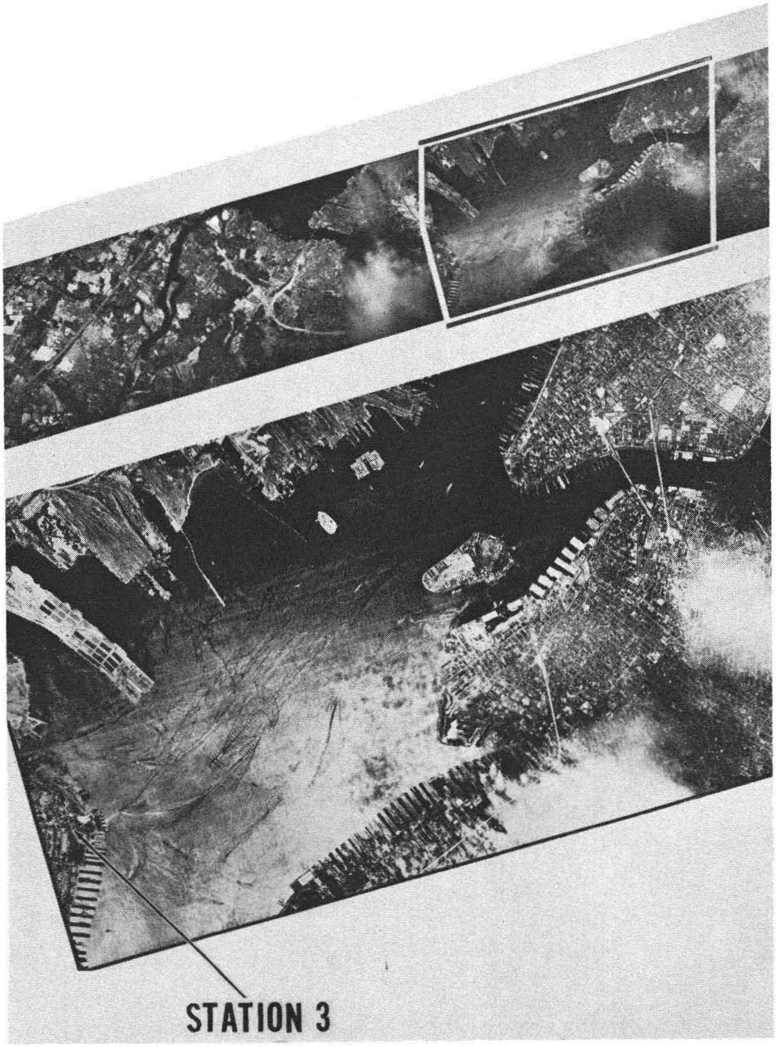

FRONTISPIECE. Interim photo map from high-altitude continuous strip photography.

SPENCER O. GULLICKSEN*
Chicago Aerial Industries, Inc.
Barrington, Illinois 60010

Continuous Strip Photography

High-altitude strip photographs can serve as a planimetric map substitute

INTRODUCTION

MOVING A GROUP OF men around, especially for the purpose of effectively combating the movement of an enemy force, requires intimate knowledge of the terrain. The need for maps of any sort has always been an important delaying factor in the timely execution of troop movements. Aerial photography has greatly reduced

* Submitted under the title "New High Altitude Interim Mapping Technique."

map preparation time, but even this improvement yields a process which is painstakingly slow by today's standards. The need for some form of map substitute has been thoroughly portrayed by Col. L. L. Haseman in a presentation in 1964.* The primary objective of the RACOM program is to provide to the using agency a map which contains topographic information developed by cartographic techniques automated to the maximum extent possible. The time of preparation is reportedly about 48 hours.†

A typical map substitute might be an uncontrolled photographic mosaic in which specific targets, terrain features, and objectives for a particular mission would be indicated in some convenient manner. Therefore, it would be required that the aerial photographs be vertical in orientation and have a specific photogrammetric quality. This would allow the use of simple linear scales for determining the terrain plane coordinates of a given location. Also, dependent on the scale of the photography, some characteristics of the elevation features of the terrain would be necessary. Naturally the amount of elevation information may or may not be important, dependent on the type of terrain being reconnoitered.

In addition to photographic information, another important aspect of the map substitute would be its geographic location. For instance, it is quite possible that a

ABSTRACT: *A definite military need often occurs for some form of map substitute. Present technology, even when augmented by aerial photography, requires that extensive time periods be allowed for final map preparation prior to publication. Programs such as RACOM are presently in effect to reduce map preparation time to a minimum. The possibilities of utilizing a photo map as interim terrain information are discussed. Typical aerial reconnaissance photography, in the form of a continuous photograph 5 inches wide by 42 inches long, is analyzed. The scale of the resulting photo map is approximately 1:60,000. A mathematical technique of transforming measurements made in the photo to yield an* RMS *error in locating various triangulation stations is presented. Other photogrammetric implications of aerial strip photography and its use as a map substitute include camera tilt, image motion compensation, different lateral and longitudinal scales, lens distortion, roll, etc.*

landing may be required at a geographic location which cannot be "tied" to a last known point by either optical or electronic triangulation. This is especially important if the strike is to be by air from a point very remote from the desired area. Although the specific requirements for accuracy pertinent to a photo map are not known at this time, the requirements would naturally consider the important fact that once the area is sighted, one can use visual observation to make final corrections. This latter thought is compatible with the use of aerial photographs as an aid to target identity in the map substitute.

In summation, it is the author's opinion that an aerial photograph can perform the function of a map substitute in areas where relief displacement can be tolerated. Further, if a strip photograph is used, all the effort required to produce a mosaic can be eliminated. However, certain characteristics of the particular photograph would have to be known. The photograph's verticality would have to be consistent with expected accuracy, probably necessitating a high-quality stabilized mount. The effect of relative motion and *IMC*‡ would have to be precisely controlled. The optical qual-

* "Rapid Military Mapping" by Col. L. L. Haseman CE presented at ASP and ACSM, 17 March 1964.

† Orientation Brochure GIMRADA, published July 1964.

‡ Image Motion Compensation.

ity of the lens would have to yield minimum distortion and high acutance. The geodetic location of the aircraft would have to be known, again to an accuracy consistent with the overall requirements. All of these various problems associated with the production of a photo map can be solved with equipment now available. The following discussion presents the results of a study program in which the cartographic features of strip photography were investigated.

Photo Analysis

In the summer of 1964, an RA-5C aircraft made an overflight of the New York City area at an altitude of 30,000 feet. Part of the objective of this flight was to display the ability of the aircraft to obtain 5-inch strip photography. The components in the system are shown in Figure 1. A camera control unit and control panel are also part of the reconnaissance system aboard this aircraft. Chicago Aerial Industries has long been of the opinion that strip photography offers an exceptionally simple solution to the problems of aerial reconnaissance. In this case, the strip camera was a simply modified KA-51A, $4\frac{1}{2}$- by $4\frac{1}{2}$-inch framing camera with a 6-inch f/2.8 lens. The imagery obtained was of excellent photographic quality.

The photo map analyzed in this study was a 42-inch-long portion of the flight negative printed on a "Cronapaque" material. The central portion of this photo map is shown in the Frontispiece together with an enlargement having the approximate scale of a standard $7\frac{1}{2}$-minute quadrangle sheet. Although the preliminary scale of the photograph, approximately 1:60,000, was thought to be too small to allow ground truth to be established, examination at 10×, and at times up to 30×, revealed an image quality suitable for positive identification of chosen targets. A file of New York and New Jersey triangulation data was obtained from Chicago Aerial Survey, and nine triangulation stations were located in the photograph. This data, shown in Figure 2, is typically used to prepare accurate maps in the usual cartographic sense.

Fig. 1. System Components.

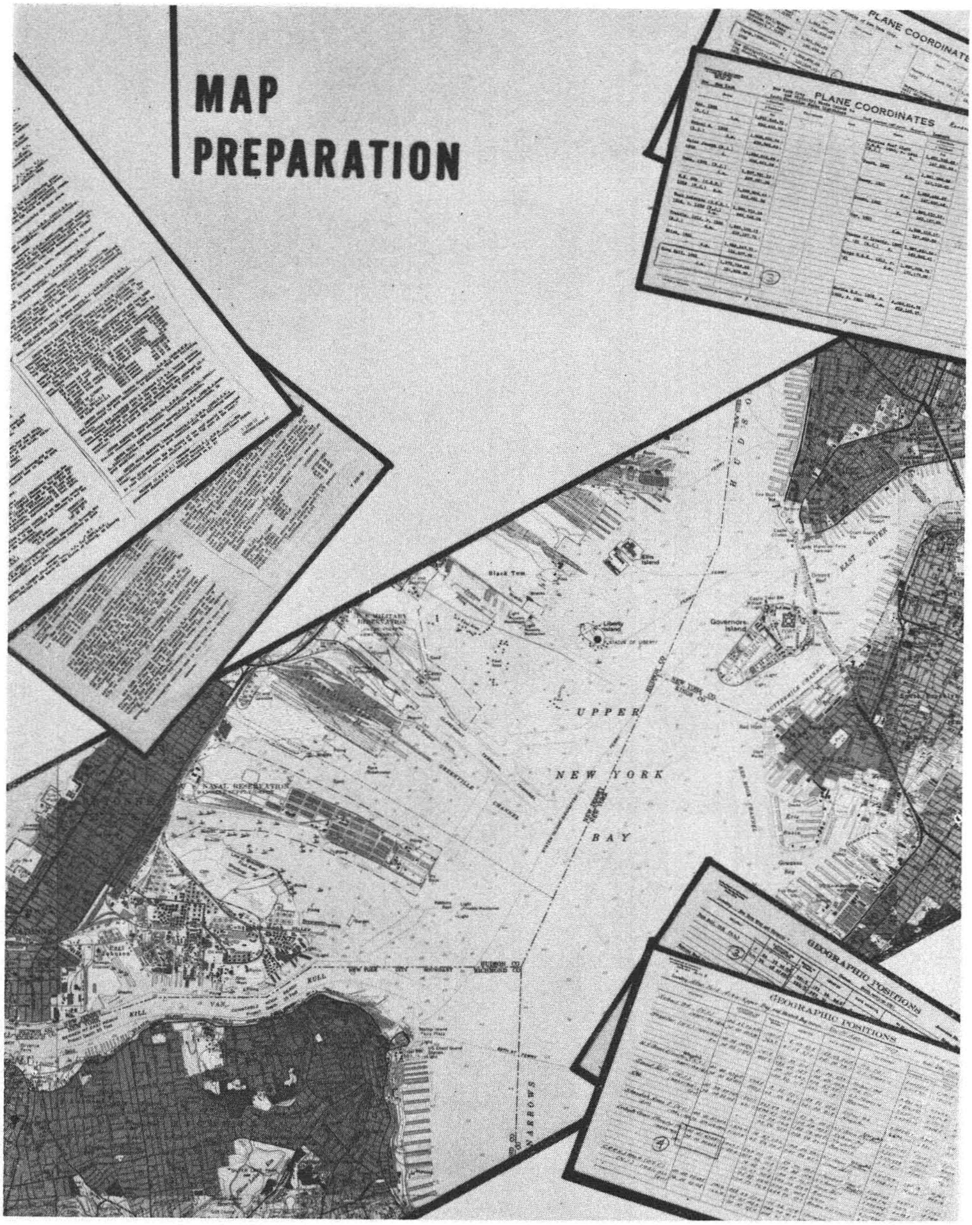

FIG. 2. Steps in map preparation from high-altitude continuous strip photography.

A measurement system was established in the photo shown in Figure 3 so that a series of eight radial distances (R_{1-8}) were obtained with respect to Station 3 (commonly known as Borough Hall, Staten Island). In addition, a reference line was established through the mechanical center of the photo map thus establishing, for the sake of the discussion, a principal line. This was necessary, since there were no provisions for fiducial marking in the modified KA-51A. Another series of measurements was obtained: the distance from the principal line to Station (y_{1-9}'). A summary of this information can be found in Table I.

This data in the photo was then subjected to a transformation process to produce

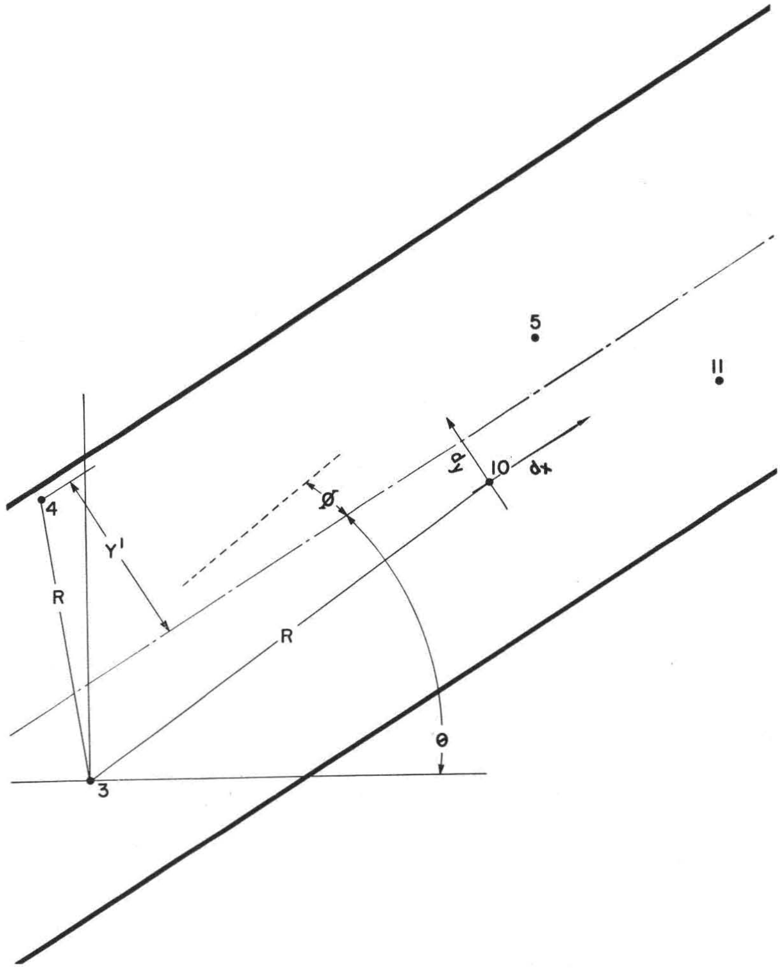

FIG. 3. Geometric elements relating to measured distances on a photo map made from a continuous strip photograph.

a best fit, on an RMS* basis, since there was a definite possibility of measurement error. The measurements were made to the nearest .001 inch with instruments adapted to the photogrammetric problem.

$$X = X_0 + S_x x_\mu' \cos\theta - S_y(y_\mu' - y_0')\sin\theta \quad (1)$$

and

$$Y = Y_0 + S_y(y_\mu' - y_0')\cos\theta + S_x x_\mu' \sin\theta \quad (2)$$

in which

X_0 and Y_0 = state plane coordinates of datum station, feet
S_x and X_y = scale in flight line and across flight line, feet
$x_\mu = \pm[R_\mu^2 - (y_\mu' - y_0')^{2\,1/2} \pm$ depending on location of μ
R = photo distance from datum station to station μ, inches
y_0' = distance from nadir line to datum station, inches
y_μ' = distance from nadir line to station μ, inches
θ = angle between nadir line and $Y=0$ line in state plane.

* Root Mean Square

TABLE I

RAW MEASURED DISTANCES IN THE PHOTO, INCHES

Station	*R*	*Distance to Nadir Line*
1. Woodbridge Church	12.1985	−0.647
2. Summerfield Church	4.6315	0.825
3. Borough Hall	0	−0.970
4. LeHigh Grain Elevator	3.9845	2.101
5. Fulton	7.9735	0.591
7. Whitestone Point Beacon	18.0485	2.087
9. Corona N. Gas Holder	12.1545	−0.606
10. Brooklyn T. H. S. Tower	6.3805	−0.727
11. St. Aloysius Church (Brooklyn)	9.9925	−1.243

The refined photo-measurement data was then compared with the state plane coordinates of the nine stations. The results of this comparison are shown in Figure 4. The information portrayed therein indicates that the nine points of interest in the photo are not precisely located with respect to their actual state plane coordinates. However, the dispersion magnitude indicates that geographic locations of points near sea level could, in all probability, be determined to an accuracy of ± 85 feet. Thus, for the photo map at hand, a radial distance from a datum station, R, and a principal line displacement, y', can be used together with Equations 1 and 2 to obtain the X and Y (state plane) coordinates.

Consider for a moment that, being situated at Station 1, it was necessary to know the distance to Station 7. The horizontal distance, from state plane data, is 152,944.7 feet. By using a suitable measurement device and a scale of 1 inch equals 5,084 feet (as the distance is almost all in the direction of flight) one obtains $30.106 \times 5{,}084 = 153{,}058.9$ feet. This represents an error of only 114 feet in a range of about 29 miles. A simple sighting on a nearby station, a base line drawn on the photo map, and the azimuth angle thus established would allow sufficient bearing and range information for accurate targeting.

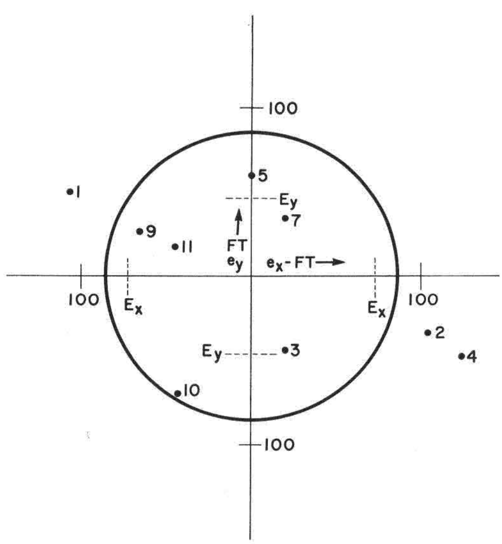

FIG. 4. Dispersion of errors obtained in determining positions through the use of high-altitude continuous strip photography.

The following dissertation involves the process of transformation, which was applied to the raw measured data obtained from the photograph under study, so that a comparison can be made with true data and thereby determine photographic accuracy capabilities in areas where the relief is small relative to the flight height. In order to establish terminology, the reader is referred to Figure 3 and Table I.

For this comparison, the true data were the state plane coordinate distances as obtained from the Plane Coordinate Index, Long Island Zone, State of New York. Two of the stations, 1 and 4, being in the New Jersey Zone, were located in subject zone by means of the "Plane Coordinate Projection Tables for New York," Publication No. 323. These coordinates were then referred to datum Station 3, and the following relationship was used to establish the value of x and y in the transformation and error analysis.

$$x_n = x_n \cos\theta + y \sin\theta$$

$$y_n = y_n \cos\theta - x \sin\theta$$

where

x_n and y_n are true coordinates rotated to the photo axes.
θ is the approximate heading (true heading is $\theta+\phi$, which is determined as a part of this analysis).

Let it be required to statistically fit the measured values in the photo to the state plane location of the nine stations in question. This results in the statement that

$$x_n'' = s_x(x_n' - d_x)$$

$$y_n'' = s_y(y_n' - d_y)$$

$$x_n' = [R_n^2 - (y_n')^2]^{1/2}$$

in which

x_n' and y_n' are the photo coordinates
d_x and d_y are the displacements required to effect matching
s_x and s_y are the scales of the photo in and across the flight line, respectively
x_n'' and y_n'' are the transformed photo coordinates.

Further let it be desirable to make the RMS errors (e_x and e_y) minimum values. We have

$$ne_x^2 = \sum (x_n'' - x_n - \phi y_n)^2$$

$$ne_y^2 = \sum (y_n'' - y_n + \phi x_n)^2.$$

As the approximate heading θ has been applied to the data, the remaining correction ϕ should be a small angle. An initial assumption therefore was made that

$$\sin\phi = \phi \quad \text{and} \quad \cos\phi = 1.$$

Then expanding we have

$$ne_x^2 = \sum (s_x^2 x_n'^2 - 2s_x^2 x_n' d_x + s_x^2 d_x^2 + x_n^2 + \phi^2 y_n^2 - 2s_x x_n' x_n + 2s_x x_n d_x - 2s_x x_n' \phi y_n + 2s_x \phi y_n d_x + 2x_n \phi y_n)$$

$$ne_y^2 = \sum (s_y^2 y_n'^2 - 2s_y^2 y_n' d_y + s_y^2 d_y^2 + y_n^2 + \phi^2 x_n^2 - 2s_y y_n' y_n + 2s_y y_n d_y + 2s_y y_n' \phi x_n - 2s_y \phi x_n d_y - 2y_n \phi x_n).$$

The total error is affected by ϕ; therefore

$$ne^2 = ne_x^2 + ne_y^2 = \sum (\text{total of all expanded terms}).$$

The derivative of this expression is made zero to find ϕ for minimum RMS error:

$$ne\frac{\partial e}{\partial \phi} = \sum (2\phi y_n{}^2 - 2s_x x_n{}' y_n + 2s_x y_n d_x + 2x_n y_n$$
$$+ 2\phi x_n{}^2 + 2s_y y_n{}' x_n - 2s_y x_n d_y - 2y_n x_n) = 0.$$

Solving for ϕ,

$$\phi = \frac{s_x \sum y_n(x_n{}' - d_x) - s_y \sum x_n(y_n{}' - d_y)}{\sum (y_n{}^2 + x_n{}^2)} .$$

In a similar manner we can solve for d and s; we have

$$ne_x\frac{\partial e_x}{\partial s_x} = \sum (2s_x x_n{}'^2 - 4s_x x_n{}' d_x + 2s_x d_x{}^2 - 2x_n{}' x_n + 2x_n d_x$$
$$- 2x_n{}' \phi y_n + 2\phi y_n d_x) = 0$$

from which

$$s_x = \frac{\sum (x_n{}' x_n - x_n d_x + x_n{}' \phi y_n - \phi y_n d_x)}{\sum (x_n{}'^2 - 2x_n{}' d_x + d_x{}^2)} .$$

Also we have

$$ne_x\frac{\partial e_x}{\partial d_x} = \sum (-2s_x{}^2 x_n{}' + 2s_x{}^2 d_x + 2s_x x_n + 2s_x \phi y_n) = 0$$

from which

$$d_x = \frac{s_x \sum (s_x x_n{}' - x_n - \phi y_n)}{s_x n s_x} .$$

Substituting, and solving for s_x and d_x, we have

$$d_x = \frac{\sum x_n{}' \sum x_n{}' x_n - \sum x_n \sum x_n{}'^2 + \phi[\sum x_n{}' \sum x_n{}' y_n - \sum y_n \sum x_n{}'^2]}{n \sum x_n{}' x_n - \sum x_n \sum x_n{}' + \phi[n \sum x_n{}' y_n - \sum y_n \sum x_n{}']}$$

$$s_x = \frac{\sum x_n + \phi \sum y_n}{\sum x_n{}' - n d_x} .$$

Then by substituting y for x, and $-\phi$ for ϕ, we have

$$d_y = \frac{\sum y_n{}' \sum y_n{}' y_n - \sum y_n \sum y_n{}'^2 - \phi[\sum y_n{}' \sum y_n{}' x_n - \sum x_n \sum y_n{}'^2]}{n \sum y_n{}' y_n - \sum y_n \sum y_n{}' - \phi[n \sum y_n{}' x_n - \sum x_n \sum y_n{}']}$$

$$s_y = \frac{\sum y_n - \phi \sum x_n}{\sum y_n{}' - n d_y} .$$

Therefore we can write the error expression which was used in plotting the dispersion magnitude of the nine stations with respect to their true state plane location

$$ex_n = s_x x_n{}' - s_x d_x - x_n - \phi y_n$$
$$ey_n = s_y y_n{}' - s_y d_y - y_n + \phi_n .$$

Photo Map Considerations

The previous discussion has only analyzed a specific photo and its inherent photogrammetric qualities. In order to proceed with the consideration of strip photography as a map substitute, the following thoughts are pertinent. Due to various system inaccuracies, the photo will contain two scales: one in the flight direction and another across the flight line. The value of s_x principally considers the variables of *IMC* and pitch perturbations. The value of s_y involves changes in altitude, lens distortion, and roll perturbations. In practice, the photo scale would no doubt be obtained only by knowing the altitude and lens focal length to yield a value of s_y, and a measured value of the *IMC*-error would provide the value of s_x as it relates to the lens scale.

A significant problem remaining in the system concept is that of positional accuracy with respect to a known reference; classically, this is the heart of the navigational problem. In today's technology the most useful method is one of radio triangulation and/or direct homing on known beacons. Naturally these techniques cannot be used during hostile action; hence, the development of elaborate bombing and navigation systems capable of passive operation. These systems embody an inertial reference platform and, in conjunction with a computer, continuously plot the position of the aircraft. Unfortunately, because of equipment inaccuracies, some form of positional updating is required. A very accurate form of guidance system can be obtained by updating the positional accuracy with data referred to celestial bodies. In addition to correcting the platform with an automatic star tracker, a system has been devised which uses both visual and radar triangulation sensors to obtain correction data. The use of these very accurate positional updating concepts are being applied to the mapping and charting problem in the form of the AN/USQ-28 system being developed for the U. S. Air Force.

A map substitute system, therefore, will have to rely on the navigational system in the particular aircraft in use. It is not the intent of this discussion to evaluate the abilities of today's bombing and navigation systems. In general, the navigational computers are programmed to update displayed longitude and latitude information every 0.1 minute. For the area of the earth, being discussed herein, this represents a rectangular portion of the earth 461 feet in an east-west direction and 607 feet in a north-south direction. Consider then, that this represents positional accuracy in which the location of the aircraft with respect to a last known navigational fix, either by visual or radio updating, is, in fact, within this area.

Thus the location of a particular datum station can be established on the photo map by means of a fiducial system in the camera. The most useful technique would be to image a pair of small crosses upon the edge of the negative. A line connecting these fiducials would lie across the flight line and be offset a small amount from the camera slit to allow implementation of the flashing lamp and imaging system. A calibrated point can then be established in each camera representing the location of the principal point with respect to the fiducial line. The fiducial system thus established becomes a series of reference points along the center of the photo map. Each point would be a specific distance from adjacent points, and would represent the geographic location of the nadir point. This, of course, presupposes that the optical axis is vertical, which can only be true if the camera is in a suitable stabilized mount. In practice, however, the degree of verticality does vary, and is a contributing factor in the dispersion data presented in Figure 4. The overall system would therefore produce a photo having as its inaccuracy the sum of the known errors plus that of the photo with respect to a last known geographic location.

Conclusion

The results of this study clearly indicate that the photo analyzed does, within the stated accuracies, possess photogrammetric quality. Further, because this photo was obtained during a routine reconnaissance mission by a proven operational system, it is appropriate to consider it as typical. In view of the fact that only one specific photo map has been analyzed, it follows that additional photography should be obtained and analyzed with the aid of adequate ground control.

A study program could be implemented considering the next phase in an interim mapping system program. Such things as the use of stereo strip imagery covering a 74° field, coordination of terrain profile recordings, and further development of ultra-precision techniques of *IMC* determination should be analyzed. In addition, preliminary designs of a strip camera embodying improved characteristics of fiducial and auxiliary data recording would be established. Along with these aspects, techniques of supplying accurate navigational information and, possibly, data on the attitude of the camera optical axis should be investigated.
