H. L. Oswal*
University of Roorkee
Roorkee/U. P., India

Comparison of Elements of Relative Orientation

The use of kappa in place of omega may yield higher precision if there is no *BY*-motion.

(Abstract on next page)

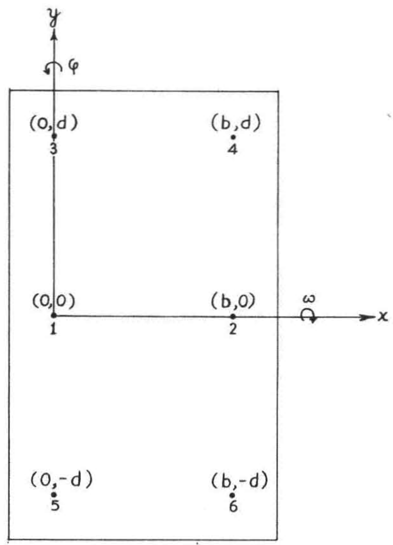

Fig. 1. Conventional location of the six points used in relative orientation.

Introduction

It is generally recognized that the precision achievable from a given method of relative orientation depends on the elements used for measuring and eliminating y-parallaxes p in a model as well as on the actual sequence of steps in the procedure. In Chapter I.11 of the I.T.C. Text Book of Photogrammetry (provisional typed notes edition), the precision cofactors of a number of methods have been worked out and tabulated.

The studies presented here aim at:

- Studying the operation of orientation elements from the aspect of their intrinsic precision;
- Selecting suitable elements for measuring residual y-parallaxes for use in numerical or semi-numerical methods; and
- Selecting orientation procedures which afford a superior degree of uniformity of precision in the relative orientation of the model considered as a whole. Obviously, an ideal orientation is one in which the quality of orientation is uniform over the entire model. It may be stated, however, that it is not at all easy to state the criteria to judge uniformity of quality in relative orientation.

Analysis

The usual criterion for establishing relative orientation is the elimination of y-parallax at five points in the model, a sixth point being used either as a check or for overdetermination. The points are distributed in the well-known pattern, as shown in Figure 1, which also shows the model coordinate system.

The approximate differential equations for y-parallaxes for the five orientation elements are:

$$(p_i)_{\Delta by} = \Delta by \tag{1}$$

$$(p_i)_{\Delta bz} = -\frac{y_i}{h_i}\Delta bz \tag{2}$$

$$(p_i)_{\Delta\kappa} = x_i\Delta\kappa \tag{3}$$

$$(p_i)_{\Delta\phi} = \frac{x_i y_i}{h_i}\Delta\phi \tag{4}$$

$$(p_i)_{\Delta\omega} = -\left[1 + (y_i^2/h_i^2)\right]h_i\Delta\omega \tag{5}$$

The subscript i is used to denote the point at which the element operates. Equations 1 to 5 may be combined into a single Equation 6,

$$p_i = \Delta b_y - \frac{y_i}{h_i}\Delta b + x_i\Delta\kappa + \frac{x_i y_i}{h_i}\Delta\phi - \left[1 + (y_i^2/h_i^2)\right]h_i\Delta\omega \tag{6}$$

Two criteria are available for studying the operation of the orientation elements:

i. Precision of y-parallax observation as the deciding factor, in which case the mechanical precision of each element should individually be made to correspond to the limiting precision of y-parallax observation in a stereo-model;

ii. Mechanical precision of each element as the deciding factor in which case the precision of y-parallax elimination would vary from point to point in the model, depending on the element used.

* Submitted under the title "Comparative Precision of Elements of Relative Orientation".

Because the elimination of y-parallax is adopted as the practical criterion for establishing relative orientation, it seems more logical to use the Criterion *i* above for our studies. It is, however, well known that the majority of manufacturers construct plotting instruments in which the various elements provided are of equal or equivalent mechanical precision, i.e., Criterion *ii* is implied. It thus becomes rather interesting to conduct a critical study into this question.

Tables 1 and 2 have been compiled on the basis of Criteria *i* and *ii*, respectively. The following basic data, normally met with in current photogrammetric practice, serves as the foundation for these tables:

a. The limiting precision of y-parallax elimination (direct optical mechanical observation) has been, from experience, assumed to be $\pm 5\mu$ in the scale of the negative;

b. The limiting mechanical precision of most instruments has been assumed to be $\pm 1^c$ for the rotational, and $\pm 10\mu$ for the translational elements. These are the values of least counts on the majority of precise plotting instruments and are assumed to reflect limiting mechanical precision;

c. Format of photography has been assumed to be 230 mm. by 230 mm. The coordinates of the six orientation points are shown in Figure 1;

d. Three cases of photography have been considered, normal angle $f=250$ mm., wide angle $f=150$ mm., and superwide angle $f=90$ mm. In each case the format is according to (*c*) above;

e. The model scale has been assumed to be 2.5× the photo scale.

ABSTRACT: *From the fundamental* y*-parallax equation of a stereomodel, the precision of* y*-parallax elimination is variable, depending on the elements that are employed for* y*-parallax measurement, or for relative orientation; in the latter case, on the actual sequence of steps in the procedure. The relative precision of different elements are compared. Also it is virtually impossible to obtain relative orientation of uniform precision in the sense that residual* y*-parallaxes in the model are of uniform weight. This holds true even if entirely numerical methods of relative orientation are employed.*

The choice of rotational elements is analyzed where orientation is performed with a non-universal plotter in which the b_y *motion is not available. The element* κ *seems to possess significant advantages over the element* ω *which is normally employed in this circumstance. Formulas are developed for using* y*-parallax measurements obtained by* κ *in relative orientation procedures.*

TABLE 1

Element	*Rotation or translation required to change 5μ of y-parallax*			*Remarks*
Angle	*Normal*	*Wide*	*Superwide*	
$\Delta\kappa$	35^{cc}	35^{cc}	35^{cc}	At points 3-1-5 or 4-2-6
$\Delta\phi$	99^{cc}	59^{cc}	35^{cc}	At points 3-5 or 4-6 No effect at 1 or 2
$\Delta\omega$	12^{cc}	21^{cc}	35^{cc}	At points 1 or 2
	11^{cc}	15^{cc}	18^{cc}	At points 3-5 or 4-6
Δb_y	$12\frac{1}{2}\mu$	$12\frac{1}{2}\mu$	$12\frac{1}{2}\mu$	At all points
$\Delta\beta_y$	35^{cc}	35^{cc}	35^{cc}	At all points
Δb_z	33μ	21μ	$12\frac{1}{2}\mu$	At points 3-4 or 4-6 No effect at 1 or 2
$\Delta\beta_z$	92^{cc}	59^{cc}	35^{cc}	At points 3-5 or 4-6 No effect at 1 or 2

TABLE 2

Element	*Y-parallax introduced for one least count of orientation element*			*Remarks*
Angle	*Normal* μ	*Wide* μ	*Superwide* μ	
$\Delta\kappa$	14	14	14	At points 3-1-5 or 4-2-6
$\Delta\phi$	5	8	14	At points 3-5 or 4-6 No effect at 1 or 2
$\Delta\omega$	39	24	14	At points 1 or 2
	44	32	28	At points 3-5 or 4-6
Δb_y	4	4	4	At all points
Δb_z	1.4	2.4	4	At points 3-5 or 4-6 No effect at 1 or 2

Similar Tables can, of course, easily be constructed for other photogrammetric data.

Table 1 (Criterion 1) shows the rotation or translation of different elements to effect a change in y-parallax at the six points by $\pm 5\mu$. The table however, discloses a disadvantage: although one may study comparatively the operations of rotational or translational groups of elements separately, it is difficult to compare the performance of rotational elements with the translational. Table 2 is free from this disadvantage. We may, however, define

$$\Delta\beta_y = \Delta b_y / b_x$$

and

$$\Delta\beta_z = \Delta b_z / b_x$$

and thus obtain angular equivalents of the translational elements for comparison with the rotational.

In Table 2, (Criterion *ii*) are shown the y-parallaxes introduced by operating the different elements through one least count on a plotter. A study of Table 2 confirms that sensitivity—defined as units of y-parallax introduced per unit of orientation element operation—is variable depending on the following factors:

A. The element itself. For normal and wide angle cases the element Δb_z (1.4μ and 2.4μ respectively) may be regarded as most sensitive. Next follow $\Delta b_y(4\mu)$ and $\Delta\phi(5\mu$ and 8μ respectively). For the superwide angle case Δb_z and Δb_y are equally effective (4μ) as also the elements $\Delta\kappa$ and $\Delta\phi$. $\Delta\omega$ has the least sensitivity—39μ to 44μ for the normal angle case, 24μ to 32μ for the wide angle, and 14μ to 28μ for the superwide.

B. Location of observation point. The elements $\Delta\phi$ and Δb_z have no effect at points 1 and 2, whereas $\Delta\omega$ is less effective at points 1 and 2 than at 3, 4, 5 or 6. $\Delta\kappa$ has the same effect at all points lying in the same cross section. Δb_y has the same effect at all points in the model.

C. Angle of camera lens. This has no effect on the operation of Δb_y or $\Delta\kappa$. Sensitivity of $\Delta\phi$ decreases as the angle increases but that of $\Delta\omega$ improves significantly.

A glance at Table 1, although drawn up on a different criterion, in general reinforces the deductions A, B, and C above. Summarily, it is interesting to note that the effect of Δb_y is independent of either focal length or format and is constant over the entire model. The effect of $\Delta\kappa$ is independent of focal length but depends on the format and is constant in any cross section parallel to the y-axis. The effect of Δb_z or $\Delta\phi$ depends on both the focal length and format and thus on the camera angle. The effect of either Δb_z or $\Delta\phi$ is equal but opposed at marginal cross sections (y = constant). The term $\Delta\omega$ exerts equal and similar influence in these sections; its effect in the basal section is reduced but is similar, and this reduction varies with the lens angle.

PROBLEMS

Two specific problems are evident:

1. Which element is best suited for measuring y-parallaxes for use in numerical relative orientation procedures, (*a*) in a universal plotter, and (*b*) in a non-universal plotter?
2. What procedures afford an orientation of maximum uniformity of precision and on what criterion would this be decided?

PROBLEM 1

An element to be considered suitable for measuring y-parallaxes must meet the following requirements:

α Its effect must be uniform in a cross section, lest observations at 1 or 2 have weights different from those at 3, 5 or 4, 6. It is ideal if the element operates with uniform precision over the entire mood.
β It should be sensitive, i.e., movement of the element through one least count should cause a small noticeable change in y-parallax.
γ Its use should not introduce large x-parallaxes as this would be a practical disadvantage. $\Delta\phi$ is unsuitable from this point of view.
δ Its operation should be directly readable on a scale with the minimum number of mechanical devices intervening. This is imperative for high precision.

$\Delta\phi$, Δb_z and $\Delta\omega$ do not fulfill requirement α but Δb_y and $\Delta\kappa$ do. The effect of $\Delta\omega$ may be regarded sufficiently uniform in cross sections perpendicular to the base for the normal and wide angle cases. Although $\Delta\phi$ and Δb_z have good sensitivity they will not be further considered since they do not meet requirement α. Δb_y then seems to be the ideal element. $\Delta\kappa$ has a uniform effect in every cross section, x is a constant, but is less sensitive than Δb_y. If it is desired to make κ as sensitive as b_y, its least count should be improved to

$$100^{cc} \times 4/14 = 30^{cc} \quad \text{or} \quad 10^{s}$$

as indicated in Table 2. The element $\Delta\omega$ is considerably less sensitive and has the additional disadvantage of not strictly meeting requirement α.

In current photogrammetric practice, if b_y is not available on an instrument (for example, non-universal instruments like Wild A8, Thompson-Watts Plotter etc.), it is usual practice to measure y-parallaxes with one of the two ω's, as indicated in Reference 2. This is difficult to explain in view of what has been said above. The obvious choice should be κ and not ω, particularly if we also take into account requirement δ. In the majority of instruments the scale for κ is given directly on the rim of the picture carrier but that for ω is arranged much farther away from the ω-axis.*

One may conclude this discussion by giving below the modified formulas of relative orientation by one of the popular seminumerical methods (Reference 3) where κ' and κ'' are used to measure y-parallaxes in the right and left sections respectively. First, parallaxes are eliminated at points 2 and 1 carefully with κ' and κ'' and record the readings on the κ-dials as κ_0' and κ_0'' respectively. Parallaxes are now eliminated at points 3 and 5 by κ'' and the readings κ_3'' and κ_5'' recorded. Similarly, parallaxes are eliminated at points 4 and 6 by κ' and readings κ_4' and κ_6'' recorded. We then have

$$\Delta\phi_1 = (h/2d)(\kappa_6' - \kappa_4')\rho;$$

$$\Delta\phi_2 = (h/2d)(\kappa_5'' - \kappa_3'')\rho;$$

$$\Delta\omega = (bh/2d^2)(\kappa_3'' + \kappa_5'' + \kappa_4' + \kappa_6' - 2\kappa_0'' - 2\kappa_0')\rho$$

In these equations ρ is the number of minutes in a radian.

PROBLEM 2

It is by no means easy to answer this question. In the first place, we have no clear concept of what constitutes precise relative orientation. Is it one in which the residual y-parallax in the model is of the same order everywhere? Or, is it one in which the five elements are determined with equal precision? Or again, is it one which gives minimum Δx, Δy and Δh errors in a model? Further, is it reasonable to assume that entirely numerical methods afford a more uniform quality of orientation within the model?

The last question only is discussed here. Relative orientation may be regarded as a problem of solution of simple matrix equations. According to Reference 4, the relative orientation may be stated by the equation.

$$\boldsymbol{Ax} = \boldsymbol{p} + \boldsymbol{P} + \boldsymbol{Q}$$

where $\boldsymbol{x}$ and $\boldsymbol{p}$ are the approximate values of orientation elements and the observed parallaxes respectively. $\boldsymbol{A}$ is a 5-by-5-coefficient matrix whereas $\boldsymbol{x}$ and $\boldsymbol{p}$ are 5-by-1-column matrices containing second and third powers of $\boldsymbol{x}$. For small values of $\boldsymbol{x}$, $\boldsymbol{P}$ and $\boldsymbol{Q}$ are small compared to $\boldsymbol{p}$; hence an iterative procedure of solution of $\boldsymbol{Ax}=\boldsymbol{p}$ is applicable, the value of $\boldsymbol{p}$ being redetermined from the preceding solution before each iteration. Essentially one is interested in the solution of the linear system

$$\boldsymbol{Ax} = \boldsymbol{p},$$

the solution of which is

$$\boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{p}.$$

In a numerical solution round-off errors occur, and as the $\boldsymbol{p}$'s are observed with uniform accuracy in a model (except where elements other than b_y or κ are used to measure the parallaxes), the round-off errors depend on the structure of the coefficient matrix $\boldsymbol{A}$ or $\boldsymbol{A}^{-1}$. Because the coefficient matrix is nearly invariable, (on account of Figure 1) it is advisable to perform relative orientation, particularly in an aerial triangulation project, by the same numerical method. This will pro-

* It may be of some interest to mention here that on the new Thompson Watts Plotter Mark II, the ω-scale has been provided right on the ω-axis.

duce data of uniform quality and contribute towards a logical approach to the problem of strip adjustment where the treatment of errors should be as rigorous as possible.

Conclusion

From the studies presented in this paper it may be concluded that, although in current practice the ω-element is used to measure y-parallaxes in a model in instruments where a b_y is not available, measurement using κ should produce better results. By improving the mechanical precision of the κ-axis, results as good as with b_y may be obtained. ω is significantly less sensitive than κ.

References

1. I.T.C. Text Book of Photogrammetry, Chapter 1.II (typewritten form)
2. Thompson, E. H., "A Note on Relative Orientation" *Photogrammetric Record*, Vol. IV, No. 24, October 1964.
3. Schwidefsky, K., "An Outline of Photogrammetry," Sir Isaac Pitman & Sons Ltd. London, 1959
4. Thompson, E. H., "An Algebraic Formulation of the Relative Orientation Problem," *Photogrammetric Record*, Vol. III, No. 14, October 1959

FORUM

Sirs:

I read with interest the article of Dr. J. M. Anderson in the September 1966 issue of PHOTOGRAMMETRIC ENGINEERING entitled "Research Interests and Capabilities in Colleges and Universities in the United States and Canada." In this connection I wish to report that this Training Center for Applied Geodesy and Photogrammetry, College of Engineering, University of the Philippines, Diliman, Quezon City, has also conducted researches in photogrammetry and photo interpretation since June 1965 as follows:

The Use of Aerial Photographs in Mapping the Soils of the U. P. Diliman Campus (*completed*);
The Application and Cost of Photo Interpretation Techniques in Soil Survey of a Portion of Mt. Makiling Area in Laguna (*completed*):
Photogeologic Study of Philippine Diorites (*completed*);
Investigation of the Efficacy of Photogrammetry for Precision and Operational Methods as Applied to Philippine Conditions (*in progress*).

We submit this report with the thought that the results and observations derived from our researches may be of value to the photo interpreters in tropical countries.

This Center was established in January 1965 primarily to fill the technical needs of the Philippine Land Reform Program and other development programs. To achieve this goal, training courses in applied geodesy, photogrammetry, and photo interpretation were organized. Since April 1965 the Center has been graduating trainees from ten government mapping agencies and the private sector. The promotion and coordination of activities, particularly those crossing Department and College lines, are facilitated by the Department of Geodetic Engineering, which is simultaneously manned by the Training Center Staff. An undergraduate ccurse in Bachelor of Science in Geodetic Engineering is offered by the Department.

—*Norberto S. Vila, Director*

Notice to Authors

1. Manuscripts should be typed, double-spaced on $8\frac{1}{2}\times 11$ or $8\times 10\frac{1}{2}$ white bond, on *one* side only. References, footnotes, captions—everything should be double-spaced. Margins should be $1\frac{1}{2}$ inches.
2. *Two* copies (the original and first carbon) of the complete manuscript and two sets of illustrations should be submitted. The second set of illustrations need not be prime quality.
3. Each article should include an abstract, which is a *digest* of the article. An abstract should be 100 to 150 words in length.
4. Tables should be designed to fit into a width no more than five inches.
5. Illustrations should not be more than twice the final print size: *glossy* prints of photos should be submitted. Lettering should be neat, and designed for the reduction anticipated. Please include a separate list of captions.
6. Formulas should be expressed as simply as possible, keeping in mind the difficulties and limitations encountered in setting type.