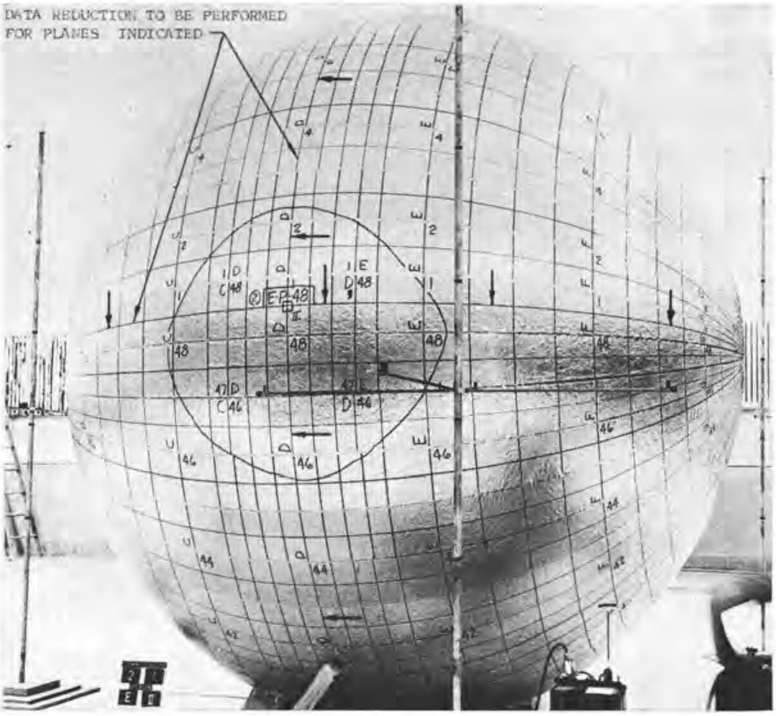

FIG. 1. Inflated sphere showing lines drawn on the surface.

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Deformation Analysis of Inflatable Spheres

A Stereometric camera and a stereoplotter are used to measure the sphericity of passive-reflector satellite models.

INTRODUCTION

THE manufacturing of space-borne inflatable balloons such as the Echo Satellites is a part of the continuing effort of the National Aeronautics and Space Administration (NASA) to provide passive reflectors for long range radio communications. The aim of the investigations of Echo-type models is to insure the manufacture of spherical wrinkle-free reflectors to achieve uniform reflective characteristics from all directions.

These are large inflatable spheres made of panels or gores of mylar polyester film covered with vapor-deposited aluminum. It was known that these spheres were not uniform in thickness and strength due to junctions along the gore edges; it was deemed necessary to investigate the sphericity to determine if wrinkles along the gores or lack of over-all sphericity would detract from the reflective characteristics (in the microwave radio spectrum) of the satellite.

ABSTRACT: *To determine the lack of sphericity for large, passive, satellite reflectors used in NASA space programs for radio communication, a test was conducted on a 12-foot model resting on a gymnasium floor using a Wild Stereometric camera and a Wild A-7 Autograph. The test showed that this procedure constitutes a feasible method for making such tests. Some difficulty was encountered because of the effect of gravity on the shape, a condition which is absent during orbit. The standard error (space vector) in the determination of the location of any point on the sphere was ±0.2 mm., the corresponding error of the center of the best-fitting sphere was ±0.5 mm., and for the radius it was ±1.33 mm.*

Some of the primary purposes for use of photogrammetry were to compare the spheres to the design diameter, to determine sphericity or lack of it, and to pinpoint material elongation accurately as a function of internal pressure. In the latter case it should make possible a definition of the unique pressure-strain relationships existing in the sphere because of the gores and tapes needed to assemble spheres.

It was believed that photogrammetry would help to develop a scaling technique for Echo-type spheres so that models could be studied with respect to physical dimensions and RF (radio frequency) reflectivity. It was believed that the results could be extrapolated to large radius-of-curvature devices. The effort was based on an analysis showing that the pressures required to obtain a sphere, constructed from flat material, depends only on the number of gores in the sphere. Therefore, an internally pressurized 12-foot diameter sphere should behave in the same manner as a 120-foot (or larger) diameter sphere if they both have the same number of gores.

The investigations reported here were made on nine spheres manufactured by the G. T. Schjeldahl Company of Northfield, Minnesota.

The specific objectives of these photogrammetric investigations were:

1. To plot contour maps of certain stereo-models on vertical planes at a scale of 1:5; contour interval of 50 millimeters for all areas excepting the flat (close to camera) areas where it was 10 millimeters.
2. To determine the best-fitting sphere, locate its center, and determine its radius.
3. To construct specified cross sections.
4. To determine the radial distances of points on the surface of every 20^g from the center of the best-fitting sphere for the specified cross sections.
5. To obtain radii of curvature at selected points in selected areas.
6. To obtain arc heights for selected areas over a span of one meter along cross sections in selected areas.

The work pattern was repeated on each of the nine spheres.

GENERAL APPROACH

PREPARATION

The sphere was inflated in a gymnasium in Northfield, Minnesota, to minimize the movement due to air currents during photography.

The surface of the sphere was previously marked with black lines into a system of meridians and parallels (Figure 1).

Every fourth meridian was lettered and every fourth parallel was numbered to help identify the points. The lines drawn with a width of approixmately 5 mm. were helpful in restoring a three dimensional photogrammetric model of the sphere. (See also Figures 4 and 5.)

PHOTOGRAPHY

Pictures were taken from four points called *EI*, *EII*, *PN* and *PS* (Figure 2) with a Wild

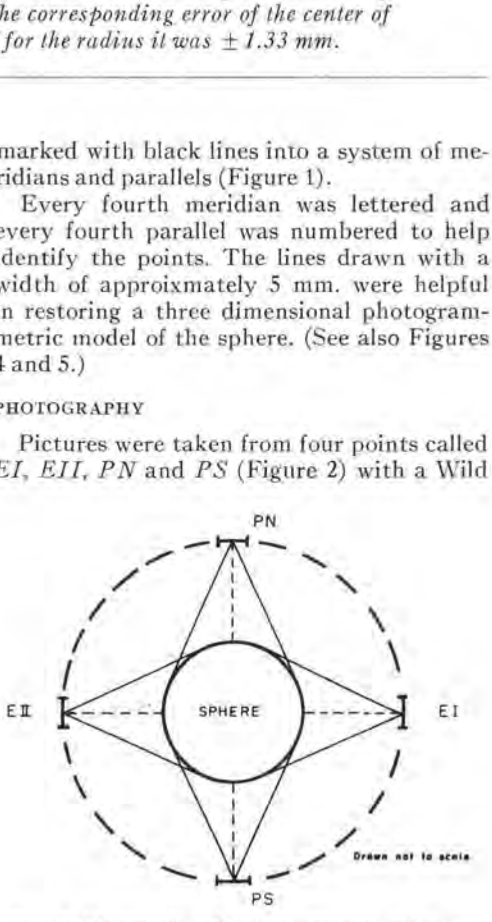

FIG. 2. Sketch plan showing camera locations.

Stereometric Camera (having a 40-cm. base) owned by the Department of Geodetic Science, The Ohio State University. (Figure 3.) The stereometric camera was located in each instance about two meters from the surface of the sphere which was about 3.5 m. in diameter. The placement of the camera and the sphere was guided with the help of pre-drawn circles and lines on the floor. Each sphere was inflated to seven different pressures for which four pairs of photographs were taken. The camera was placed on a triangular dolly for convenience in movement between photographs.

Features considered which resulted in the choice of this camera (Stereometric Camera) include:

a. Camera axes are parallel and the axes are perpendicular to the base which is fixed. This makes plotting and data reduction simpler.
b. The base being fixed, errors in the determination of scale of model are eliminated.
c. Ease of transport.
d. Availability of quality photographic material.

All pictures were taken with zero tilt in the camera (i.e., horizontal photography). The pictures were taken with an aperture of $f/12$ using an exposure of one second (this was determined through experimentation). Kodak Metallographic plates were used. These are orthochromatic, antihalation plates of high green-sensitivity and very high resolving power so necessary for such work.

As the photographic work was done indoors, extra care had to be taken for the lighting.

FIG. 3. Wild Stereometric Camera.

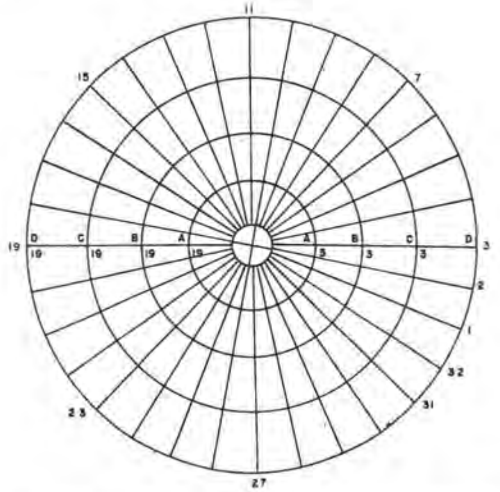

FIG. 4. System of marking major grid lines in polar view.

The lighting in the gymnasium was used, consisting of eight 350-watt and eight 200-watt light bulbs arranged in four rows of four lights each. A canopy of white muslin was hung under the two central rows of lights to avoid direct lighting. All windows were covered to avoid direct natural light. Finally, the balloon was placed on white paper spread on the floor to give a second source of diffused light. All these precautions were taken to eliminate reflections off the highly reflective aluminum surface.

The base-distance ratio was kept between $\frac{1}{5}$ and $\frac{1}{6}$. Although this ratio may not be considered as the best situation from the accuracy point of view, it was used because of the fact that closer range photography, giving higher base-height ratio, would give unsharp images. Therefore, a compromise had to be made in this respect.

GROUND CONTROL

Ground control was established in each pair of photographs with three rods, mutually perpendicular, with holes drilled at equal distances along them for insertion of straight paper pins. The pinheads of about 1 mm diameter provided the control points. This arrangement helped not only to determine the scale of the stereomodel but also to check affine deformation originating from using a focal length which might not be entirely accurate.

To join the adjacent models in the final analyses, the overlapping four or five lines at the edge of each pair of photographs were used. (Figures 4 and 5.)

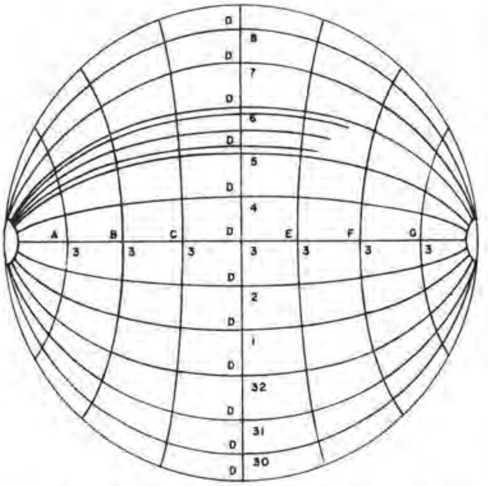

FIG. 5. System of marking major grid lines in equatorial view.

STEREO MEASUREMENTS AND DATA REDUCTION

WORK AT THE STEREO PLOTTING INSTRUMENT

The Wild Autograph A7 of the Department of Geodetic Science, The Ohio State University was used for this work.

The original negatives were placed (emulsion down) on non-compensating plate holders and inserted into the autograph cameras. The effect of objective distortion was ignored for this work as the radial distortion was less than 10 microns at the negative, which is the least count of any coordinate at the stereo instrument.

INTERIOR ORIENTATION

The camera was calibrated for the focal length separately by three researchers. The average of these three calibrations, i.e., 92.00 mm., was considered for this project. The acquisition of numerical data was at the Wild A7 which permits direct introduction of any focal length from 98 to 215 mm. The focal length introduced was 98.13 mm. i.e., an increase of 1/15 of the calibrated value. This would cause affine deformation of the model and would affect the Z-scale (along the camera axes) in the normal case (where camera axes are parallel to each other and are perpendicular to the camera base) that applies here.

Further, because the stereometric camera is provided with only two fiducial marks, in order to ensure accuracy in placing the negatives on the plate carriers, the fiducial centers were predetermined and marked on the negatives before they were inserted in the plate carriers. Residual errors in interior orientation, if any, are largely compensated by relative orientation of the model. Because each model was adequately controlled, and because extra care was taken in orienting each model, it was considered that no future precautions in interior orientation were necessary, especially in view of the fact that the stereoplotting instrument (Wild A7) was calibrated and was in excellent working condition.

RELATIVE ORIENTATION

Numerical relative orientation using six points by the method of reading Y-parallaxes was performed for each model. The operations were reiterated until a standard error of ± 0.02 mm. of residual Y-parallax was obtained in the model.

SCALING AND ABSOLUTE ORIENTATION

Each model was scaled in X and Y to the distances on the control rods (the pinheads being the control points). The scale along Z, being different from X and Y, was considered after being divided by the scale factor 0.93632. The model scale was 1:10. Apart from scaling, no absolute orientation was accomplished. Rotation of the model, wherever required, was done analytically.

CONTOUR PLOTTING

The contour maps were plotted at the Wild A7. This involved plotting the grid lines and the contours at the scale 1:5, the projection being orthogonal in a plane (XY-plane) normal to the camera axes. The elevation counter (Z-counter) transmission gears at the Wild A7 were set in positions Ba so that the elevations in the model were read directly in millimeters. A contour interval of 50 mm. on the sphere was selected for all areas excepting the topmost flat area, where it was 10 mm. The proper instrument interval was derived by using the following relation:

$$\Delta H' = \frac{\Delta H}{\lambda} \cdot M_M$$

where

$\Delta H'$ is the contour interval at the (A7) instrument.

ΔH is the actual contour interval on sphere.

λ is the scale factor (0.93632 in this case).

M_M is the model scale (1:10 in this case).

The reading for the highest point (nearest to the camera) was arbitrarily set at 1,000.00 mm.

It may be emphasized here that the numbers assigned to the contours are purely arbitrary because it is impossible to establish a common absolute datum until after the analytical computation of the absolute center of the sphere is complete. Actually there was no need in this problem to convert the coordinates to an absolute system.

READING COORDINATES FOR DATA PROCESSING

Coordinates were read in millimeters from the counters of the Wild A7 for the large scale data processing. The least count of reading each coordinate in the model space was 0.01 mm. In order to increase the accuracy of reading the cordinates, each point was observed in two rounds (by moving the instrument carriage in opposite directions). The average of such readings of each coordinate at each point was considered for the subsequent data processing.

GENERAL DATA PROCESSING

All measured data were key-punched on data cards for use in the IBM 7094A computer located in The Ohio State University Research Center. SCATRAN source language was used as this permitted the maximum use of library extensions associated with the computer.

RADII OF CURVATURE

Data processing was primarily limited to deriving radii of curvature at specific locations. The relationship between three known points and the radius of the circle passing through them is solved to obtain the coordinate of the center. These are solved to get the radius ρ as shown in Figure 6 and Table 1.

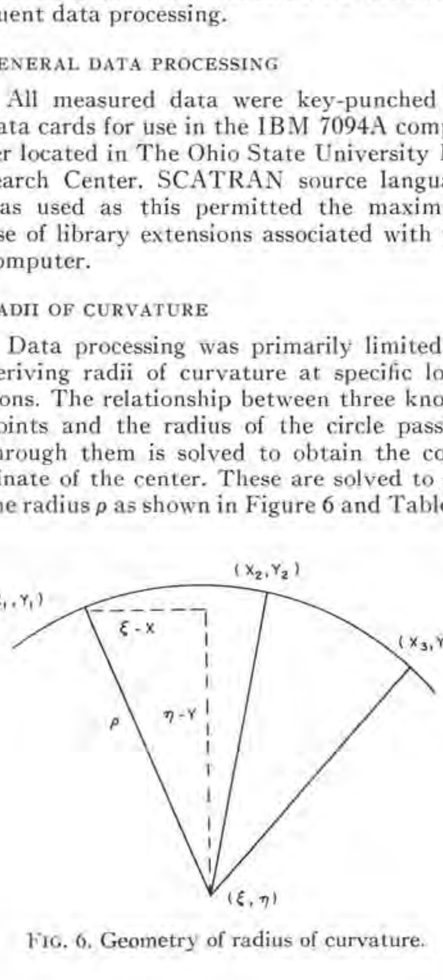

FIG. 6. Geometry of radius of curvature.

TABLE 1. Formulas for computing the radius of curvature ρ from the coordinates of three points on a circle. (See Figure 6)

$$\xi = + \frac{\dfrac{x_3^2 - x_2^2}{y_3 - y_2} - \dfrac{x_2^2 - x_1^2}{y_2 - y_1} + \dfrac{y_3^2 - y_2^2}{y_3 - y_2} - \dfrac{y_2^2 - y_1^2}{y_2 - y_1}}{2\left(\dfrac{x_3 - x_2}{y_3 - y_2} - \dfrac{x_2 - x_1}{y_2 - y_1}\right)}$$

$$\eta = + \frac{\dfrac{x_3^2 - x_2^2}{x_3 - x_2} - \dfrac{x_2^2 - x_1^2}{x_2 - x_1} + \dfrac{y_3^2 - y_2^2}{x_3 - x_2} - \dfrac{y_2^2 - y_1^2}{x_2 - x_1}}{2\left(\dfrac{y_3 - y_2}{x_3 - x_2} - \dfrac{y_2 - y_1}{x_2 - x_1}\right)}$$

$$\rho = \left[(\xi - x_i)^2 + (\eta - y_i)^2\right]^{1/2}$$

The model scale was 1:10, but 1:8 also occurred. The computer program used scale as an input parameter.

Extreme deviations in radii of curvature (e.g., 30 times or greater than the average) were eliminated. They may be caused by reading errors in the Wild A7 or by other blunders. A careful check was made to eliminate all such errors.

ERROR COMPUTATION

For one least square computation a 78-by-78 matrix was inverted in the 7094A computer with The Ohio State University computer center. A SCATRAN library function exists for matrix inversion so this was an extremely simple program.

RESULTS

The detailed results were presented as:

(1) Contour maps of the relevant stereo models of a scale of 1:5.
(2) Plots of horizontal sections through poles showing (a) location of the center of the best fitting sphere, and (b) radial directions at every 20^g along which the radial distances are computed.
(3) Tabulated radial distances of the specific points at every 20^g on the surface of the sphere from the center of the best-fitting sphere. The computed radius of the best-fitting sphere for each pressure is also given in these tables.
(4) Computer output giving the radii of curvature of selected points with listing of instruments and ground X, Y, Z coordinates to define the location of each point.
(5) One graphical plot for each view of each sphere to serve as reference to the tabulated radii of curvature values to help locate the points easily.
(6) Graphical plots at a scale of 1:2, giving, for the equatorial view, horizontal sections with height of arc for one meter length, and vertical section with height at each gore mid-point for a span of one meter.

FINAL ACCURACY

The various results are the end products of a complex system including the Wild Stereometric Camera system, its operating conditions and circumstances; the Wild Autograph A7, its operator and operating conditions; the draftsman and the drafting conditions of the final output. The resulting errors (indicating the resulting accuracy of the output data) were analyzed by Captain Forrest L. Hicks and submitted to The Ohio State University in his M.S. Thesis. These are presented below for ready reference (all are expressed in terms of the object scale, *not* photo scale, model scale, nor plotting scale):

Contour Map. The accuracy of locating any point, expressed in terms of the standard error of each coordinate:

$$m_X = m_Y = m_Z \leqq \pm 0.2 \text{ mm}.$$

Cross Section. The graphical output at the scale of 1:5 had standard errors:

$$m_Z = m_X \leqq \pm 0.5 \text{ mm}.$$

Best-Fitting Sphere. The method is based upon the fact that a perpendicular bisector of a chord on a circle passes through the center of the circle. By taking pairs of such perpendicular bisectors, their points of intersection, after an adjustment, help obtain the center of the circle concerned. The standard error of locating the center is:

$$m_{\text{point}} \leqq \pm 0.5 \text{ mm}.$$

The standard error of establishing the radius of the best-fitting sphere is:

$$m_\rho \leqq \pm 1.33 \text{ mm}.$$

Radii of Curvature. The standard error of locating a point is:

$$m_{\text{point}} \leqq \pm 0.2 \text{ mm}.$$

whence

$$m_\rho \leqq \pm 0.9 \text{ mm}.$$

Arc Height. These are graphically constructed from values used in the radii of curvature derivations. Here

$$m_{\text{point}} \leqq \pm 0.2 \text{ mm}.$$

and

$$m_{\text{height}} \leqq \pm 0.3 \text{ mm}.$$

CONCLUSIONS AND RECOMMENDATIONS

The pictures were taken with the spheres resting at a particular attitude (polar axis horizontal in each case). The investigations show the deformations, etc. for each sphere only for that attitude. It is possible that the shape of the sphere may change if it rested on a different side.

Because the spheres were always resting on the floor, gravity would cause an elliptical shape on the vertical section, whereas the horizontal section may be expected to be circular. This was studied in one case. The best-fitting sphere, in this case, obtained from the vertical sections was found to have its diameter four centimeters shorter than that of the best-fitting sphere obtained from the horizontal sections. A thorough analysis of the systematic errors due to gravity would eliminate this elliptical effect. This was not done due to lack of time and funds.

The photography of the present investigations still had circular gaps of radius 2.5 ft., approximately, at the top and on the bottom. Unless the spheres are also photographed from above and below, the gaps may not be entirely covered. However, an indirect solution to this problem might have been obtained by having extra sets of photographs with a changed attitude of the sphere (viz., horizontal polar axis).

Results of these investigations indicate that a feasible method of measurement of such spheres for the purpose of analysis and calibration has been developed based on purely photogrammetric procedures. This method, unlike other previously developed complicated and time-consuming methods, does not require very special equipment, often unavailable in either governmental or commercial organizations.

ACKNOWLEDGMENTS

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