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Block Adjustment Operation at C&GS

Using the collineation principle, 90 super-wide angle photos on film at 1:70,000 gave 2-foot accuracy and required 17 minutes on a CDC 6600

(*Abstract on page 1268*)

INTRODUCTION

ANALYTIC AEROTRIANGULATION has been in routine operation at the Coast and Geodetic Survey since 1961. The system permits the accurate determination of the ground coordinates of objects appearing on a block of overlapping aerial photographs using relatively few known ground positions. The digital calculations involved depend on comparator measurements of pertinent image positions on each photograph in contrast with he analog approach where measurements are made with a stereoscopic plotting instrument. The analytic solution offers certain worthwhile advantages occurring from automation, digital accuracy, least-squares adjustment, and freedom from the mechanical discrepancies contributed by the plotting instruments.

The principle of collinearity developed by Dr. Hellmut Schmid provides the basis for this method of analytic computations. Every object, its image, and the camera lens must lie on a common straight line as defined by the method of least squares in which the sum of the squares of the residual errors of image coordinate measurement are minimized. The Coast Survey system utilizes the collinearity condition in an iterative manner to determine incremental corrections to initial approximations for the unknowns, which are reasonably close to the correct values.

The computer programs for aerotriangulation have been written in Fortran because of the large number of computers that will accept this language. The Coast and Geodetic Survey has trained several of its photogrammetric personnel in Fortran programming because of the relative ease with which it can be mastered. The electronic data processing equipment available to the Coast and Geodetic Survey, which is now a component of the Environmental Science Services Administration, consists of an IBM 1620, an IBM 360/30, and a CDC 6600. The Bureau is faced, however, with ever-increasing demands for data processing related to scientific computing and fiscal and administrative operations. Consequently, it has been necessary to rent time on an outside IBM 7030 (STRETCH) computer (belonging to the Naval Weapons Laboratory and located at Dahlgren, Virginia) to perform its analytic aerotriangulation computations.

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The Coast and Geodetic Survey System

The system developed at the Coast and Geodetic Survey consists of four parts (Figure 1): (1) coordinate refinement and three-photo orientation, (2) strip adjustment to ground control, (3) secant plane coordinate transformation, and (4) space resection and block adjustment, which comprises the climax of the analytic solution and is the subject for this paper.

Computations may terminate after strip adjustment or continue through block adjustment depending on the desired accuracy. Although block adjustment can be performed without using the three-photo orientation and strip adjustment programs, these "preliminary" programs are employed to furnish improved and complete data for the block adjustment in an effort to reduce computer costs by minimizing the number of iterations of the block program.

Coordinate Refinement and Three-Photo Orientation

Computations begin with the refinement of image coordinates that have been measured with a comparator. This consists of corrections for the calibration of the comparator, the distortion of the photographic film, the symmetrical and asymmetrical radial distortion introduced by the camera lens, and atmospheric refraction. Although a correction for earth curvature can be introduced into this program, the effect of earth curvature is best treated by means of a secant plane coordinate transformation later in the solution. The refined image coordinates are punched out to serve as input for the block adjustment.

After all coordinate observations have been refined, the program proceeds to the camera orientation computation. In essence the photogrammetric orientation comprises only an inter-related geometric fitting of photographs based on the refined image coordinates and is entirely independent from any ground control data. The three-photo iterative solution derives an orientation of each photograph relative to the previous two in a strip and determines the positions of all pertinent objects in a three-dimensional coordinate system at the approximate scale of the photography. The analysis of three photographs at a time automates the joining of the separate triplets into a continuous strip and allows the detection of discrepancies in the x- coordinate observation of an image; otherwise, the discrepancies would be interpreted as elevation differences by a computer programmed to make a two-photo solution.

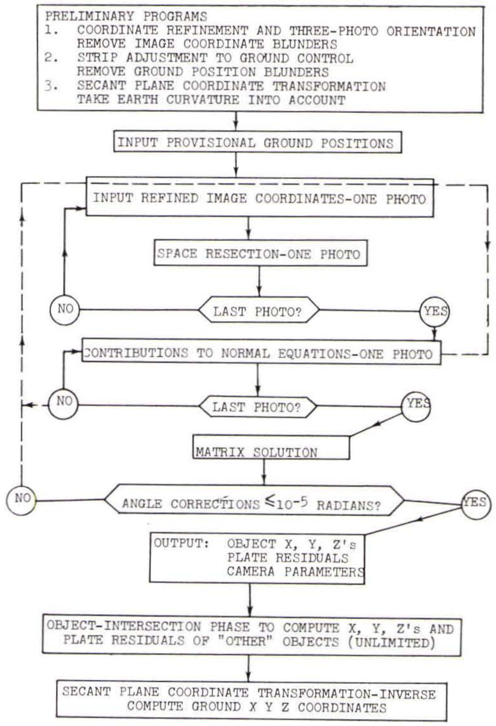

Fig. 1. Flow diagram for block adjustment program.

The collineation principle is imposed so that discrepancies in the observed image coordinates are minimized through the application of least squares. These residual errors are analyzed by the computer which discards those images exhibiting excessively large discrepancies. The removal of the blunders provides "clean" image coordinate data for all subsequent computations.

Strip Adjustment to Ground Control

The model coordinates developed by the three-photo orientation program are analogous to the product obtained from conventional stereoscopic plotting instruments. A small number of the objects have their true ground positions already known and can therefore serve as control stations for a horizontal and vertical strip adjustment of the data. The strip adjustment program is used to transform the three-photo model coordinates of objects into the prevailing ground-control coordinate system by fitting to control data through the application of polynominal formulas (which are empirical in a sense) and least squares. The results constitute provisional positions and elevations of all the objects, including elevations of horizontal control stations and horizontal coordinates of vertical control stations. This same program can also be used to adjust a strip which contains insufficient control points by fitting to common points of an adjacent strip. If the resultant adjustment contains large residual discrepancies, the difficulties are detected by human inspection and corrected. This leaves the provisional ground position data free of blunders prior to entering the block adjustment routine.

Secant-Plane Coordinate Transformation

The provisional ground coordinates of the objects are transformed into a special secant plane system before beginning the block adjustment. This is a geocentric system which utilizes a convenient, geometrically correct, three-dimensional Cartesian coordinate system that takes earth curvature into account. The block adjustment is completed using secant plane coordinates for the objects. The adjusted secant plane coordinates are then transformed back into the original ground coordinate system by applying the transformation program in its inverse mode.

Block Adjustment

If maximum accuracy is required for a photogrammetric survey, the block adjustment program is applied using the refined image coordinates and the previously obtained provisional object coordinates. Block adjustment may be defined as the simultaneous solution of the absolute orientation (three linear elements of position and three angular elements of orientation) of all the photographs, together with corrections to the three provisional coordinates of each object. The simultaneous observation equations impose the collineation principle on all these unknowns in a single, very large solution that minimizes the discrepancies of the image coordinates through the application of least squares. The major chore of the block adjustment computation relates to the solution of this set of equations in a manner which efficiently utilizes the memory capacity of the computer.

Formulation

The well-known equations of collineation comprising the projective transformation are:

$$\frac{x}{z} = \frac{a_{11}(X - X_0) + a_{12}(Y - Y_0) + a_{13}(Z - Z_0)}{a_{31}(X - X_0) + a_{32}(Y - Y_0) + a_{33}(Z - Z_0)}$$
$$\frac{y}{z} = \frac{a_{21}(X - X_0) + a_{22}(Y - Y_0) + a_{23}(Z - Z_0)}{a_{31}(X - X_0) + a_{32}(Y - Y_0) + a_{33}(Z - Z_0)} \qquad (1)$$

where the a-terms are the nine elements of the rotation matrix relating the x, y, z-image coordinate system to X, Y, Z-ground coordinate system of the objects, and X_0, Y_0, Z_0 are the coordinates of the camera station expressed in the ground coordinate system.

Partial differentiation is applied to the transcendental collinearity equations to obtain linearized observation equations. These equations can then be solved for the unknowns through the application of least squares. Their complete form is:

$$\begin{aligned} v_x = (&p_{11} + p_{12}d\omega + p_{13}d\phi + p_{14}d\kappa - p_{15}dX_0 \\ &- p_{16}dY_0 - p_{17}dZ_0 + p_{15}dX + p_{16}dY + \\ &p_{17}dZ)/A_3B \\ v_y = (&p_{21} + p_{22}d\omega + p_{23}d\phi + p_{24}d\kappa - p_{25}dX_0 \\ &- p_{26}dY_0 - p_{27}dZ_0 + p_{25}dX + p_{26}dY \\ &+ p_{27}dZ)/A_3B \end{aligned} \qquad (2)$$

where the nine terms dw through dZ are incremental corrections to be applied to initial approximations of the unknowns. These two equations occur for each image on each photograph. Block adjustment requires the presence of all nine terms, whereas the space

Abstract: *The Coast and Geodetic Survey is publishing its analytical aerotriangulation computer programs in the Fortran language for the purpose of expediting the application by the public of analytical methods. The analytic system includes: systematic correction of observed image coordinates; three-photo orientation; horizontal and vertical strip adjustment; and block adjustment. All of these programs, with the exception of block adjustment, have been documented in a series of technical bulletins available on request. The block adjustment program was put into operation in 1966 and documentation is expected in the spring of 1968. The purpose of this paper is to discuss the philosophy and programming techniques used in the block adjustment solution, and to describe some practical applications.*

resection computations use only the first six terms dw through dZ_0. Sufficient photographs and images are required to provide at least as many equations as there are unknowns to be computed. The solution is iterative and terminates when the incremental corrections to the angular parameters are smaller than the observational precision.

Considerations in the Design of the Block Adjustment Program

The block adjustment program was designed to minimize the pre-handling and arrangement of card data and to maximize the practical application of the block routine to the many kinds of photogrammetric surveys that can occur. Accordingly, the area to be block adjusted may have any shape provided each pertinent object appears on at least two photographs within the block. The photographic strips can be of variable length and may have any overlap with other strips. Thus, diagonal cross-flights may be included in the computation if desired. The photographs can be entered into the block adjustment program in any order. The routine also permits the mixing of photographs taken by aerial cameras having different focal lengths.

The maximum number of pictures that can be incorporated in the block program is limited by the capacity of the computer. In addition, the time and cost of computer operation are affected by the number of photographs for which the program is designed. As a consequence of these considerations, the Coast Survey has prepared three versions of the block adjustment program for the IBM 7030 (STRETCH) computer of the Naval Weapons Laboratory in Dahlgren, Virginia. The core memory consists of 48,000 words of 64-bit length (14 digits), of which approximately 39,000 are available for this application.

The first version was designed to operate within the limits of core memory and can accommodate only 20 photographs. The second version introduced the use of auxiliary disk memory and can accommodate a maximum of 200 photographs. The introduction of disk memory, however, was found to increase the time and cost of the computer operation by a ratio of about 5 to 1 when an 18 photo strip was run through both block adjustment versions. By restricting the maximum number of pictures to be accommodated in the block to 185, a third version was written which reduced the calls on disk memory and lowered the time-cost ratio to 4:1. It is believed that a maximum size of 185 and/or 200 photographs in a block adjustment is sufficiently large to encompass most normal applications. Additional versions will be prepared to accommodate maximum sizes of 50 to 100 pictures in a further effort to conserve computer time and costs. The design of efficient programs capable of accommodating more than 200 pictures appears to await the availability to the Coast and Geodetic Survey of computers possessing larger memories.

Provision exists in the programs for the introduction of various weight factors with the input data. These include: resolution weights for weighting the images relative to the resolution of the camera lens based on the image radius; collinearity weights used to enforce the collinearity condition for control stations thereby minimizing their image coordinate residuals; and position weights used to enforce the ground coordinates of the control stations. The resolution and collinearity weighting is accomplished by multiplying both observation equations by the assigned weight factor for each image. Control-position weighting is performed by multiplying the appropriate diagonal term of the normal equation system by a preassigned factor. Additional provisions exist for limiting the maximum number of iterations of the block routine, and for limiting the maximum acceptable image coordinate residual for objects whose ground coordinates are computed in the object intersection phase to be discussed later.

Contents of the Block Program

The block adjustment program consists of three phases: (1) space resection, (2) block orientation, (3) object intersection.

Space Resection

Computations begin with reading into storage the true ground coordinates of weighted control stations and the provisional ground coordinates of objects whose ground coordinates are to be finalized in the block orientation phase. The ground positions may be entered in any order and in excess of the actual number of these objects. The 200-photo version of the program will accept 1706 such positions, and the 185-photo version will accept 2142 positions.

The refined coordinates of each image are entered for one photograph at a time. The routine accepts a maximum of 49 images per photograph of which the first 18 images are the pass-point images associated with the photo (two pass-point images in each of the nine relative orientation locations) and are entered in a prescribed order. The image identification number consists of nine digits which contain the number of the corresponding object, the number of the photograph center nearest the object, the number of the photograph on which the image coordinates are observed, the category of the object (type of control, pass point, etc.) and a general pass point location number. This identification number allows the program to extract the appropriate ground coordinates for each image from the previously entered ground position data. As each object enters the solution for the first time, its six-digit identification number is added to an order list of unknown objects for use later in the least-squares block orientation phase.

The space-resection routine then fits the image data from each photograph to the corresponding provisional ground coordinates in order to determine initial values for the camera parameters. This involves the formation of a set of equations having the six incremental corrections to the camera parameters as unknowns, and solving repeatedly (a maximum of five iterations is permitted by the program) until further corrections are insignificant.

The resection phase is applied and solved for each photograph in turn only once; if a second solution of the block is required, the resection phase is circumvented.

BLOCK ORIENTATION

The program then proceeds to the least-squares block orientation phase for the simultaneous solution of the absolute orientation of all the photographs and the computation of the final ground coordinates for each object. These objects normally consist of the pass-point objects and the weighted control stations. The maximum number of objects whose positions can be finalized in the orientation phase is limited to 1500 in the 200-photo version. The 200 pictures require 1200 relative orientation pass-point objects if none of these objects can be used for relative orientation on more than one photo. Thus, as many as 300 control stations can be introduced into the problem. This number increases if some of the objects service more than one photograph.

It is in the orientation phase that the use of disk memory becomes significant. Inasmuch as computations take place using data stored in core memory, it is necessary to transfer the contents of several large arrays from core to disk storage, thereby leaving the core arrays free to hold new information. As the computations progress, data will move from core storage to disk storage and back again hundreds of times depending on the number of photographs in the block. The transfer of data between disk and core is extremely rapid, but the locating of the reading heads to the desired disk address is relatively slow (disk being a random access device) thereby materially increasing the time of computer operation.

After the resection solution is completed, observation equations for the block are formed for each image and the resolution and collinearity weights applied. These equations contain all of the 9 unknowns shown in equation 2. The contribution of these equations to the system of normal equations is computed and added to the corresponding terms. The diagonal terms associated with control stations are then modified by the assigned position weights.

A typical system of normal equations for block adjustment is shown in Figure 2, where each line contains the coefficients for one normal equation. The solid portions represent the presence of non-zero numbers and the open portions consist entirely of zeros. The 3-by-3 "stairsteps" shown at *Z1* are related to the correction terms to be computed for the provisional ground coordinates of the objects; the 6-by-6 "stairsteps" at *Z3* refer to correction terms which will be added to the initial approximations of the camera parameters determined during the space resection phase; the column shown at *Z2* represents the list of constant terms in the normal equations; and the solid portions at *Z* are intermediate cross-products of the normal equation procedure. Although the submatrix of camera

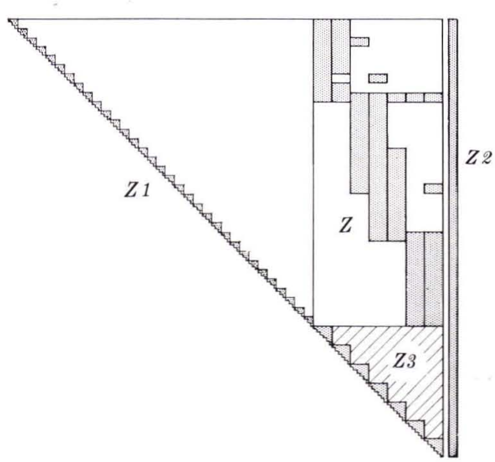

FIG. 2. Typical system of normal equations for block adjustment.

parameters shown at *Z3* contains blocks of zeros in the cross-hatched area during the formation of normal equations, large portions of the cross-hatched area become non-zero during the solution. It is the solution of this submatrix that requires the predominant heavy burden of arithmetic operations.

Storage of the Normal Equations

A 200-photo block containing 1500 objects whose coordinates are to be adjusted in the least squares orientation solution would contain 5700 unknowns. A conventional storage of a 5700-by-5701 normal equation matrix would require over 32 million words of computer memory. However, by taking advantage of the large blocks of zeros whose positions in the matrix are fixed by the nature of the photogrammetric problem, this storage requirement is reduced to less than one million. A solution of this size is feasible on the STRETCH computer because the combined core and disk memory exceeds 1.5 million.

To accomplish the solution, the normal equation matrix is divided into the four irregular submatrices *Z*, *Z1*, *Z2* and *Z3* shown in Figure 1, and stored in the computer. Only the solid sections are stored for *Z*, *Z1*, and *Z2*, thereby eliminating the zero portions of the matrix. However, storage area is provided for both the solid portion and the cross-hatched section of *Z3* because of the non-zero numbers that occur in this section during the forward and back solution. This type of compression technique demands a strict "housekeeping" program procedure to keep track of all the elements comprising the normal-equation matrix.

Solution of the Equations

The normal-equation solution is accomplished by a Gauss-Cholesky elimination method. Mathematically the solution is a rigorous least-squares adjustment involving as many as 5700 unknowns. Due to the elimination of the large zero portions in the matrix, the computation is vastly less arduous and time consuming than a classical solution of this size. Although it is impressive to speak of a solution containing 5700 unknowns, it should be realized that this problem constitutes a special application involving far less computing than a standard solution.

It has been suggested that large systems of equations treated by this method may not result in a satisfactory solution because of the rounding off of numbers arising from repeated multiplications. This difficulty is not significant for several reasons. The solution determines only relatively small corrections to data which are already fairly accurate. Each repetition or iteration is based on a new set of coefficients which, in turn, are based on the corrected values of the parameters. Experience has shown the average size of the corrections on any iteration to be from one-tenth to one-thousandth of the magnitude on the previous iteration. Consequently, such a system will continue to converge rapidly as long as the leading one or two significant digits of the corrections remain undamaged by the encroachment of round-off discrepancies. The floating arithmetic and 14-digit word-length of the STRETCH computer make this possible. Similar solutions in the adjustment of very large networks in classical geodesy have always been satisfactory and, finally, no difficulties have been encountered in block adjustment solutions involving as many as 180 photographs.

The product of the forward and back solution consists of corrections to all the camera parameters and the provisional ground coordinates of the relative orientation pass point objects. These corrections are added to their respective initial values. The dX, dY, and dZ computed for the weighted control stations are not added to the true ground coordinates of these stations as it is necessary to re-use the true positions for each iteration of the solution.

If none of the angular corrections exceeds 10^{-5} radians (about two arc-seconds), the least square orientation computation is considered finished. If any value is larger than this limit, the solution is repeated to determine further corrections. However, the space-resection phase is by-passed on the repeated solution. The new observation equations and their contributions to the normal equations are based only on the corrected data. The number of iterations will be minimum if the preliminary programs have been used to provide clean improved data for the block adjustment.

Output from the Orientation Phase

The output from the orientation phase consists of: (1) the maximum angular correction required by each program pass; (2) the ground coordinates of all the relative orientation pass point objects; (3) the misfit of the solution to the weighted control stations; (4) the residual discrepancies of all images on each photograph and the RMS-value for the entire block; (5) the sines and cosines of the final three orientation angles of each photograph; and (6) the final three coordinates of each camera station.

Object-Intersection Phase

In addition to the pass-point objects and the weighted control stations, a number of other objects usually exist in the block-adjustment area for which accurate ground coordinates are to be computed. As a result of their nature, these objects may occur in clusters rather than being uniformly distributed throughout the block as is the case for the pass-point objects. It is not desirable to include these additional objects in the orientation phase solution as their very presence would weight the solution according to their location. For this reason an object-intersection phase was incorporated into the program utilizing the finalized camera parameters from the orientation phase to compute ground coordinates for objects not appearing in the block orientation solution.

Because the ground coordinates X, Y, Z, of the object are the only unknowns, the collinearity equations may be arranged to provide observation equations as follows:

$$\begin{aligned} v_x &= (p_{15}X + p_{16}Y + p_{17}Z - P_{15}X_0 \\ &\quad - p_{16}Y_0 - p_{17}Z_0)/A_3B \\ v_y &= (p_{25}X + p_{26}Y + p_{27}Z - p_{25}X_0 \\ &\quad - p_{26}Y_0 - p_{27}Z_0)/A_3B \end{aligned} \tag{3}$$

These equations are written for each photograph on which the object appears. Least squares is applied to obtain a solution for X, Y, and Z, minimizing the residual image discrepancies. The computation is iterated until the change in X and Y does not exceed one micron on the smallest-scale photograph in the block. The maximum residual image discrepancy is determined and compared with the maximum acceptable value imposed by the input data. If the residual is excessive, the image is discarded and the object-intersection solution is repeated using the remaining images. The ground coordinates of an unlimited number of objects may be determined in this manner.

Analytic Aerotriangulation Accuracy

As noted earlier, analytic computations may terminate after strip adjustment or continue through block adjustment depending on the desired accuracy. The best root-mean-square accuracy that can be expected after strip adjustment is probably in the neighborhood of 1/10,000 to 1/20,000 of the flight altitude where film is used in the super-wide angle aerial camera. If glass plates are used in a wide-angle camera, and if the results are refined by a subsequent block adjustment technique, present results suggest that 1/40,000 of the altitude can be approached.

Earth Crustal Movement Survey of Anchorage, Alaska

A rectangular area of approximately six square miles within the city of Anchorage, Alaska, has been selected for an intensive analysis of earth crustal movements through the repetitive location of more than 50 points distributed throughout the area. Aerial photography was obtained with a 6-inch focal length Wild RC-8 camera from a flight altitude of 3000 feet. Four strips of photography were taken in which the first strip consisted of nine pictures and the remaining strips each contained ten photographs. The sidelap between strips ranged from 50 to 80 per cent.

Nine stations having known ground X, Y, Z-coordinates served as basic control. In addition, seven other vertical control stations were available for use. These stations together with 35 other recoverable ground objects were premarked in the field prior to photography. The white circular targets were recorded as 50-micron images on the 1:6,000 scale photographs. An additional 18 recoverable ground objects were selected in the office and included in the study. The Wild PUG 2 stereoscopic point transfer device was used first to select and transfer suitable images to adjacent photographs. The coordinates of pertinent images were then measured to micron accuracy on a Wild STK Stereocomparator and the observed data submitted for processing through the complete analytic aerotriangulation system.

The block orientation required the least square solution of some 3,500 observation equations containing nearly 800 unknowns. Analysis of the results from the 39-photograph block adjustment revealed a horizontal coordinate standard error for all nine horizontal control points of 1.1 inches and a standard error of elevation for all 16 vertical control points of also 1.1 inches. Stated in terms of the flying height, the coordinate accuracy for both the horizontal and vertical fit was one part in 33,000.

Kansas Control Test Area

The successful development of a feasible block adjustment technique is considered a major advancement toward the goal of mak-

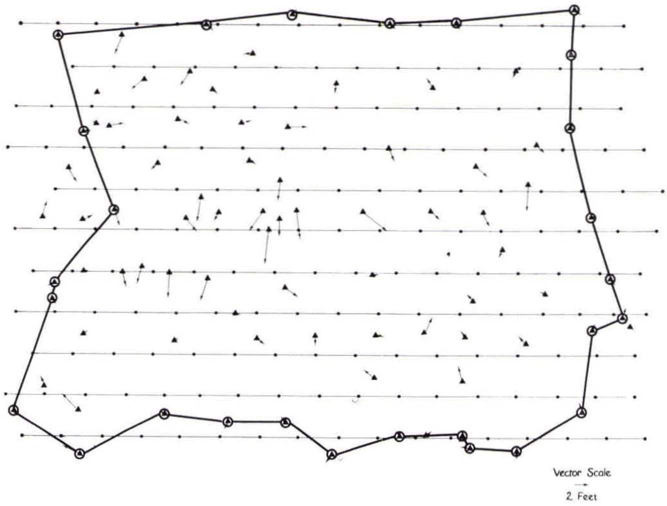

FIG. 3. Differences between photogrammetric and geodetic results obtained on the Parsons, Kansas test area.

ing aerial photogrammetry a tool of geodesy capable of establishing ground control adequate for large-scale mapping. The Coast and Geodetic Survey is now conducting tests for the purpose of evaluating the present block aerotriangulation capability of the Bureau in this endeavor.

The site selected for these tests is an uncontrolled 40 by 45 mile area in the southeast corner of Kansas, bounded on the four sides by arcs of first-order triangulation. In 1964 approximately 50 new geodetic control stations were established throughout the area by triangulation and/or electronic traverse. By using block aerotriangulation methods to determine X, Y, Z-coordinates for these new geodetic points, comparative data was obtained relating to the cost and accuracy of establishing area control separately by triangulation, electronic traverse, and analytic photogrammetry.

All geodetic stations, both existing and new, were premarked with targets prior to photography to assure maximum accuracy in the measurement of their photographic images. The entire test area was then photographed with the super-wide-angle (120 degree) RC-9 camera from a flying height of 20,000 feet above terrain. The 1:70,000 scale photography consisted of 180 pictures arranged in 11 parallel flight lines of approximately 16 photographs each. An overlap of 60 per cent was maintained along and between successive lines. The northwest quadrant of the test area was also photographed with the wide-angle (90 degree) RC-8 film camera and the RC-7 glass plate camera at the altitude of 20,000 feet. This provided approximately 90 photographs at 1:40,000 scale and 1:60,000 scale respectively.

The 1:70,000 scale photography shown in Figure 3 was measured on the Wild STK stereocomparator and the data processed through the analytic aerotriangulation system. The geodetic control for the 180-photograph block consisted of the 27 peripheral geodetic stations circled on the figure and several vertical positions within the area as identified from available quadrangles. A listing of the geodetic positions of the more than 50 interior test points was withheld until after the analytic computations were completed. The block orientation phase required the least square solution of more than 15,000 observation equations containing nearly 4500 unknowns.

Figure 3 shows the vector differences between the positions obtained from the photogrammetric block adjustment and the positions provided by classical triangulation or traverse methods. The RMS difference in position for those stations not used to control the block adjustment was 2.10 feet with a maximum position difference of 4.7 feet occurring near the center of the 1800-square-mile area. Even though this maximum difference is near the center of the area, it should not be inferred that error propagation was the major contributor to this difference because the differences for other points in this vicinity are less than the RMS difference. In addition, it should be noted that the RMS differ-

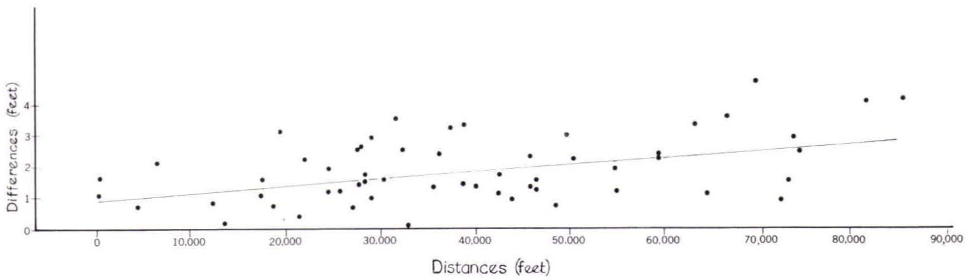

FIG. 4. Differences as a function of distance from control.

ence in position is approximately 1/45,000 of the maximum distance from any interior test point to the peripheral geodetic control (about 18 miles). The fact that 4 microns on 1:70,000 scale photography is nearly equivalent to 1 foot on the ground further testifies to the accuracy of the analytic computations.

Figure 4 is a scatter diagram in which the vector differences are plotted against the distance of the interior test points from the nearest peripheral control station. As a consequence of the linear relationship appearing to exist between the variables, a regression line of vector differences on distance was fitted to the data by least square methods. The slope of the line revealed a difference propagation of one part in 44,000 with distance from control. The computed standard deviation of the differences from the least square line was ± 0.9 feet. However, an actual count showed 60 per cent of the differences to be within ± 0.9 feet of the magnitude predicted by the regression line. The discrepancy that should not be exceeded 68 per cent of the time between any two interior points A and B is given by an expression obtained by coupling the station noise, as provided by the standard deviation, with the slope of the regression line. This expression is:

$$\pm \left(\frac{\overline{AB}}{44{,}000} + 1.3 \right) \text{ feet}$$

Later this year, it is planned to process the 1:40,000 scale RC-8 photography and the 1:60,000 glass plate RC-7 pictures through the analytic solution in order to make comparative studies of the three photographic systems. The study will also evaluate the block adjustment vertical accuracy provided by each system.

COMPUTATION TIME REQUIRED ON THE IBM 7030 (STRETCH) COMPUTER

The time and cost of computer operation for block adjustment have been found to be relatively inexpensive. For example, two passes through the 185-photo version of the program required 4.5 minutes for an 18-photo strip-block and cost \$22.50. Two passes through the same program of a 50-photo block required 20 minutes at a cost of \$100. Because the second pass did not significantly affect the results, the solution could have been terminated after the first pass. This would reduce the time-cost factor to under three minutes (\$12) for the 18-photo strip-block and to about 10 minutes (\$50) for the 50-photo block.

At the other end of the scale, the 180 photo block from the Kansas test requires 2.5 hours for a single pass through the program and costs \$750. These figures, however, include the time and cost of a check-point and restart procedure incorporated into the program for use with very large blocks. This procedure guards against lost time due to machine malfunction during the lengthy computation by periodically dumping all computer data onto tape. In the event of such a malfunction, computations may be restarted at the check point just prior to the malfunction. It is estimated that the check point and restart procedure added 10 minutes (\$50) to the time required to process the 180-photo block.*

CONCLUSION

The block adjustment program was put into operation at the Coast and Geodetric Survey in 1966 and has furnished good results to-date. The program is being constantly modified in our office as new thoughts, new equipment, and other advances in-the-art

* Since the preparation of the paper, a 33-photo block of the same photographs in the northwest corner was adjusted using the U.S. Navy STRETCH computer at Dahlgren, Virginia. The time required on the central processor was 12 minutes, and the total computing charges were \$60. In addition, an assessment of vertical accuracy indicated that it was essentially equal to the horizontal accuracy: ± 2 feet.

make periodic revisions necessary. For instance, a modification in the program is contemplated in the near future that will permit the enforcement of a taped length and/or a known azimuth during the block solution. The availability of still larger and faster computers will allow further program additions to be made so that the analytic method can be used to solve an increasing variety of photogrammetric problems.

The Coast and Geodetric Survey is publishing its analytical aerotriangulation computer programs in the Fortran language for the purpose of expediting the application by the public of analytical methods. Documentation of the block adjustment program is expected in the spring of 1968. The analytic programs, preceding block adjustment, have been documented and are available from the Bureau on request.

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Postscript. In the last few months, the 185-photo version of the block adjustment program has been converted to operate on the ESSA CDC 6600 computer at Suitland, Maryland. During this conversion, the program was modified somewhat to reduce the calls on disk memory by taking advantage of the increased core storage. The handling of card data has been minimized in this program by having the mass of the input data and all of the output data stored on magnetic tapes.

Two passes through the program for an 18-photo strip block required 64 seconds and cost $6.00. A single pass of a 90 photo block used 17 minutes and cost $85.00. However, the same significant savings were not experienced when the 180-photo Kansas block was processed through a single pass of the program; it required slightly over two hours on the CDC 6600 (as compared with 2.5 hours on the IBM (STRETCH) 7030 computer) and cost $615.00.